Swampland conjectures from Black Holes and Finiteness

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Based on 2111.00015 with Hamada, Montero and Vafa

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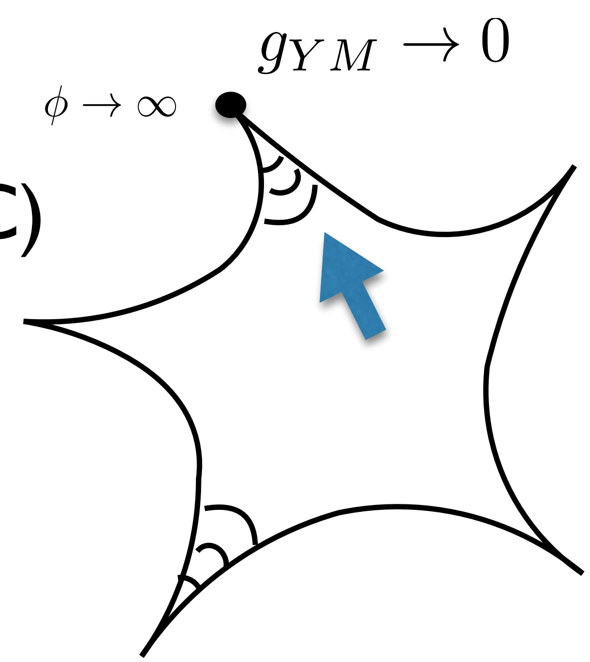
Swampland Program

Recall: EFT expectations can fail in the presence of gravity

For instance, EFT breaks down at a scale much below Mp if we have:

- ❖ Small gauge couplings $\Lambda \sim g M_p$ (WGC)
- ❖ Large field ranges $\Lambda \sim M_p \exp(-\alpha \Delta\phi)$ (SDC)
- ❖ Approximate global symmetries

…



because of the presence of an infinite tower of states becoming light.

Surprising from EFT point of view, but very natural from string theory perspective (linked to string dualities)

Asymptotic Towers of States

String Theory Evidence:

❖ More than 8 supercharges: $\mathcal{M} = G/H$ [Cecotti'15] see Timo's talk

❖ Theories with 8 supercharges:

- *5d/6d N=1 theories: F-theory CY compactifications*
- *4d N=2 theories: Calabi-Yau compactifications of Type II*

Systematic classification of limits ✓

[Lee,Lerche,Weigand'18-20]
[Grimm, Palti, IV'18][Gendler,IV'20]
[Grimm,Palti,Li'18] …
[Baume,Marchesano,Wiesner'19]
[Corvilain, Grimm, IV'18]
…

❖ AdS/CFT [Perlmutter,Rastelli,Vafa,IV'20]
[Luest,Palti,Vafa'19] [Baume,Calderon-Infante'20]

❖ Theories with 4 supercharges
[Lanza,Marchesano,Martucci, IV'20] [Klaewer,Lee,Weigand,Wiesner'20]

Asymptotic Towers of States

A bottom-up explanation independent of string theory is missing

Promising avenue: think about **black hole physics**

❖ Distance Conjecture ??

❖ Weak Gravity Conjecture: to allow extremal black holes to decay

It only supports a mild version of WGC

(a single particle is needed, not a tower)

In this talk:

I will provide new arguments in support of these conjectured asymptotic towers of states based on finiteness of black hole entropy

… or more generally, finiteness of quantum gravity amplitudes

Very nice connection with Emergence Proposal

Finiteness

Many Swampland conditions somehow emerge from a suitable replacement of infinity by a finite number, achieved when Mp is finite

❖ WGC: $m \leq g M_p$ (finite mass, or finite gauge kinetic function)

❖ SDC: $\Delta\phi \leq \frac{1}{\alpha} \log \frac{M_p}{\Lambda}$ (finite scalar field range)

❖ Finite number of degrees of freedom below a cut-off

$$N \leq \left(\frac{M_p}{\Lambda}\right)^{d-2}$$

❖ Finite number of quantum gravity vacua

[Arkani-Hamed'05][Dvali'07]…

We want to initiate a program to view all the Swampland criteria from the prism of finiteness, thus taking a step in unifying the conjectures.

Outline

❖ Introduction

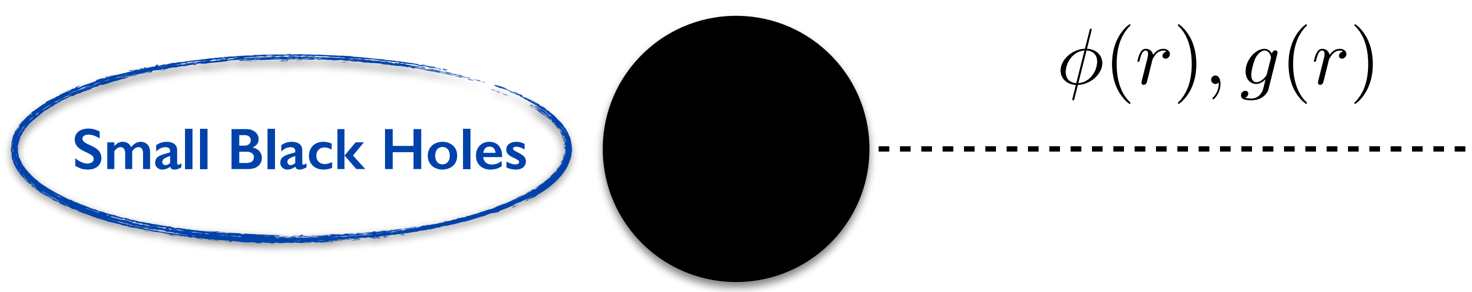
❖ Asymptotic towers of states from black hole entropy bounds

❖ UV compactness and Emergence

❖ Finiteness of quantum gravity amplitudes (and vacua)

WGC and SDC from Entropy Bounds

Let us engineer some large field range and weak coupling limit induced by the backreaction of a solitonic object in a given EFT



What goes wrong then?

Very useful avenue in [Klaewer, Palti'16] charged dilatonic black holes

[Draper,Farkas'19] bubble of nothing, dilatonic BHs…

[Bonnefoy et al'19] large black holes

[Lanza,Marchesano,Martucci,IV'20-21] low codimension objects

WGC and SDC from Entropy Bounds

Take Einstein-Maxwell-Dilaton theory:

$$S = \int d^4x \sqrt{-g} \left[R + 2|d\phi|^2 + \frac{1}{2g(\phi)^2} |F|^2 \right]$$

such that $g(\phi) \to 0$ as $\phi \to \infty$

Look for electrically charged extremal black hole solutions

$$ds^2 = -e^{2U} dt^2 + e^{-2U} \left(\frac{d\tau^2}{\tau^4} + \frac{1}{\tau^2} d\Omega_2^2 \right)$$

$$F = \frac{g^2}{4\pi} Q e^{2U} \tau^2$$

WGC and SDC from Entropy Bounds

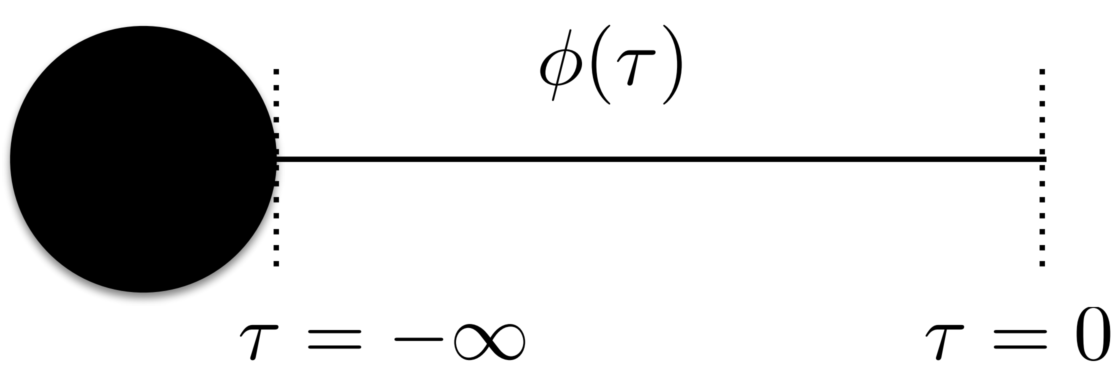
Due to the attractor mechanism, the dynamics is captured by the eom of:

$$\mathcal{L}_{1d} = \frac{1}{2}\left(\dot{U}^2 + \dot{\phi}^2\right) + g^2 Q^2 e^{2U}$$

extremal BHs

subject to the constraint $\dot{U}^2 + \dot{\phi}^2 - g^2 Q^2 e^{2U} = 0$

BH induces a running of the scalar field and gauge coupling as approaching the horizon



WGC and SDC from Entropy Bounds

Extremal black holes:

BH area

$$\dot{U}^2 \leq g^2 Q^2 e^{2U} \quad \Rightarrow \quad A(\tau) = \frac{e^{-2U(\tau)}}{\tau^2} \leq \frac{1}{\tau^2}\left(1 + Q\int_\tau^0 g(\tau)\, d\tau\right)^2$$

❖ If $g(\tau) = \text{constant}$ **then** $A(-\infty) > 0$: Reissner-Nordstrom BH

❖ If $g(-\infty) \to 0$ **then** $A(-\infty) \to 0$: Small BH

example: $g = e^{-\phi}$

$$\phi = \frac{1}{2}\log\frac{A}{r^2} = \log\left(1 - \frac{2M}{r}\right) \to \infty \quad \text{at} \quad r = r_h = 2M$$

$$\tau = -\frac{1}{r - 2M}$$

WGC and SDC from Entropy Bounds

Large field variations associated with weak coupling limits are confined near small regions in space and associated to small BHs

infinitely many nearly point objects??

The area receives corrections in string theory embeddings

We will see that they can lead to a violation of the Bekenstein bound, unless the EFT cutoff decreases as dictated by the SDC and WGC.

WGC and SDC from Entropy Bounds

Entropy Bound:

A region of size L cannot have more entropy than a Schwarzschild black hole of the same area $A = L^2$

The number of distinct BH states that fit in the box should not grow larger than L^2

This is violated if they are point-like, since we can then fit infinitely many.

If the EFT breaks down, they will have an effective area dictated by the cut-off, thus solving the problem.

Goal: determine how the cut-off must behave to avoid violation of entropy bounds

WGC and SDC from Entropy Bounds

The gradient of the scalar field diverges at the core for small black holes:

$$|d\phi|^2 = \tau^4 e^{2U} \dot{\phi}^2 \geq \frac{\Delta g^2 Q^2}{A^2} \to \infty$$

EFT breaks down whenever: $|d\phi|^2 = \tau^4 e^{2U} \dot{\phi}^2 \sim \Lambda^2$

Using $\dot{\phi}^2 \leq g^2 Q^2 \, e^{2U}$ and $A(\tau) = \dfrac{e^{-2U(\tau)}}{\tau^2}$

we get $\boxed{\Lambda \leq g \dfrac{Q}{A}}$

WGC and SDC from Entropy Bounds

How many BH states fit in a box of size L before the system collapses to a BH?

The entropy at $E \sim L$ for light N_{species} is:

$$S = N_{\text{species}}^{1/4} E^{3/4} L^{3/4} = N_{\text{species}}^{1/4} L^{3/2}$$

At sufficiently weak coupling, any small black hole of charge Q effectively counts as an additional specie

(gravitational and gauge interactions become arbitrarily small)

$$N_{\text{species}} = Q_{\text{max}} \lesssim L^2 = A$$

WGC and SDC from Entropy Bounds

To avoid violation with entropy bounds, EFT must break down at

$$\Lambda \leq g\frac{Q}{A} \quad \Longrightarrow \quad \Lambda \lesssim g \quad \text{in Planck units}$$

$$Q \lesssim A$$

This is precisely the cut-off dictated by the WGC!

- It supports a strong version of WGC since it requires the existence of infinitely many charged states becoming light (the local EFT description must break down)
- It makes quantitative the trouble with charged remnants for small gauge couplings (which didn't work for RN BHs)

WGC and SDC from Entropy Bounds

It also provides a BH argument for the SDC:

EFT must break down by **infinite many fields** becoming massless at large field ranges associated to weak coupling points

If $\quad g \sim e^{-a\phi} \quad$ then $\quad \Lambda \lesssim g \sim e^{-a\phi}$

This is the asymptotic behaviour for gauge couplings at infinite distance limits in string theory

Lamppost effect or fundamental reason?

Supported by Emergence Proposal (exponential behaviour emerges from integrating out tower of states)

[Heidenreich et al'17] [Palti,Grimm,IV'18]

(see also [Klaewer,Palti'16])

WGC and SDC from Entropy Bounds

Open question:

Any infinite distance limit corresponds to a weak coupling limit?

So far, in all string theory examples, there is a vanishing p-form gauge coupling asymptotically

- Proposed to be general in [Gendler,IV'20]
- Fits with Emergent String Conjecture [Lee,Lerche,Weigand'19]

(either KK photon or B-field)

UV compactness and Emergence

Let us next argue that non-compact scalars are not allowed in QG regardless of having a gauge field theory.

Take a free scalar field theory and compactify to 1d:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \varphi\right)^2 \quad \Rightarrow \quad H = \frac{1}{2}\hat{p}^2$$

Partition function diverges due to the continuum of eigenstates, unless the scalar field is compact (so the momenta spectrum is discrete)

$$Z = \sum_n^{E_n=\Lambda} e^{-\beta E_n} \simeq \frac{\phi_{\max}}{\sqrt{2\pi\beta}} \operatorname{erf}\left(\sqrt{\beta\Lambda}\right)$$

To keep partition function finite: $\Lambda\left(\phi_{\max}\right) \leq \frac{1}{\phi_{\max}^2} \quad \text{for} \quad \phi_{\max} \to \infty$

UV compactness and Emergence

What about the exponential behaviour of the cut-off?

Emergence Proposal: the dynamics (all kinetic terms) emerge upon integrating out states up to QG cut-off

[Palti,Grimm,IV'18] [Heidenreich et al'18]

In particular, infinite distance emerges from integrating out an infinite tower of states up to species scale

$$N \sim \frac{1}{\Lambda^{d-2}} \qquad N = \frac{\Lambda}{\Delta m}$$

Quantum corrections to field metric:

$$\delta g_{\phi\phi} \simeq \sum_{n=1}^{N} m_n^{d-4} \left(\partial_\phi m_n\right)^2 = \left(\frac{\partial_\phi \Delta m}{\Delta m}\right)^2$$

$$\Delta\phi \sim \int d(\log \Delta m) = \log \Delta m$$

Masses behave exponentially!

UV compactness and Emergence

Emergence proposal assumes that the field space is compact in the UV, so the infinite distance is an IR artefact

Finiteness of partition function can be used to motivate this 'UV compactness'

Why this partition function should be finite?

- Associated to entropy of a black (d-1)-brane [Hamada, Vafa'20] [Bedroya et al'21]

  (generalization of finiteness of probe brane moduli space) see Miguel's talk

- Proposal: swampland conditions emerge from finiteness of quantum gravity amplitudes

Finiteness of Quantum Gravity Amplitudes

On the quest of a general underlying principle behind the swampland conjectures!

Finiteness of quantum gravity amplitudes

(S-matrix elements, CFT correlators…)

EFT amplitudes can diverge, but not in the UV complete theory

Generalization of finiteness of probe brane moduli space

[Hamada, Vafa'20] [Bedroya et al'21]

Finiteness of Quantum Gravity Vacua

Finiteness of QG amplitudes can be used to motivate finiteness of the string landscape:

- Compactify to 2d in arbitrary manifold. The wavefunction spreads over all topologies, so the partition function diverges if there are infinite many vacua.
- More concretely, the partition function of a scalar field probing all (infinitely many) vacua diverges.

Proposal:

The number of low energy EFTs (after quotienting by moduli spaces) consistent with quantum gravity that are valid (at least) up to a fixed finite energy cutoff is finite.

Finiteness of Quantum Gravity Vacua

The number of low energy EFTs (after quotienting by moduli spaces) consistent with quantum gravity that are valid (at least) up to a fixed finite energy cutoff is finite.

- ❖ It cuts-off the infinite distance tails of vacua asymptotically
- ❖ Infinite families of AdS vacua must come together with non-decoupled extra dimensions
- ❖ It fits with Cheeger's theorem: finite number of diffeomorphisms types of Riemenannian manifolds with bounded curvatures, volumes and diameters

see also [Archarya,Douglas'06] finiteness of 4d vacua with bounded KK scale, volumes and vacuum energy

[Grimm'20] finiteness of self-dual flux vacua in CYs

Conclusions

My time is up
and conclusions must come,
and just like Miguel,
I am afraid I have to rap.

You cannot fit too many states in a box
because that would violate Bekenstein entropy bound.
The cut-off must decrease
as the Weak Gravity predicts
or small black holes
can be a problem indeed.

For the future we want to analyse
whether finiteness of amplitudes
can the swampland conjectures unify.
Asymptotic towers, finite number of vacua,
if you violate it, you are in the swampland.

Thank you for your attention
I hope you enjoyed my humble presentation.

Thank you!

back-up slides

Approximate global symmetries,
Weakly coupled gauge theories,
Large field ranges…

…come at a price.

(Swampland) Distance Conjecture (SDC):

There is an infinite tower of states becoming exponentially light at every infinite field distance limit of the moduli space

$$m(P) \sim m(Q)e^{-\alpha\Delta\phi} \quad \text{when} \quad \Delta\phi \to \infty$$

(geodesic distance)

[Ooguri-Vafa'06]

Weak Gravity Conjecture (WGC):

Given a gauge theory, there must exist an electrically charged state with

$$\frac{Q}{M} \geq \left(\frac{Q}{M}\right)_{\text{extremal}} = \mathcal{O}(1)$$

Q=q g : charge
m : mass in Planck units

Strong version: *there is a sublattice/tower of superextremal states*

[Montero et al.'16][Heidenreich et al.'15-16][Andriolo et al'18]

UV cut-off goes to zero
due to new light states

$$\Lambda \sim gM_p$$

$$\Lambda \sim M_p \exp(-\alpha\Delta\phi)$$

What is the value of the exponential rate?

- AdS_{d+1}/CFT_d with $d > 2$ [Perlmutter,Rastelli,Vafa,IV'20]

$$\alpha = \sqrt{\frac{2c}{\mathrm{dim}G}} \begin{array}{ll} \geq \frac{1}{\sqrt{3}} & \text{for 4d N=2} \\ \geq \frac{1}{2} & \text{for 4d N=1} \end{array}$$

[Grimm, Palti, IV'18] [Gendler,IV'20]

- Lower bound for BPS states in CY compactifications: $\alpha \geq \frac{1}{\sqrt{2n}}$ for CY_n

$$K = -n \log \phi + \ldots$$

- 4D N=2 theories: $\alpha^2 \geq \left.\frac{Q_{\rm ext}^2}{T_{\rm ext}^2}\right|_{\rm BPS\,particles} - \frac{1}{2}$

- 4D N=1 theories: $\alpha \geq \frac{1}{2} \left.\frac{Q_{\rm ext}}{T_{\rm ext}}\right|_{\rm BPS\,string}$

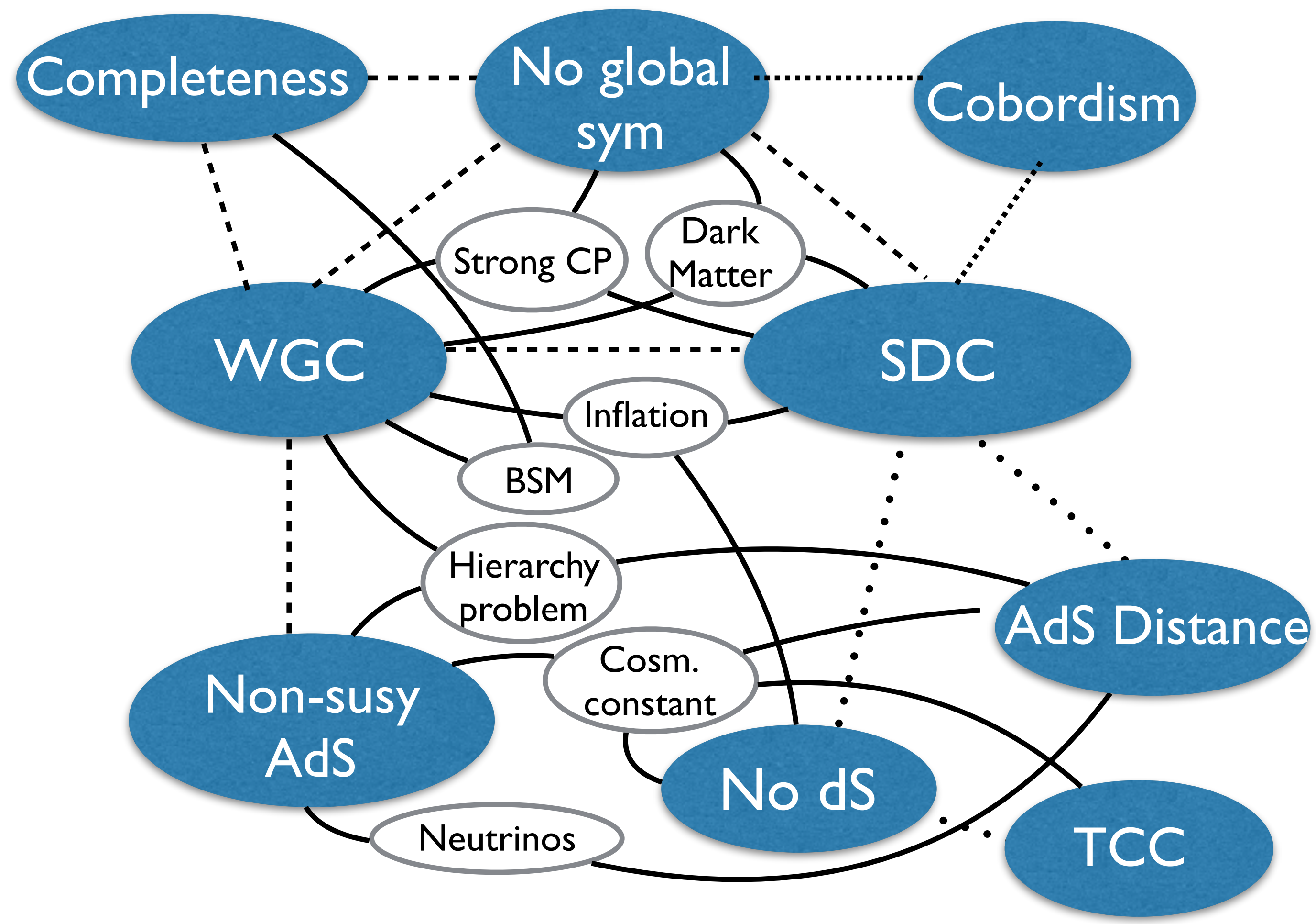
→ bounded by scalar contribution to WGC/extremality bound!

[Lee,Lerche,Weigand'19] [Gendler,IV'20] [Bastian, Grimm, Van de Heisteeg'20]

- TCC $\alpha \geq \frac{1}{\sqrt{(d-2)(d-3)}}$ [Bedroya,Vafa'19] [Andriot et al'20]

- Consistency with quantum gravity can have important phenomenological implications. Not every EFT is allowed!

- Web of interconnected swampland conjectures. Different faces of the same principles?



- Significant new evidence in favour of some conjectures in the past years from different research areas. Much more work to do!

CFT Distance Conjecture

☑ AdS_{d+1}/CFT_d with $d>2$ [Perlmutter,Rastelli,Vafa,IV'20] (see also [Baume,Calderon-Infante'20])

Bulk moduli space ⟷ *Conformal manifold* *(space of exactly marginal couplings)*

field metric ⟷ *Zamolodchikov metric* $|x-y|^{2d}\langle O_i(x)O_j(y)\rangle = g_{ij}(t^i)$

tower of light states ⟷ *tower of operators saturating unitarity bound*

Our proposal:

$\exists$ tower of HS with $\gamma_J \sim e^{-\alpha d(\tau,\tau')}$ as $d(\tau,\tau')\to\infty$ in the conformal manifold

(measured by Zamolodchikov metric)

not proven (supported by all known examples)

[Maldacena,Zhiboedov'11]

Infinite distance ⇄ Free point ⇄ Higher Spin tower

proven in SCFTs $g_{YM}\to 0$ *proven in SCFTs*

Consequence: 3d conformal manifolds are compact

CFT Distance Conjecture

☑ AdS_{d+1}/CFT_d with $d > 2$ [Perlmutter,Rastelli,Vafa,IV'20] (see also [Baume,Calderon-Infante'20])

In the free limit $g_{YM} \to 0$ $\longrightarrow$ $\mathcal{O}_\tau = \mathrm{Tr}(F^2 + \ldots)$

By perturbation theory:

$$ds^2 = \beta^2 \frac{d\tau d\bar{\tau}}{(\mathrm{Im}\tau)^2} \quad \text{as} \quad \mathrm{Im}\tau \to \infty \qquad \beta^2 = 24 \, \mathrm{dim} G$$

gauge group getting free

If there is a weakly coupled AdS dual, it implies a stronger version of SDC:

Infinite distance limits at fixed AdS5 radius $\Longleftrightarrow$ Tower of higher spin fields with an exponential rate:

$$\alpha = \sqrt{\frac{2c}{\mathrm{dim}G}} \quad \begin{array}{ll} \geq \frac{1}{\sqrt{3}} & \text{for 4d N=2} \\ \geq \frac{1}{2} & \text{for 4d N=1} \end{array}$$

Lower bound for α !

SDC in 4d N=1 EFTs

1) There could be other towers of states getting light, but the leading one scales like

$$m_*^2 \simeq M_{\mathrm{P}}^2 A \left(\frac{T}{M_{\mathrm{P}}^2} \right)^w$$

for some positive integer $w = 1, 2, \ldots$

T= string tension

In all analysed string theory examples: $w = 1, 2, 3$??

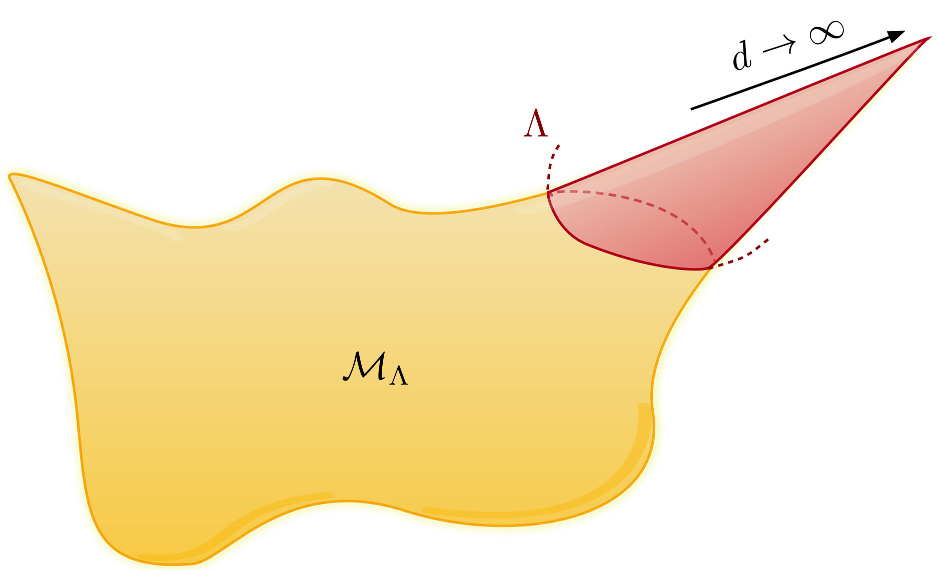
dualities?

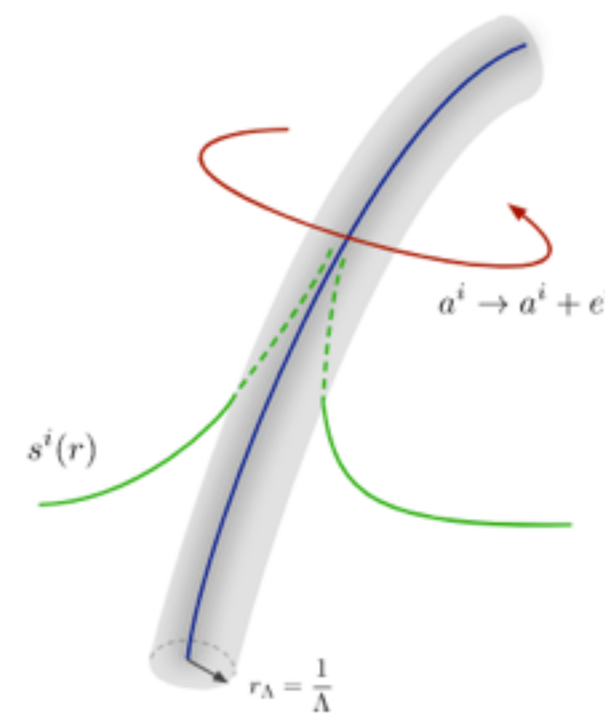
SDC in 4d N=1 EFTs

2) Distant Axionic String Conjecture:

All infinite distance limits in 4d EFTs can be realised as RG flow endpoints of an axionic EFT string

Bottom-up derivation of SDC using WGC: $\alpha \geq \left.\frac{Q}{T}\right|_{extr.}$





Same philosophy than [Klaewer,Palti'16],[Draper,Farkas'19]

What happens in the presence of a potential?

Everything holds as long as the energy scale remains below the cut-off

(so the infinite distance is not obstructed by the potential)

$$W = W_{\text{flux}}(\phi) + \mathcal{O}(e^{2\pi i m_i t^i})$$

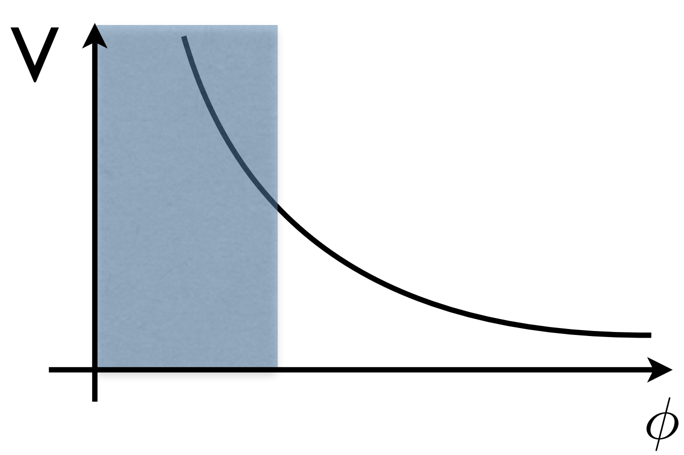
EFT strings still get tensionless inducing the EFT breakdown, but they get attached to membranes

We can translate properties of the flux potential to the membranes:

$$T_{\text{mem}} = 2M_p^3 e^{K/2}|W(q)| \qquad Q^2 = 2V(q) \quad \Rightarrow \quad \|\partial T_{\text{mem}}\|^2 - \frac{3}{2}T_{\text{mem}}^2 = M_{\text{P}}^2 Q_{\mathbf{q}}^2$$

They induce a runaway for the scalar potential

[Lanza,Marchesano,Martucci,IV'20]



- *It follows from a membrane saturating the WGC*
- *Asymptotic version of de Sitter conjecture* [Obied et al'18] [Ooguri et al'18]
- *Tests in string theory (classical no-go's)*

[Hertzberg et al'07] [Wrase et al'10]… [Grimm,Li,IV'19]