

Reducing the $N = 1$, E_8 , 10-dim gauge theory over a modified flag manifold

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- ① Short reminder of the Kaluza - Klein programme
- ② Higher-Dimensional Unified Gauge Theories and Coset Space Dimensional Reduction (CSDR)
- ③ The model
- ④ Embedding in the heterotic 10D Superstring

- ① Fuzzy extra dimensions \rightarrow **renormalizable** realistic 4-d GUTs
- ② Reduction of couplings in $\mathcal{N} = 1$ gauge theories \rightarrow **predictive** GUTs, Finite Unified Theories, reduced MSSM
- ③ Noncommutative (fuzzy) Gravity

- Kaluza-Klein observation: **Dimensional Reduction** of a pure gravity theory on $M^4 \times S^1$ leads to a $U(1)$ gauge theory coupled to gravity in four dimensions. The **extra dimensional** gravity provided a **geometrical unified picture** of gravitation and electromagnetism.
- Generalization to $M^D = M^4 \times B$, with B a compact Riemannian space with a non-abelian isometry group S leads after dim. reduction to gravity coupled to Y-M in 4 dims.

Kerner '68

Cho - Freund '75

Problems

- No classical ground state corresponding to the assumed M^D .
- Adding fermions in the original action, it is impossible to obtain chiral fermions in four dims.

Witten '85

- However by adding suitable matter fields in the original action, in particular Y-M one can have a classical stable ground state of the required form and massless chiral fermions in four dims.

Horvath - Palla - Cremmer - Scherk '77

Original motivation

Use higher dimensions

- to unify the gauge and Higgs sectors
- to unify the fermion interactions with gauge and Higgs fields
- ★ Supersymmetry provides further unification (fermions in adj. reps)

Forgacs - Manton '79, Manton '81, Chapline - Slansky '82

Kubyshev - Mourao - Rudolph - Volobuev '89

Kapetanakis - Z '92, Manousselis - Z '01 – '08

Further successes

- (a) chiral fermions in 4 dims from vector-like reps in the higher dim theory
- (b) the metric can be deformed (in certain non-symmetric coset spaces) and more than one scales can be introduced
- (c) Wilson flux breaking can be used
- (d) Softly broken susy chiral theories in 4 dims can result from a higher dimensional susy theory

Theory in D dims \rightarrow Theory in 4 dims

1. Compactification

$$\begin{array}{ccc} M^D & \rightarrow & M^4 \times B \\ | & & | \quad | \\ x^M & & x^\mu \quad y^a \end{array}$$

B - a compact space

$$\dim B = D - 4 = d$$

2. Dimensional Reduction

Demand that \mathcal{L} is independent of the extra y^a coordinates

- One way: Discard the field dependence on y^a coordinates
- An elegant way: Allow field dependence on y^a and employ a symmetry of the Lagrangian to compensate

Obvious choice: Gauge Symmetry

Allow a non-trivial dependence on y^a , but **impose** the condition that a symmetry transformation by an element of the isometry group S of B is compensated by a gauge transformation.

$\Rightarrow \mathcal{L}$ independent of y^a just because is gauge invariant.

Integrate out extra coordinates

CSDR: $B = S/R$

$$S: \quad \mathcal{Q}_A = \{\mathcal{Q}_i, \mathcal{Q}_a\}$$

$$\begin{array}{cc} | & | \\ R & S/R \end{array}$$

$$[\mathcal{Q}_i, \mathcal{Q}_j] = f_{ij}^k \mathcal{Q}_k, \quad [\mathcal{Q}_i, \mathcal{Q}_a] = f_{ia}^b \mathcal{Q}_b,$$

$$[\mathcal{Q}_a, \mathcal{Q}_b] = f_{ab}^i \mathcal{Q}_i + f_{ab}^c \mathcal{Q}_c,$$

where f_{ab}^c vanishes in symmetric S/R

Consider a Yang-Mills-Dirac theory in D dims based on group G defined on $M^D \rightarrow M^4 \times S/R$, $D = 4 + d$

$$g^{MN} = \begin{pmatrix} \eta^{\mu\nu} & 0 \\ 0 & -g^{ab} \end{pmatrix} \quad \eta^{\mu\nu} = \text{diag}(1, -1, -1, -1)$$

$$d = \dim S - \dim R \quad g^{ab} - \text{coset space metric}$$

$$A = \int d^4x d^d y \sqrt{-g} \left[-\frac{1}{4} \text{Tr}(F_{MN} F_{K\Lambda}) g^{MK} g^{N\Lambda} + \frac{i}{2} \bar{\psi} \Gamma^M D_M \psi \right]$$

$$D_M = \partial_M - \theta_M - A_M \quad , \quad \theta_M = \frac{1}{2} \theta_{MN\Lambda} \Sigma^{N\Lambda}$$

where θ_M is the spin connection of M^D and ψ is in rep F of G

We require that any transformation by an element of S acting on S/R is compensated by gauge transformations.

$$\begin{aligned}
A_\mu(x, y) &= g(\mathbf{s}) A_\mu(x, \mathbf{s}^{-1} y) g^{-1}(\mathbf{s}) \\
A_a(x, y) &= g(\mathbf{s}) J_a^b A_b(x, \mathbf{s}^{-1} y) g^{-1}(\mathbf{s}) \\
&\quad + g(\mathbf{s}) \partial_a g^{-1}(\mathbf{s}) \\
\psi(x, y) &= f(\mathbf{s}) \Omega \psi(x, \mathbf{s}^{-1} y) f^{-1}(\mathbf{s})
\end{aligned}$$

g, f - gauge transformations in the adj, F of G corresponding to the \mathbf{s} transformation of S acting on S/R

J_a^b - Jacobian for \mathbf{s}

Ω - Jacobian + local Lorentz rotation in tangent space

Above conditions imply **constraints** that D -dims fields should obey.

Solution of constraints:

- 4-dim fields
- Potential
- Remaining gauge invariance

Taking into account all the constraints and integrating out the extra coordinates, we obtain in 4 dims:

$$A = C \int d^4x \left(-\frac{1}{4} \text{Tr} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \sum_a \text{Tr} (D_\mu \phi_a D^\mu \phi^a) \right. \\ \left. + V(\phi) + \frac{i}{2} \bar{\psi} \Gamma^\mu D_\mu \psi - \frac{i}{2} \bar{\psi} \Gamma^a D_a \psi \right)$$

\downarrow
 kinetic terms

\downarrow
 mass terms

$$D_\mu = \partial_\mu - A_\mu, \quad D_a = \partial_a - \theta_a - \phi_a, \quad \theta_a = \frac{1}{2} \theta_{abc} \Sigma^{bc}$$

C— volume of cs, θ_a — spin connection of cs

$$V(\phi) = -\frac{1}{4} g^{ac} g^{bd} \text{Tr} \{ (f_{ab}^C \phi_C - [\phi_a, \phi_b]) (f_{cd}^D \phi_D - [\phi_c, \phi_d]) \}$$

$A = 1, \dots, \dim S$, f — structure constants of S .

Still $V(\phi)$ only formal since ϕ_a must satisfy $f_{ai}^D \phi_D - [\phi_a, \phi_i] = 0$.

1) The 4-dim gauge group

$$H = C_G(R_G)$$

$$\text{i.e. } G \supset R_G \times H$$

where G is the higher-dim group and H is the 4 dim group.

2) Scalar fields

$$S \supset R$$

$$\text{adj}S = \text{adj}R + v$$

$$G \supset R_G \times H$$

$$\text{adj}G \supset (\text{adj}R, 1) + (1, \text{adj}H) + \Sigma(r_i, h_i)$$

If $v = \Sigma s_i$

when $s_i = r_i \Rightarrow h_i$ survives in 4 dims.

3) Fermions

$$G \supset R_G \times H$$

$$F = \sum (t_i, h_i)$$

spinor of $SO(d)$ under R

$$\sigma_d = \sum \sigma_j$$

for every $t_i = \sigma_i \Rightarrow h_i$ survives in 4 dims.

Possible to obtain a chiral theory in 4 dims starting from Weyl fermions in a complex rep.

However, even starting with Weyl (+ Majorana) fermions in vector-like reps of G in $D = 4n + 2$ dims we are also led to a chiral theory.

If D is even:

$$\Gamma^{D+1}\Psi_{\pm} = \pm\Psi_{\pm} \quad \text{Weyl condition}$$

$$\Psi = \Psi_+ \oplus \Psi_- = \sigma_D + \sigma'_D,$$

where σ_D, σ'_D are non-self conjugate spinors of $SO(1, D-1)$.

The $(SU(2) \times SU(2)) \times SO(d)$ branching rule is:

$$\sigma_D = (2, 1; \sigma_d) + (1, 2; \sigma'_d)$$

$$\sigma'_D = (2, 1; \sigma'_d) + (1, 2; \sigma_d)$$

Starting with Dirac fermions

equal number of left and right-handed



reps of the 4-dim group H

Weyl condition selects either σ_D or σ'_D

Weyl condition cannot be applied in odd dims. In that case:

$$\sigma_D = (2, 1; \sigma_d) + (1, 2; \sigma_d),$$

where σ_d is the unique spinor of $SO(d)$

equal number of left and right-handed

↪

reps in 4 dims

Most interesting case is when $D = 4n + 2$ and we start with a **vectorlike** rep. In that case σ_d is non-self-conjugate and $\sigma'_d = \bar{\sigma}_d$.

Then the decomposition of $\sigma_d, \bar{\sigma}_d$ of $SO(d)$ under R is:

$$\sigma_d = \sum \sigma_k, \quad \bar{\sigma}_d = \sum \bar{\sigma}_k.$$

Then:

$$G \supset R_G \times H$$

$$\text{vectorlike} \leftarrow F = \sum_i (r_i, h_i) \rightarrow \text{either self-conjugate or}$$

have a partner (\bar{r}_i, \bar{h}_i) .

Then according to the rule from σ_d we will obtain in 4 dims left-handed fermions $f_L = \sum h_k^L$.

Since σ_d is non-self-conjugate, f_L is non-self-conjugate.

Similarly, from $\bar{\sigma}_d$, we obtain the right-handed rep $\sum \bar{h}_k^R = \sum h_k^L$.

Moreover since F **vectorlike**, $\bar{h}_k^R \sim h_k^L$, i.e. H is **chiral** theory with **double** spectrum.

We can still impose **Majorana** condition (Weyl and Majorana are compatible in $4n + 2$ dims) to eliminate the doubling of the fermion spectrum.

Majorana condition (reverses the sign of all int. qu. nos) forces f_R to be the charge conjugate of f_L .

If F **complex** \rightarrow **chiral theory** just \bar{h}_k^R is different from h_k^L .

An easy case in calculating the potential, its minimization and SSB:

If $G \supset S \Rightarrow H$ breaks to $K = C_G(S)$:

$G \supset S \times K \leftarrow$ gauge group after SSB

$\cup \quad \cap$

$G \supset R \times H \leftarrow$ gauge group in 4 dims

But

fermion masses

$$M^2 \Psi = D_a D^a \Psi - \frac{1}{4} R \Psi - \frac{1}{2} \underbrace{\Sigma^{ab} F_{ab}}_{=0} \Psi > 0$$

if $S \subset G$

comparable to the compactification scale.

Supersymmetry breaking by dim reduction over symmetric CS (e.g $SO(7)/SO(6)$)

Consider $G = E_8$ in 10 dims with Weyl-Majorana fermions in the adjoint rep of E_8 , i.e. a susy E_8 .

Embedding of $R = SO(6)$ in E_8 is suggested by the decomposition:

$$E_8 \supset SO(6) \times SO(10)$$
$$248 = (15, 1) + (1, 45) + (6, 10) + (4, 16) + (\bar{4}, \bar{16})$$

$$\text{adj}S = \text{adj}R + v$$

$$21 = 15 + 6 \leftarrow \text{vector}$$

Spinor of $SO(6)$: 4

In 4 dims we obtain a gauge theory based on:

$$H = C_{E_8}(SO(6)) = SO(10),$$

with scalars in 10 and fermions in 16.

- *Theorem:* When S/R symmetric, the potential necessarily leads to spontaneous breakdown of H .
- Moreover in this case we have:

$$\begin{array}{ccc}
 E_8 & \supset & SO(7) \times SO(9) \\
 & \cup & \cap \\
 E_8 & \supset & SO(6) \times SO(10)
 \end{array}$$

⇒ Final gauge group after breaking:

$$K = C_{E_8}(SO(7)) = SO(9)$$

CSDR over symmetric coset spaces breaks completely original supersymmetry.

Soft Supersymmetry Breaking by CSDR over non-symmetric CS.

We have examined the dim reduction of a supersymmetric E_8 over the 3 existing 6-dim CS:

$$G_2/SU(3), \quad Sp(4)/(SU(2) \times U(1))_{\text{non-max}}, \quad SU(3)/U(1) \times U(1)$$

\Rightarrow Softly Broken Supersymmetric
Theories in 4 dims without any
further assumption

Non-symmetric CS admit torsion and the two latter more than one radii.

Consider supersymmetric E_8 in 10 dims and $S/R = G_2/SU(3)$.

We use the decomposition:

$$E_8 \supset SU(3) \times E_6$$
$$248 = (8, 1) + (1, 78) + (3, 27) + (\bar{3}, \bar{27})$$

and choose $R = SU(3)$

$$\text{adj}S = \text{adj}R + v$$
$$14 = 8 + \underbrace{3 + \bar{3}}_{\text{vector}}$$

Spinor: $1 + 3$ under $R = SU(3)$

\Rightarrow In 4 dim theory: $H = C_{E_8}(SU(3)) = E_6$ with:
scalars in $27 = \beta$ and fermions in $27, 78$

i.e.: spectrum of a supersymmetric E_6 theory in 4 dims.

The Higgs potential of the genuine Higgs β :

$$\begin{aligned}
 V(\beta) = & 8 - \frac{40}{3}\beta^2 - [4d_{ijk}\beta^i\beta^j\beta^k + h.c.] \\
 & + \beta^i\beta^jd_{ijk}d^{klm}\beta_\ell\beta_m \\
 & + \frac{11}{4}\sum_\alpha\beta^i(G^\alpha)_i^j\beta_j\beta^k(G^\alpha)_k^\ell\beta_\ell
 \end{aligned}$$

which obtains F-terms contributions from the superpotential:

$$W(B) = \frac{1}{3}d_{ijk}B^iB^jB^k$$

D-term contributions:

$$\frac{1}{2}D^\alpha D^\alpha, \quad D^\alpha = \sqrt{\frac{11}{2}}\beta^i(G^\alpha)_i^j\beta_j$$

The rest terms belong to the SSB part of the Lagrangian:

$$\begin{aligned}
 \mathcal{L}_{scalar}^{SSB} = & -\frac{1}{R^2}\frac{40}{3}\beta^2 - [4d_{ijk}\beta^i\beta^j\beta^k + h.c.]\frac{g}{R} \\
 M_{gaugino} = & (1 + 3\tau)\frac{6}{\sqrt{3}}\frac{1}{R}
 \end{aligned}$$

Reduction of 10-dim, $\mathcal{N} = 1$, E_8 over $S/R = SU(3)/U(1) \times U(1) \times Z_3$

Irges - Z '11

We use the decomposition:

$$E_8 \supset E_6 \times SU(3) \supset E_6 \times U(1)_A \times U(1)_B$$

and choose $R = U(1)_A \times U(1)_B$,

$$\rightsquigarrow H = C_{E_8}(U(1)_A \times U(1)_B) = E_6 \times U(1)_A \times U(1)_B$$

$$E_8 \supset E_6 \times U(1)_A \times U(1)_B$$

$$248 = 1_{(0,0)} + 1_{(0,0)} + 1_{(3,1/2)} + 1_{(-3,1/2)}$$

$$1_{(0,-1)} + 1_{(0,1)} + 1_{(-3,-1/2)} + 1_{(3,-1/2)}$$

$$78_{(0,0)} + 27_{(3,1/2)} + 27_{(-3,1/2)} + 27_{(0,-1)}$$

$$\overline{27}_{(-3,-1/2)} + \overline{27}_{(3,-1/2)} + \overline{27}_{(0,1)}$$

$$\text{adj}S = \text{adj}R + v \quad \leftarrow \text{vector}$$



$$8 = (0, 0) + (0, 0) + (3, 1/2) + (-3, 1/2) \\ + (0, -1) + (0, 1) + (-3, -1/2) + (3, -1/2)$$

$$SO(6) \supset SU(3) \supset U(1)_A \times U(1)_B$$

$$4 = 1 + 3 = (0, 0) + (3, 1/2) + (-3, 1/2) + (0, -1)$$



spinor

4-dim theory

$$\mathcal{N} = 1, E_6 \times U(1)_A \times U(1)_B$$

with chiral supermultiplets:

$$A^i : 27_{(3,1/2)}, B^i : 27_{(-3,1/2)}, C^i : 27_{(0,-1)}, A : 1_{(3,1/2)}, B : 1_{(-3,1/2)}, C : 1_{(0,-1)}$$

Scalar potential:

$$\begin{aligned} \frac{2}{g^2} V = & \frac{2}{5} \left(\frac{1}{R_1^4} + \frac{1}{R_2^4} + \frac{1}{R_3^4} \right) + \left(\frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2} \right) \alpha^i \alpha_i + \left(\frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2} \right) \bar{\alpha} \alpha \\ & + \left(\frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2} \right) \beta^i \beta_i + \left(\frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2} \right) \bar{\beta} \beta + \left(\frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2} \right) \gamma^i \gamma_i + \left(\frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2} \right) \bar{\gamma} \gamma \\ & + \sqrt{280} \left[\left(\frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_2 R_1} \right) d_{ijk} \alpha^i \beta^j \gamma^k + \left(\frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_2 R_1} \right) \alpha \beta \gamma + h.c \right] \\ & + \frac{1}{6} \left(\alpha^i (G^\alpha)_i^j \alpha_j + \beta^i (G^\alpha)_i^j \beta_j + \gamma^i (G^\alpha)_i^j \gamma_j \right)^2 \\ & + \frac{10}{6} \left(\alpha^i (3\delta_i^j) \alpha_j + \bar{\alpha} (3) \alpha + \beta^i (-3\delta_i^j) \beta_j + \bar{\beta} (-3) \beta \right)^2 \\ & + \frac{40}{6} \left(\alpha^i (\tfrac{1}{2} \delta_i^j) \alpha_j + \bar{\alpha} (\tfrac{1}{2}) \alpha + \beta^i (\tfrac{1}{2} \delta_i^j) \beta_j + \bar{\beta} (\tfrac{1}{2}) \beta + \gamma^i (-1\delta_i^j) \gamma_j + \bar{\gamma} (-1) \gamma \right)^2 \\ & + 40 \alpha^i \beta^j d_{ijk} d^{klm} \alpha_l \beta_m + 40 \beta^i \gamma^j d_{ijk} d^{klm} \beta_l \gamma_m + 40 \alpha^i \gamma^j d_{ijk} d^{klm} \alpha_l \gamma_m \\ & + 40 (\bar{\alpha} \bar{\beta}) (\alpha \beta) + 40 (\bar{\beta} \bar{\gamma}) (\beta \gamma) + 40 (\bar{\gamma} \bar{\alpha}) (\gamma \alpha) \end{aligned}$$

where $\alpha^i, \beta^i, \gamma^i, \alpha, \beta, \gamma$ are the scalar components of A^i, B^i, C^i, A, B, C .

Superpotential: $W(A^i, B^j, C^k, A, B, C) = \sqrt{40}d_{ijk}A^iB^jC^k + \sqrt{40}ABC$

D-terms: $\frac{1}{2}D^\alpha D^\alpha + \frac{1}{2}D_1 D_1 + \frac{1}{2}D_2 D_2$ **where:**

$$D^\alpha = \frac{1}{\sqrt{3}} (\alpha^i (G^\alpha)_i^j \alpha_j + \beta^i (G^\alpha)_i^j \beta_j + \gamma^i (G^\alpha)_i^j \gamma_j)$$

$$D_1 = \frac{\sqrt{10}}{3} (\alpha^i (3\delta_i^j) \alpha_j + \bar{\alpha}(3) \alpha + \beta^i (-3\delta_i^j) \beta_j + \bar{\beta}(-3) \beta)$$

$$D_2 = \frac{\sqrt{40}}{3} \left(\alpha^i \left(\frac{1}{2} \delta_i^j \right) \alpha_j + \bar{\alpha} \left(\frac{1}{2} \right) \alpha + \beta^i \left(\frac{1}{2} \delta_i^j \right) \beta_j + \bar{\beta} \left(\frac{1}{2} \right) \beta + \gamma^i (-1\delta_i^j) \gamma_j + \bar{\gamma}(-1) \gamma \right)$$

Soft scalar supersymmetry breaking terms, $\mathcal{L}_{scalar}^{SSB}$:

$$\begin{aligned} & \left(\frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2} \right) \alpha^i \alpha_i + \left(\frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2} \right) \bar{\alpha} \alpha + \left(\frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2} \right) \beta^i \beta_i + \\ & \left(\frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2} \right) \bar{\beta} \beta + \left(\frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2} \right) \gamma^i \gamma_i + \left(\frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2} \right) \bar{\gamma} \gamma + \\ & \sqrt{280} \left[\left(\frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_2 R_1} \right) d_{ijk} \alpha^i \beta^j \gamma^k + \left(\frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_2 R_1} \right) \alpha \beta \gamma + h.c. \right], \end{aligned}$$

Gaugino mass, $M = (1 + 3\tau) \frac{R_1^2 + R_2^2 + R_3^2}{8\sqrt{R_1^2 R_2^2 R_3^2}}$, τ torsion coeff.

Potential, $V = V_F + V_D + V_{soft}$

The Wilson flux breaking

$$M^4 \times B_0 \rightarrow M^4 \times B, B = B_0/F^{S/R}$$

$F^{S/R}$ - a freely acting discrete symmetry of B_0 .

1. B becomes multiply connected
2. For every element $g \in F^{S/R}$,

$$\rightsquigarrow \mathcal{V}_g = P \exp \left(-i \int_{\gamma_g} T^a A_M^a(x) dx^M \right) \in H$$

3. If the contour is non-contractible $\rightsquigarrow \mathcal{V}_g \neq 1$ and then $f(g(x)) = \mathcal{V}_g f(x)$, which leads to a breaking of H to $K' = C_H(T^H)$, where T^H is the image of the homomorphism of $F^{S/R}$ into H .
4. Matter fields invariant under $F^{S/R} \oplus T^H$.

In the case of $SU(3)/U(1) \times U(1)$ a freely acting discrete group is:

$$F^{S/R} = \mathbb{Z}_3 \subset W, W = \frac{W_S}{W_R},$$

$W_{S,R}$: Weyl group of S, R .

$$\rightsquigarrow \gamma_3 = \text{diag}(\mathbb{1}, \omega \mathbb{1}, \omega^2 \mathbb{1}), \quad \omega = e^{2i\pi/3} \in \mathbb{Z}_3$$

The fields that are invariant under $F^{S/R} \oplus T^H$ survive, i.e.:

$$\begin{aligned} A_\mu &= \gamma_3 A_\mu \gamma_3^{-1} \\ A^i &= \gamma_3 A^i, \quad B^i = \omega \gamma_3 B^i, \quad C^i = \omega^2 \gamma_3 C^i \\ A &= A, \quad B = \omega B, \quad C = \omega^2 C \end{aligned}$$

$$\rightsquigarrow \mathcal{N} = 1, \quad SU(3)_c \times SU(3)_L \times SU(3)_R,$$

Recall that $27 = (1, 3, \bar{3}) + (3, \bar{3}, 1) + (\bar{3}, 1, 3)$

with matter superfields in:

$$\begin{array}{ccc}
 (1, 3, \bar{3})_{(3, 1/2)}, & (3, \bar{3}, 1)_{(0, -1)}, & (\bar{3}, 1, 3)_{(-3, 1/2)} \\
 \updownarrow & \updownarrow & \updownarrow \\
 L = \begin{pmatrix} H_d^0 & H_u^+ & \nu_L \\ H_d^- & H_u^0 & e_L \\ \nu_R^c & e_R^c & S \end{pmatrix}, & q^c = \begin{pmatrix} d_R^{c1} & u_R^{c1} & D_R^{c1} \\ d_R^{c2} & u_R^{c2} & D_R^{c2} \\ d_R^{c3} & u_R^{c3} & D_R^{c3} \end{pmatrix}, & Q = \begin{pmatrix} -d_L^1 & -d_L^2 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 \\ D_L^1 & D_L^2 & D_L^3 \end{pmatrix}
 \end{array}$$

and the surviving singlet

$$\theta \rightarrow (1, 1, 1)_{(3, 1/2)} .$$

Introducing non-trivial windings in R can appear 3 identical flavours in each of the bifundamental matter superfields and singlet superfield.

Further Gauge Breaking of $SU(3)^3$

*Babu - He - Pakvasa '86; Ma - Mondragon - Z '04;
Leontaris - Rizos '06; Sayre - Wiesenfeldt - Willenbrock '06*

Two generations of L acquire vevs that **break the GUT**:

$$\langle L_s^{(3)} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & V \end{pmatrix}, \quad \langle L_s^{(2)} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V & 0 & 0 \end{pmatrix}$$

each one alone is not enough to produce the (MS)SM gauge group:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)$$

$$SU(3)_c \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_c \times SU(2)_L \times SU(2)'_R \times U(1)'$$

Their **combination** gives the desired breaking:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$$

Electroweak breaking then proceeds by:

$$\langle L_s^{(3)} \rangle = \begin{pmatrix} v_d & 0 & 0 \\ 0 & v_u & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Choice of Radii

- Soft **trilinear** terms $\sim \frac{1}{R_i}$
- Soft **scalar masses** $\sim \frac{1}{R_i^2}$

Manolakos - Patellis - Z '20

Two main possible directions:

- **Large** $R_i \rightarrow$ calculation of the Kaluza-Klein contributions of the 4D theory
 × Eigenvalues of the **Dirac** and **Laplace** operators unknown.
- **Small** $R_i \rightarrow$ **high scale** SUSY breaking
- **Small** $R_i \sim \frac{1}{M_{GUT}}$ with R_3 **slightly different** such that

$$m_3^2 \sim -\mathcal{O}(\text{TeV}^2), \quad m_{1,2}^2 \sim -\mathcal{O}(M_{GUT}^2), \quad a_{abc} \sim M_{GUT}$$

where $m_{1,2,3}^2$ are the squared soft scalar masses and a_{abc} are the soft trilinear couplings.

- **supermassive** squarks
- **TeV-scale** sleptons
- **TeV-scale** soft Higgs squared masses

Reminder: in this scenario $M_{Comp} = M_{GUT}$

Lepton Yukawas and μ terms

At the GUT scale

$$\begin{aligned} SU(3)^3 &\xrightarrow{V} SU(3)_c \times SU(2)_L \times U(1)_Y \\ &\xrightarrow{v_{u,d}} SU(3)_c \times U(1)_{em} \end{aligned}$$

$$\rightarrow \langle \theta^{(3)} \rangle \sim \mathcal{O}(\text{TeV}), \quad \langle \theta^{(1,2)} \rangle \sim \mathcal{O}(M_{GUT})$$

The GUT breaking vevs and the $\langle \theta^{(1,2)} \rangle$ vevs **break** the two $U(1)$ s.

- The two global $U(1)$ s **forbid** Yukawa terms for **leptons**

→ introduce **higher-dimensional** operators: $L \bar{e} H_d \left(\frac{\bar{K}}{M} \right)^3$

- μ **terms** for each generation of Higgs doublets are **absent**

→ solution through **higher-dim** operators: $H_u^{(3)} H_d^{(3)} \bar{\theta}^{(3)} \frac{\bar{K}}{M}$

- \bar{K} is the **vev** of the conjugate scalar component of either S , ν_R or θ , or any combination of them

Approximate Scale of Parameters

Parameter	Scale
soft trilinear couplings	$\mathcal{O}(GUT)$
squark masses	$\mathcal{O}(GUT)$
slepton masses	$\mathcal{O}(TeV)$
$\mu^{(3)}$	$\mathcal{O}(TeV)$
unified gaugino mass M_U	$\mathcal{O}(TeV)$

Gauge Unification

There exist three basic scales: M_{GUT} , M_{int} and M_{TeV} .

- Squarks, Higgsinos of the two first families and the new exotic quarks decouple at an intermediate scale M_{int}
- Every other high-scale parameter decouples at M_{GUT}

Concerning the 1-loop gauge couplings:

- $\alpha_{1,2}$ are used as input to determine M_{GUT}
- α_3 is found within 2σ of the experimental value

$$\alpha_s(M_Z) = 0.1218$$

$$\alpha_s^{EXP}(M_Z) = 0.1187 \pm 0.0016$$

Scale	GeV
M_{GUT}	$\sim 10^{15}$
M_{int}	$\sim 10^{14}$
M_{TeV}	~ 1500

✓ No proton decay problem due to the global symmetries.

1-loop Results

1-loop β -functions used throughout the analysis that change between the three scales M_{GUT} , M_{int} and M_{TeV} .

→ $m_b(M_Z)$ and \hat{m}_t are found within 2σ of the experimental values

$$\bullet m_b(M_Z) = 3.00 \text{ GeV} \quad m_b^{EXP}(M_Z) = 2.83 \pm 0.10 \text{ GeV}$$

$$\bullet \hat{m}_t = 171.6 \text{ GeV} \quad \hat{m}_t^{EXP} = 172.4 \pm 0.7 \text{ GeV}$$

→ m_h is found within 1σ of the experimental value

$$\bullet m_h = 125.18 \text{ GeV} \quad m_h^{EXP} = 125.10 \pm 0.14 \text{ GeV}$$

$$- \text{Large } \tan \beta \sim 48$$

$$- M_A > 2000 \text{ GeV} \checkmark$$

CSDR and the Einstein-Yang-Mills system

EYM theory with cosmological constant in $4 + d$ dimensions:

$$L = -\frac{1}{16\pi G}\sqrt{-g}R^{(D)} - \frac{1}{4g^2}\sqrt{-g}F_{MN}^a F^{aMN} - \sqrt{-g}\Lambda$$

The corresponding equations of motion are:

$$D_M F^{MN} = 0, \quad R_{MN} - \frac{1}{2}Rg_{MN} = -8\pi GT_{MN}$$

Spontaneous compactification: Solutions of the coupled EYM system corresponding to $M^4 \times B$ - B a coset space and α, β coset indices + demanding M^4 to be **flat Minkowski**:

$$\Lambda = \frac{1}{4}\text{Tr}(F_{\alpha\beta}F^{\alpha\beta})$$

Λ is absent in 4 dims: eliminates the vacuum energy of the gauge fields

Λ equal to the **minimum of the potential** of the theory

The potential of the reduced low-energy limit of 10-d heterotic string over $SU(3)/U(1) \times U(1)$

Low-energy effective action of $E_8 \times E_8$ heterotic string (bos part):

$$\mathcal{S}_{het} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-|g|} \left(R - \frac{1}{2} \partial_M \tilde{\Phi} \partial^M \tilde{\Phi} - \frac{e^{-\tilde{\Phi}}}{12} \tilde{H}_{MN\Lambda} \tilde{H}^{MN\Lambda} + \frac{\alpha' e^{-\frac{1}{2}\tilde{\Phi}}}{4} \text{Tr} F_{MN} F^{MN} \right)$$

- $\kappa^2 = 8\pi G^{(10)}$ the 10-d gravitational constant
- α' the Regge slope parameter
- R the Ricci scalar of the 10-d (target) space
- $\tilde{\Phi}$ the dilaton scalar field
- \tilde{H} the field strength tensor of the 2-form B_{MN} field
- F the field strength tensor of the $E_8 \times E_8$ gauge field

Also, $g_s^2 = e^{2\tilde{\Phi}_0}$ is the string coupling constant ($\tilde{\Phi}_0$ is the constant mode of the dilaton)

Application of the CSDR over $SU(3)/U(1) \times U(1)$ leads to a
 $4 - d$ scalar potential Chatzistavrakidis - Z '09

The contributions of the three sectors after the CSDR:

$$\begin{aligned}
 V_{gr} &= -\frac{1}{4\kappa^2} e^{-\tilde{\phi}} \left(\frac{6}{R_1^2} + \frac{6}{R_2^2} + \frac{6}{R_3^2} - \frac{R_1^2}{R_2^2 R_3^2} - \frac{R_2^2}{R_1^2 R_3^2} - \frac{R_3^2}{R_1^2 R_2^2} \right) \\
 V_H &= \frac{1}{2\kappa^2} e^{-\tilde{\phi}} \left[\frac{(b_1^2 + b_2^2 + b_3^2)^2}{(R_1 R_2 R_3)^2} + \sqrt{2} i \alpha' \frac{1}{R_1 R_2 R_3} (b_1^2 + b_2^2 + b_3^2) (d_{ijk} \alpha^i \beta^j \gamma^k - h.c.) \right] \\
 V_F &= \frac{\alpha'}{8\kappa^2} e^{-\frac{\tilde{\phi}}{2}} \left[c + \left(\frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2} \right) \alpha^i \alpha_i + \left(\frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2} \right) \beta^i \beta_i + \left(\frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2} \right) \gamma^i \gamma_i \right. \\
 &\quad + \sqrt{2} 80 \frac{R_1^2 + R_2^2 + R_3^2}{R_1 R_2 R_3} (d_{ijk} \alpha^i \beta^j \gamma^k + h.c.) + \frac{1}{6} (\alpha^i (G^\alpha)_i^j \alpha_j + \beta^i (G^\alpha)_i^j \beta_j + \gamma^i (G^\alpha)_i^j \gamma_j)^2 \\
 &\quad + 5 (\alpha^i \alpha_i - \beta^i \beta_i)^2 + \frac{10}{3} (\alpha^i \alpha_i + \beta^i \beta_i - 2 \gamma^i \gamma_i)^2 \\
 &\quad \left. + 40 \alpha^i \beta^j d_{ijk} d^{klm} \alpha_l \beta_m + 40 \beta^i \gamma^j d_{ijk} d^{klm} \beta_l \gamma_m + 40 \alpha^i \gamma^j d_{ijk} d^{klm} \alpha_l \gamma_m \right]
 \end{aligned}$$

Possible *compensation* to the *negative* gravity contribution by the
 presence of *gauge* and *3-form* sectors.

Gibbons '84; De Wit - Smit - Dass '87;
 Maldacena - Nuñez '01, Manousselis - Prezas - Z '06

THANK YOU!

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