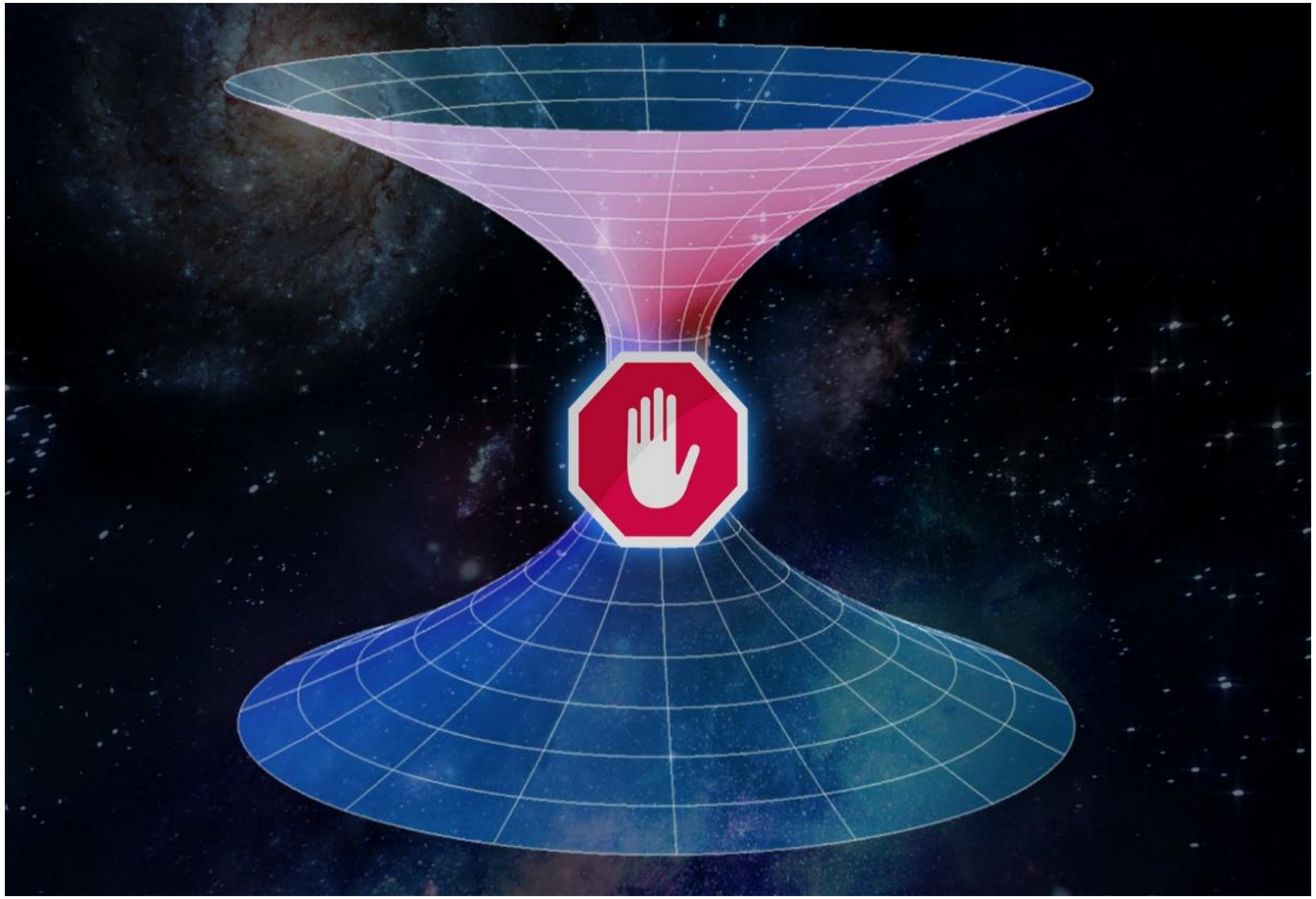
Euclidean wormholes and the Swampland



Thomas Van Riet – Leuven U.

Ringberg
2021

Based on

- VR, arXiv:2004.08956
- Katmadas, Trigiante, Ruggeri, VR: 1812.05986 & Astesiano, Trigiante, Ruggeri, VR, arXiv: 2112.XXXX.
- Hertog, Truijen, VR, arXiv: 1811.12690 & Hertog, Maenaut, Tielemans, VR, 2112.XXXX.

Euclidean Wormholes & Axions

Action: $$S = -\frac{1}{2\kappa^2}\int\sqrt{|g|}\left(\mathcal{R} - \frac{1}{2}\frac{1}{(D-1)!}F_{\mu_1\ldots\mu_{D-1}}F^{\mu_1\ldots\mu_{D-1}}\right)$$

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Ansatz $$ds^2 = f(\tau)^2 d\tau^2 + a^2(\tau)^2 d\Omega_3^2$$
$$F_3 = Q\epsilon_3$$

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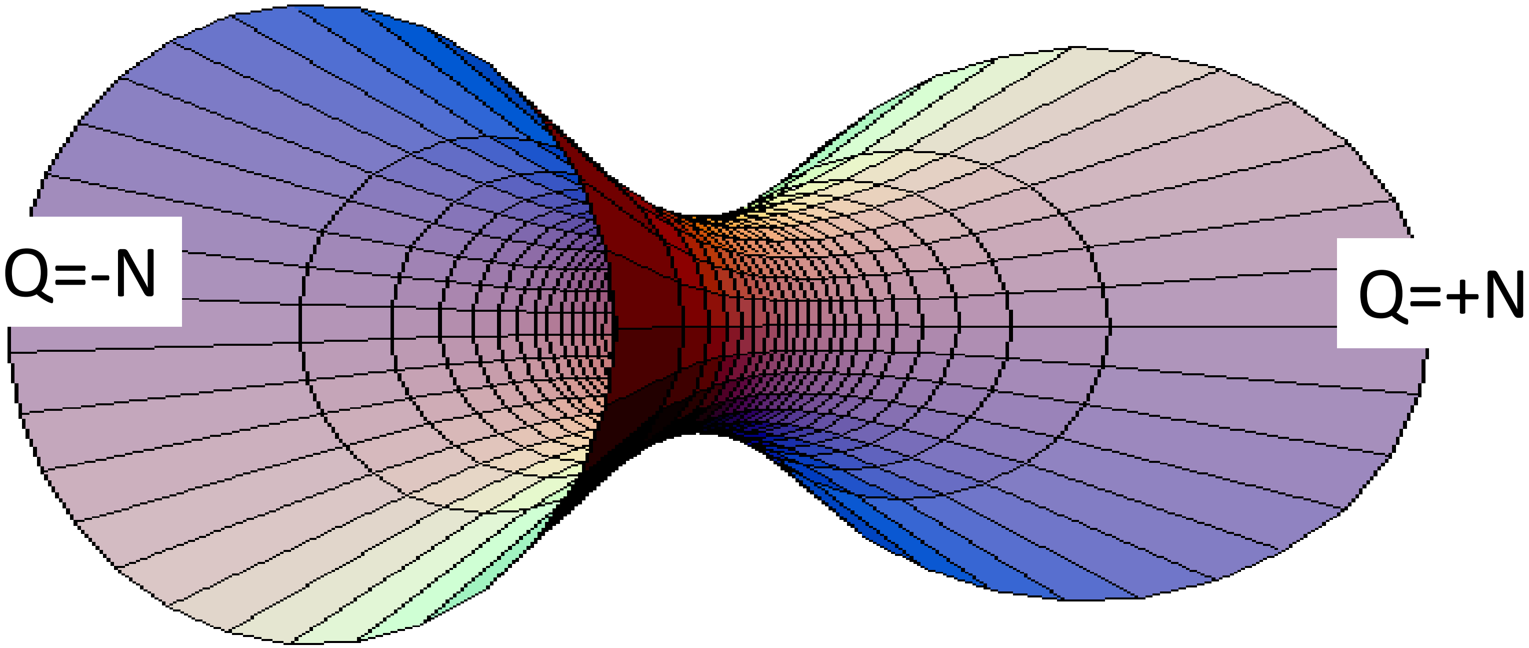
Wormhole? In gauge f=1, a(t) should grow, reach a minimum and then grow again.
Other gauge is easier:

$$\boxed{ds^2 = \left(1 + \frac{\tau^2}{\ell^2} - \frac{Q^2}{\tau^8}\right)^{-1} d\tau^2 + \tau^2 d\Omega_3^2} \qquad \Lambda = -\frac{1}{\ell^2}$$

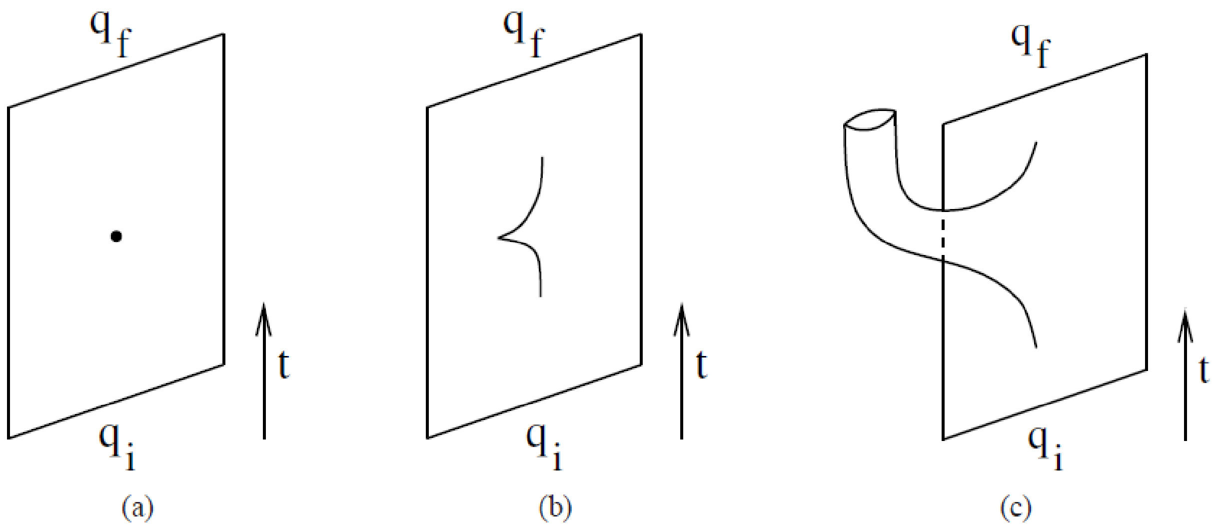


Wormhole is a dipole. There is no monopole axion charge, only locally at one side.

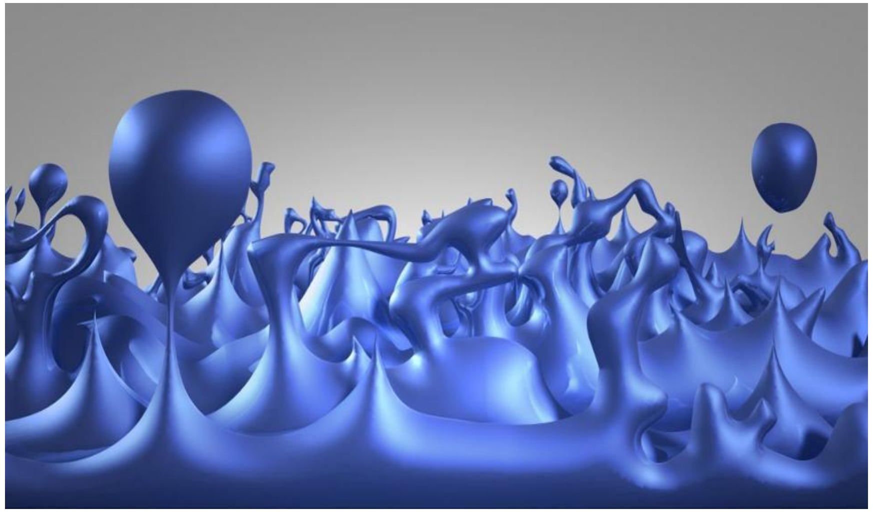
Finite action: $\boxed{S \sim |Q|}$

Very rich and long history in quantum gravity, prior to string theory. Recent revival in string theory due to Swampland discussions & holography. See [Hebecker, Mikhail, Soler 2018] for comprehensive review

Interpretation as tunneling instantons describing nucleation of baby universes :



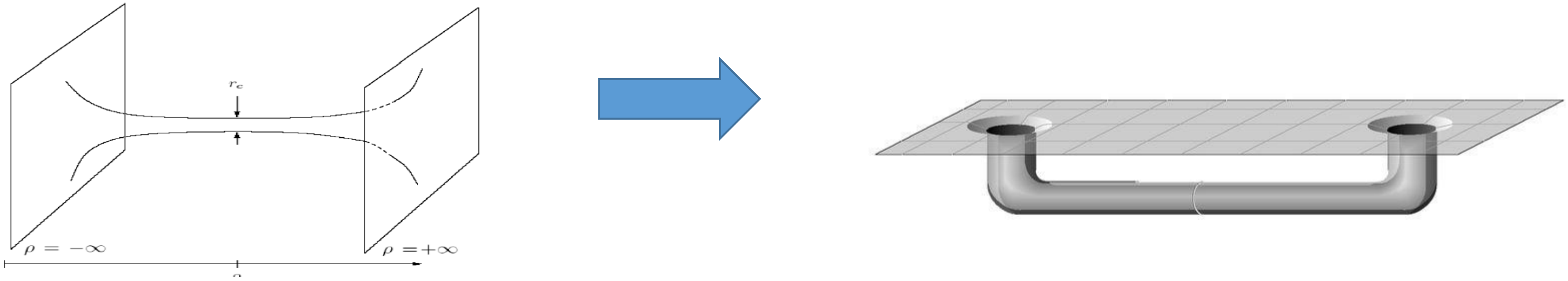
[Giddings/Strominger 1987,
Lavrelashvili/Tinyakov/Rubakov 1998,
Hawking 1987, …]



→Full wormhole describes emission *and* subsequent absorption of baby universe. Tunneling probability Planckian suppressed.

An observer detects a violation of axion charge conservation. Related phenomenom of ***non-unitarity***.

If one glues the two boundaries into one space-time:



then wormholes represent a breakdown of (macroscopic) locality : the effective action would include operators of the form

$$S_{WH} = -\frac{1}{2}\sum_{IJ}\int d^D x\, d^D y\, \mathcal{O}_I(x) C_{IJ} \mathcal{O}_J(y)\ ;$$

[Coleman 1989]: Not really since

$$e^{-S_{WH}} = \int d\alpha_I\, e^{-\frac{1}{2}\alpha_I (C^{-1})_{IJ}\alpha_J} e^{-\int d^D x \sum_I \alpha_I \mathcal{O}_I(x)}\ .$$

ENSEMBLES

Swampland?

- Breaking global symmetries by Planck suppressed terms (axion potential).
- Wormholes & axion/instanton WGC & large field inflation [Montero-Valenzuela-Uranga 2015, Brown-Cottrell-Shiu-Soler 2015, Heidenreich-Reece-Rudelius 2015, Hebecker-Mangat-Theissen-Witkowski 2016, ….]

$$S_1 f \lesssim M_P.$$

- (-1)-form global symmetries [McNamara-Vafa 2020]
- Derivative corrections lower wormhole actions (WGC like reasoning). [Andriolo-Huang-Noumi-Ooguri-Shiu 2020]

→ ARE AXION WORMHOLES IN THE SWAMPLAND?

Axion Wormholes In String Theory

Clean embedding means no “phenomenological compactification“. Then saxions are unavoidable.

$$\mathcal{L}_{\rm kin} = -\tfrac{1}{2}(\partial\phi)^2 + \tfrac{1}{2}e^{b\phi}(\partial\chi)^2$$

b small enough for regular wormhole $$b^2 < \frac{2(D-1)}{D-2}$$

For general sigma model regularity condition implies condition on length of timelike geodesics on sigma model [Arkani-Hamed-Orgera-Polchinski 2007]. Easily achieved in flat space compactifications [Bergshoeff-Collinucci-Roest-Vandoren 2004]

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→ But we want AdS. So, truncate to AdS moduli space, if any. Then:

$$\boxed{S = -\frac{1}{2\kappa^2}\int \sqrt{g}\left(\mathcal{R} - \tfrac{1}{2}G_{ij}\partial\phi^i\partial\phi^j - \Lambda\right)}$$

Solutions?:

$$\boxed{\mathrm{d}s^2 = \left(1+\frac{\tau^2}{\ell^2}+\frac{c}{2(D-1)(D-2)}\tau^{-2(D-2)}\right)^{-1}\mathrm{d}\tau^2+\tau^2\mathrm{d}\Omega^2}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}h^2}\phi^i+\Gamma^i_{jk}\frac{\mathrm{d}}{\mathrm{d}h}\phi^j\frac{\mathrm{d}}{\mathrm{d}h}\phi^k=0 \quad \Longrightarrow \quad \boxed{G_{ij}\frac{\mathrm{d}}{\mathrm{d}h}\phi^i\frac{\mathrm{d}}{\mathrm{d}h}\phi^j=c}$$

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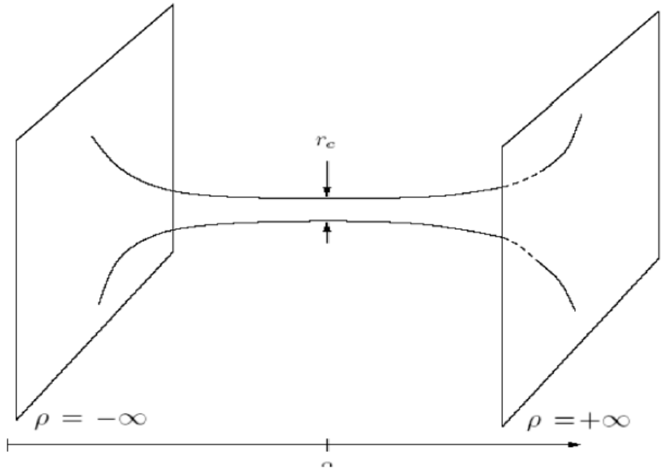
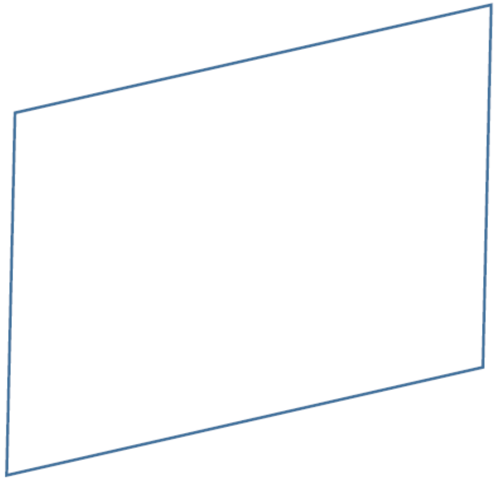
$$\frac{d^2}{dh^2}\phi^i + \Gamma^i_{jk}\frac{d}{dh}\phi^j\frac{d}{dh}\phi^k = 0 \quad \Rightarrow \quad \boxed{G_{ij}\frac{d}{dh}\phi^i\frac{d}{dh}\phi^j = c}$$

"Over-extremal" c < 0



``Time-like" geodesics

"Extremal" c = 0 :



``Light-like'' geodesics

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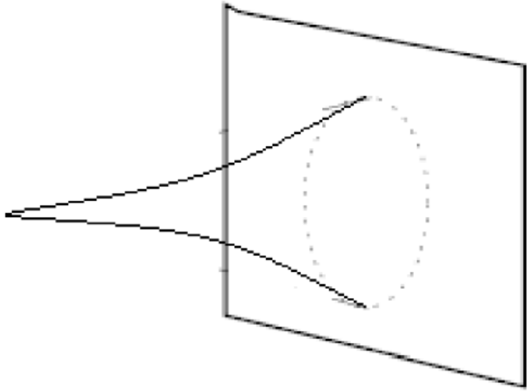

``Space-like" geodesics

Extremality for instantons, how exactly?

- On-shell action (or direct dimensional reduction of black holes)

$$\boxed{S \sim |Q| e^{-b\phi(\infty)/2} \sqrt{1 + \frac{c}{Q^2} e^{b\phi(\infty)}},}$$

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- Probe extremal instantons show "repulsion" away from over-extremal instantons. Wormholes have "positive binding energy". [VR 2020]

$$c > 0 : \quad S_{probe} = \frac{\sqrt{c}}{|Q|} \frac{1}{\sinh(\frac{1}{2}\sqrt{c} b h(\tau))} \Big(1 - \cosh(\tfrac{1}{2}\sqrt{c} b h(\tau))\Big) \,,$$

$$c < 0 : \quad S_{probe} = \frac{\sqrt{|c|}}{|Q|} \frac{1}{\sin(\frac{1}{2}\sqrt{|c|} b h(\tau))} \Big(1 - \cos(\tfrac{1}{2}\sqrt{|c|} b h(\tau))\Big) \,.$$

We studied [Hertog, Trigiante, VR 2017, Katmadas, Ruggeri, Trigiante, VR, 2018]

$$\mathrm{AdS}_5 \times \mathrm{S}^5/\mathbb{Z}_\mathrm{k}$$

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Dual theory is N=2 “necklace quiver CFT” [Kachru, Silverstein ‘98] and has k gauge nodes → hence k complex couplings (k theta-angles),

$$\mathcal{L} \ni \sum_{\alpha=0}^{k-1} \left(-\frac{1}{4g_\alpha^2} \mathrm{Tr}[F_\alpha^2] - i\frac{\theta_\alpha}{32\pi^2} \mathrm{Tr}[F_\alpha \wedge F_\alpha] \right)$$

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Conf manifold $\frac{\mathrm{SU}(1,k)}{\mathrm{S}[\mathrm{U}(1)\times\mathrm{U}(\mathrm{k})]} \Longrightarrow \frac{\mathrm{SL}(k+1,\mathbb{R})}{\mathrm{GL}(k,\mathbb{R})}$. 2k real scalars.

→ Consistent with SUGRA moduli space [Corrado-Gunaydin-Warner-Zagermann 2002]

$$\boxed{S = -\frac{1}{2\kappa^2}\int \sqrt{g}\left(\mathcal{R} - \frac{1}{2}G_{ij}\partial\phi^i\partial\phi^j - \Lambda\right)}$$

- Moduli inside AdS are coupling constants for exactly marginal operators in the dual field theory: they label the family of CFT's = *conformal manifold.*

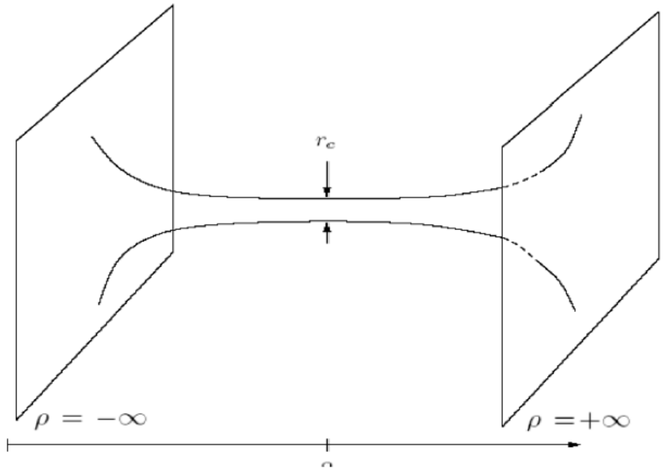
- Metric Gij on moduli space corresponds to the `Zamolodchikov' metric gij defined by the two-point functions:

$$g_{ij}(\varphi) = x^{2\Delta}\langle O_i(x)O_j(0)\rangle_{S[\varphi]}$$

- **Holography suggest that some *geodesic curves on the conformal manifold correspond to instantons of the CFT*.**

We constructed all (instantons) geodesics. Results?

“Over-extremal” c < 0



``Time-like” geodesics

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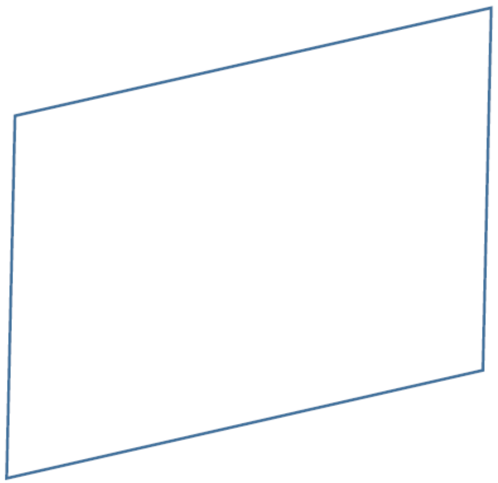

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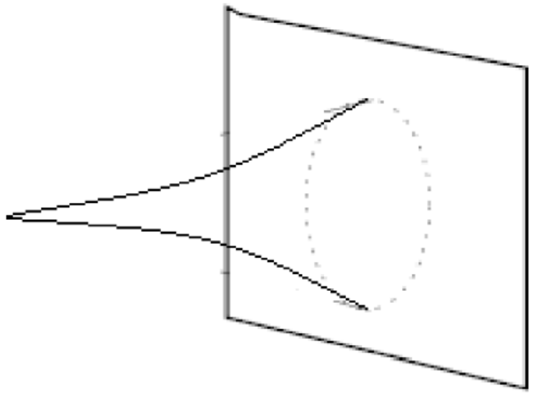
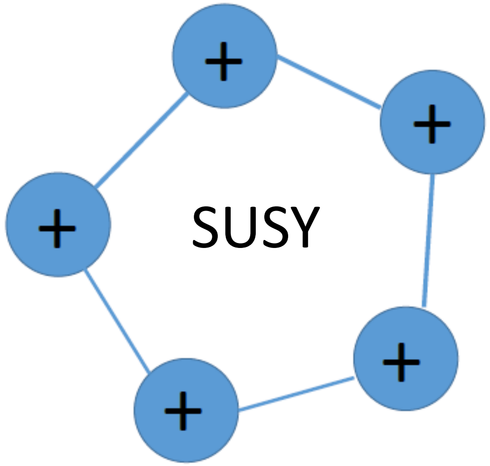


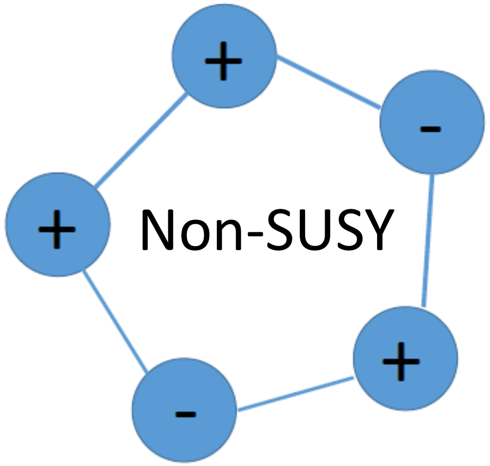
``Space-like” geodesics

Main results are

- **<u>SUSY solutions</u>** match SUSY gauge theory instantons. (One point functions & on-shell actions)

- **<u>non-SUSY solutions but extremal</u>**: Some of them can be interpreted and match so called "quasi-instantons" [Imaanpur 2008]. *These are solutions which are self-dual in each separate gauge node, but orientations differ from node to node.*





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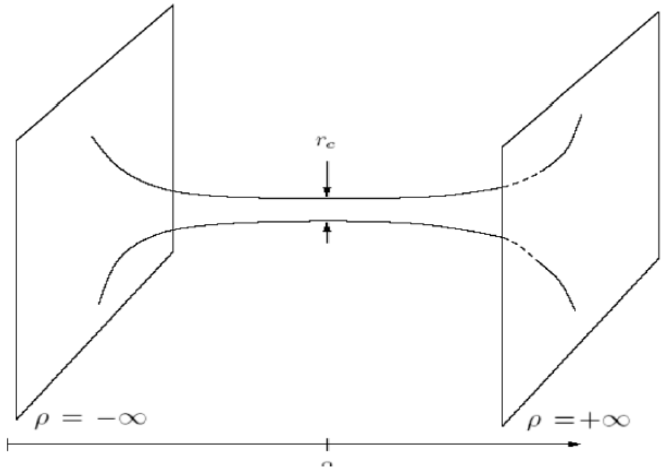

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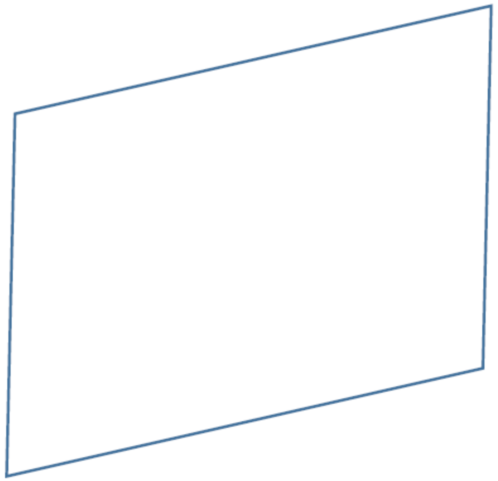

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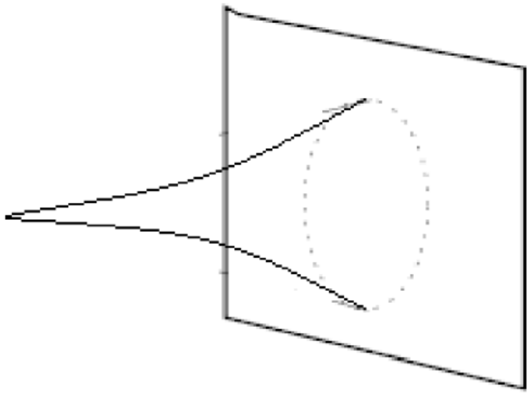

``Space-like” geodesics

- Solution is singular, but singularity seems ok?

- Suggestion for holographic dual from computing one point functions & action.

non-self dual YM instantons…
[Bergshoeff, Collinucci, Ploegh, Vandoren, VR 2005]

$$A_\mu^{\mathrm{SU(N)}} = \begin{pmatrix} A_\mu^{\mathrm{SU}(2)} & 0 & \cdots & 0 \\ 0 & A_\mu^{\mathrm{SU}(2)} & & 0 \\ \vdots & & \ddots & \\ 0 & & & \overline{A}_\mu^{\mathrm{SU}(2)} \end{pmatrix} .$$

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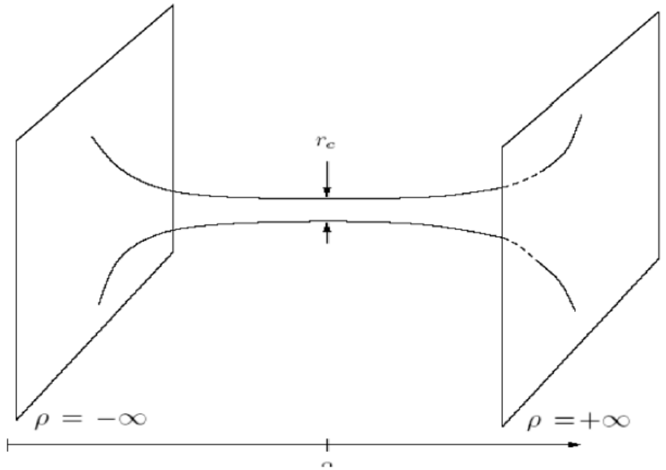

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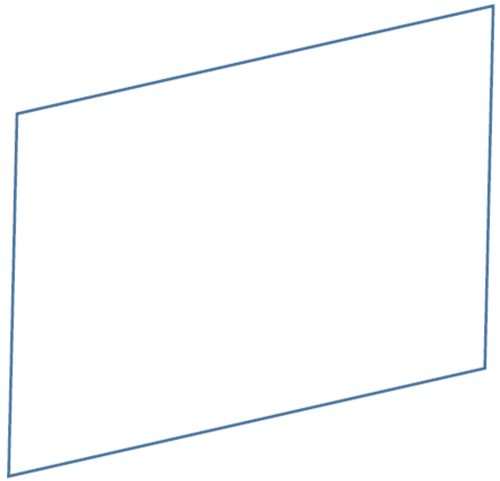

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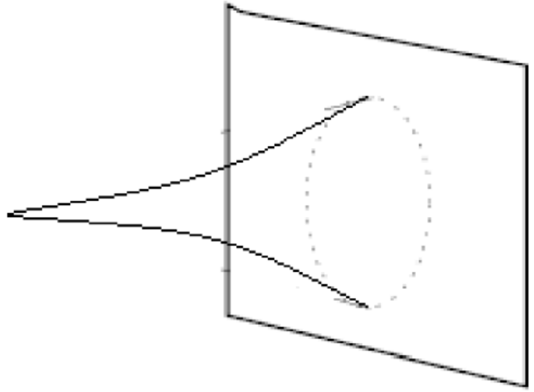

``Space-like” geodesics

Wormholes clash with AdS/CFT. Dual field theory has no sign of Coleman's α parameters [Maldacena-Maoz 2004] + factorization paradox.

Over-extremal black holes unphysical. Not over-extremal particles. What about over-extremal instantons (Colemans wormholes)? There is no naked singularity to warn us.

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Our explicit embedding provides **another paradox**: holographic one-point function give violation of positivity:

$$\boxed{|\mathrm{Tr}[F_\alpha^2]| < |\mathrm{Tr}[F_\alpha \wedge F_\alpha]| \,.}$$

Clear evidence for spurious nature of wormholes.

Euclidean Stability

Interpretation of instantons depends on **stability**

Perform Gaussian approximation around saddle point:

$$Z = e^{-S[\Phi_0]} \int \mathrm{D}\phi \, e^{-\delta^2 S[\Phi_0,\phi] + \mathcal{O}(\phi^3)} \qquad \delta^2 S = \tfrac{1}{2}\int \phi \hat{\mathrm{M}} \phi$$

Solve eigenvalue problem: $$\frac{1}{X}\hat{\mathrm{M}}\phi_n = \lambda_n \phi_n \,, \qquad \int X \, \phi_n \phi_m = \delta_{nm}$$

To find: $$Z \sim e^{-S[\Phi_0]} \int \Pi_n \mathrm{d}z_n \, e^{-\frac{1}{2}\sum_n \lambda_n z_n^2} \sim \frac{e^{-S[\Phi_0]}}{\sqrt{\Pi_n \lambda_n}} \,.$$

*Coleman: in QM & QFT we have standard instantons (all eigenvalues positive) or "bounces" with **one** negative eigenvalue. The latter describe tunneling amplitudes. **Multiple** negative eigenvalues means instanton is spurious.*

Literature: there is possibly one negative eigenmode, which is expected from tunneling interpretation [Rubakov 1989, Kim&Lee&Myung 1997, Kim&Kim&Hetrick2003, Alonso&Urbano 2017].

→ [Hertog, Truijen, VR 2018] Computations did not use the right gauge-invariant variables + Interpretation as path integral for axion-charge transitions is crucial.

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Boundary conditions

We want matrix elements from charge eigenstates = momentum eigenstates

$$|\Pi\rangle = |Q\rangle$$

So we wish to evaluate

$$K \equiv \langle \Pi_F | \exp(-HT) | \Pi_I \rangle \qquad |\Pi\rangle = \int \mathrm{d}[\chi]\, e^{i\int_\Sigma \chi \Pi} |\chi\rangle$$

Saddles obey: $\Box\chi = 0$ $(\star \mathrm{d}\chi - i\Pi)|_{\Sigma_{I,F}} = 0$

→ "Euclidean free field action with wrong sign kinetic term"

Equivalent to

$$Z = \int_{bc} d[F]d[\chi]\mathrm{e}^{-\frac{1}{\hbar}\int \star F\wedge F + i\chi \mathrm{d}F} \qquad F_{I,F} = \star\Pi_{I,F}$$

→ Dirichlet bc for momentum.

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Taking this into account gives well-behaved quadratic action. ***No conformal factor problem*! Reason: homogenous modes non-dynamical.**

HOW?

Use formalism of cosmological perturbation theory [Gratton-Turok 1999]

$$\mathrm{d}s^2 = b^2\Big(-(1+A)^2\mathrm{d}\eta^2 + \partial_i B \mathrm{d}x^i \mathrm{d}\rho + [(1-2\psi)\gamma_{ij} + \partial_i\partial_j E]\mathrm{d}x^i\mathrm{d}x^j\Big) \,,$$

Scalar perturbations & gauge invariant observable:

$$\boxed{\mathcal{X} = \psi + \frac{b'}{b\chi'}\delta\chi}$$

After a mode decomposition and lengthy computation (software) :

$$S_2 = \frac{\mathrm{Vol}(S^3)}{\kappa^2}\int \mathrm{d}\rho\big(A_n\dot{\mathcal{X}}_n^2 - B_n\mathcal{X}_n^2\big)$$

With A_n, B_n certain functions of Euclidean time.

Crucially we need the quadratic action for the momenta instead! Formal manipulation;

$$\boxed{S_2 = \frac{\mathrm{Vol}(S^3)}{\kappa^2} \int \mathrm{d}\rho \left(-B_n^{-1} (\dot{\Pi}^n_{\mathcal{X}})^2 + A_n^{-1} (\Pi^n_{\mathcal{X}})^2 \right) .}$$

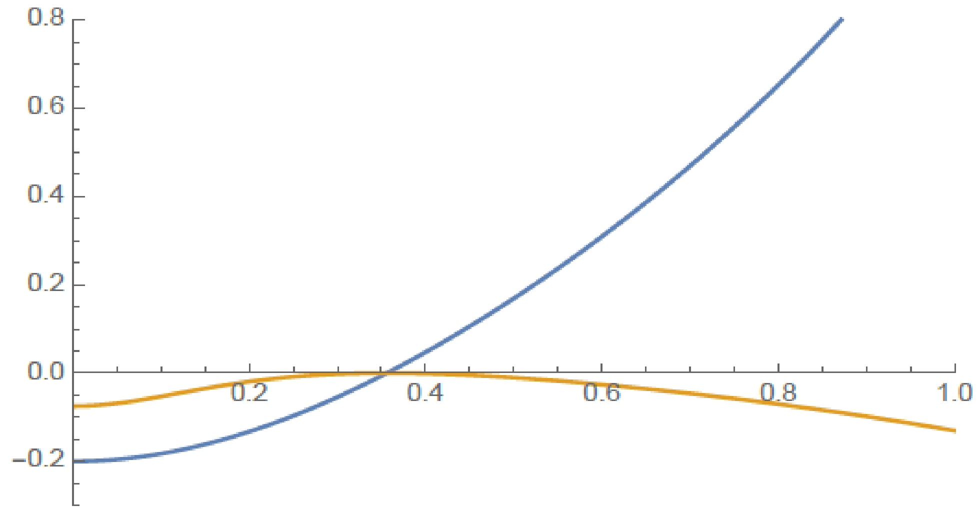

FIG. 1: *The coefficients* A_n^{-1} *(blue) and* B_n^{-1} *(orange) entering in the action for perturbations about axion wormholes, shown here for* $n = 3$ *(and with* $c = 1$*).*

Kinetic term positive. Potential bounded from below and negative only near neck. But enough to find square integrable test functions that lower the total action. Only for n>2.

Infinitely many modes lower the action. All centered close to the neck and probe the non-trivial topology. For very small wormholes those modes become sub-planckian.

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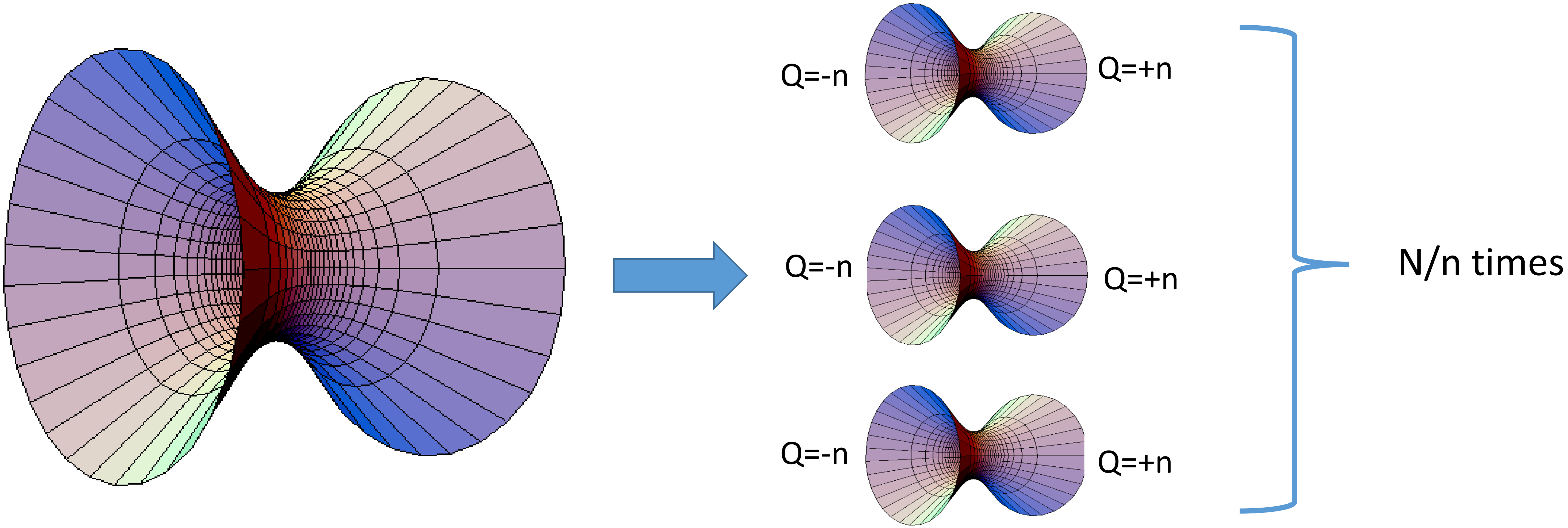
→ Macroscopic wormholes do not contribute. There is a lower action saddle with same boundary conditions? Which one? → wormhole fragments into smaller wormholes.



Wormhole defragmentation

- Consistent with picture of [McNamara&Vafa, 2020].
- **Microscopic** instantons cannot be argued to be spurious. But they are not wormholes

Over-extremal black holes unphysical. Not over-extremal particles. What about over-extremal instantons? (There is no naked singularity to warn us.)

It is the instability in the path integral that makes them unphysical. Instability is in non-homogenous sector: signals fragment into smaller pieces to lower action. Just like super-extremal "black holes" shatter into super-extremal particles that cannot decay anymore. *<u>Macroscopic</u> wormholes are unphysical!*

Outlook

- <u>Hodge dual analysis.</u>

We Wickrotated **Lorentzian** perturbation theory and also the **dressed** axion fluctation:

$$\chi \to i\chi \qquad \underset{?}{\Longrightarrow} \qquad \mathcal{X} \to i\mathcal{X}$$

Fluctuation theory directly in Euclidean space with form field having normal kinetic term can be shown to yield identical results. [Hertog, Maenaut, Tielemans, VR, in progress]

- <u>*Multiple field analysis*</u>.

We have done a completely general multi-field analysis. Wormholes remain unstable in general sigma models.

AdS_3 x S^3 x CY_2

[Astesiano, Ruggeri, Trigiante, VR, to appear soon]

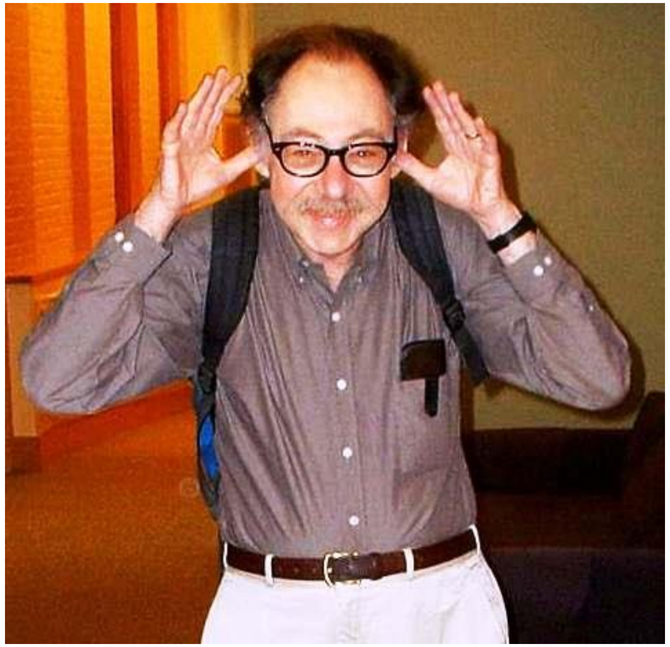
Conformal manifold
$$\frac{SO(4,n)}{SO(4)\times SO(n)}$$

→We found all geodesics and there are **no regular wormholes**.

→SUSY extremal instantons are all Euclidean strings wrapping 2 cycles inside CY2

$$\begin{array}{lcccccccccc}
D1 & \times & \times & - & - & - & - & - & - & - & - \\
D5 & \times & \times & - & - & - & - & \times & \times & \times & \times \\
D1 & - & - & - & - & - & - & - & - & \times & \times \\
D1 & \multicolumn{6}{c}{\underbrace{-\ \ -\ \ -\ \ -\ \ -\ \ -}_{AdS_3\times S^3}} & \times & \times & - & -
\end{array}$$

Are Coleman's Euclidean wormholes real?



(S. Coleman 1937-2007)

No, they are self-repulsive in a Euclidean sense. So macroscopic wormholes won’t contribute in path integral.

- We gave direct evidence through computation.
- And then a GR-like interpretation
- Plus evidence from top-down holography: violation positivity bounds.

→ Fragmentation.

EXTRA

$$S[A] = \int \star R - \tfrac{1}{2} \star F_p \wedge F_p$$

$$S[F,B] = \int \star R - \tfrac{1}{2} \star F_p \wedge F_p + dF_p \wedge B_{D-p-1} \qquad dB = G_{D-p}$$

With partial integration, and dropping a boundary term, we can get:

$$S[F,B] = \int \star R - \tfrac{1}{2} \star F_p \wedge F_p + (-1)^{p+1} F_p \wedge G_{D-p}$$

the EOM for F gives:

$$\star F_p = (-1)^{p(D-p)} G_{D-p} \quad \Rightarrow \quad S = \int \star R + \tfrac{1}{2}(-1)^t \star G_{D-p} \wedge G_{D-p}$$

$$
\begin{aligned}
Z &= \int \mathcal{D}q \; exp\left[-\int dt\left(-\frac{A}{2}\dot{q}^2 + \frac{B}{2}q^2\right)\right] \\
&= \int \mathcal{D}q\mathcal{D}p \; exp\left[-\int dt\left(\frac{A^{-1}}{2}(p - A\dot{q}^2) - \frac{A}{2}\dot{q}^2 + \frac{B}{2}q^2\right)\right] \\
&= \int \mathcal{D}q\mathcal{D}p \; exp\left[-\int dt\left(\frac{A^{-1}}{2}p^2 + \dot{p}q + \frac{B}{2}q^2\right)\right] \\
&= \int \mathcal{D}q\mathcal{D}p \; exp\left[-\int dt\left(\frac{A^{-1}}{2}p^2 + \frac{B}{2}(q + B^{-1}\dot{p})^2 - \frac{B^{-1}}{2}\dot{p}^2\right)\right] \\
&= \int \mathcal{D}p \; exp\left[-\int dt\left(-\frac{B^{-1}}{2}\dot{p}^2 + \frac{A^{-1}}{2}p^2\right)\right]
\end{aligned}
$$

Boundary

$$\mathcal{Z}_{\mathrm{QG}}(X) = \mathcal{Z}_{\mathrm{CFT}}(X).$$

$$\mathcal{H}_{\mathrm{QG}}(M_1 \sqcup M_2) = \mathcal{H}_{\mathrm{QG}}(M_1) \otimes \mathcal{H}_{\mathrm{QG}}(M_2)$$

$$\mathcal{H}_{\mathrm{BU}} = \mathcal{H}_{\mathrm{QG}}(\varnothing)$$

$$\varnothing \sqcup M = M,$$

$$\mathcal{H}_{\mathrm{QG}}(M) = \mathcal{H}_{\mathrm{BU}} \otimes \mathcal{H}_{\mathrm{QG}}(M)$$

$$\left(\frac{n}{S_n}\right)_{WGC} \geq \left(\frac{N}{S_N}\right)_{ext} \qquad S_N/N \ \sim \ M_p/f$$

$$S_1 f \ \lesssim \ M_P.$$