OFF-SHELL CONIFOLD POTENTIALS

in progress
with L. Randall

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Geometry, Strings and the Swampland

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de Sitter vacua

$\rightarrow$ challenge for String Phenomenology

- notoriously difficult to construct (contrary to $\Lambda \leq 0$)
- constrained by various no-go theorems, require stringy ingredients (O-planes, D-branes)
- conjectured not to exist at all (de Sitter conjecture) [Obied et al. '18, Ooguri et al. '18]

KKLT DE SITTER VACUA

Three step procedure [KKLT '03]

1. warped IIB with CS-moduli stabilized by three-form fluxes
   + region with strong warping [GKP]
   described by Klebanov-Strassler throat [Klebanov, Strassler '00]
   → large hierarchy of scales

2. Stabilize Kähler moduli by non-perturbative effets
   → supersymmetric AdS-vacuum

3. Supersymmetry breaking by an anti-D3-brane at the bottom of the throat
   → exp. suppressed uplift to dS due to strong warping

CHALLENGES OF KKLT

- Bottom-up (EFT) construction with top-down ingredients!
- general scenario, concrete realization more difficult (flux landscape, tadpole problem, …)
- interaction between ingredients not always well understood

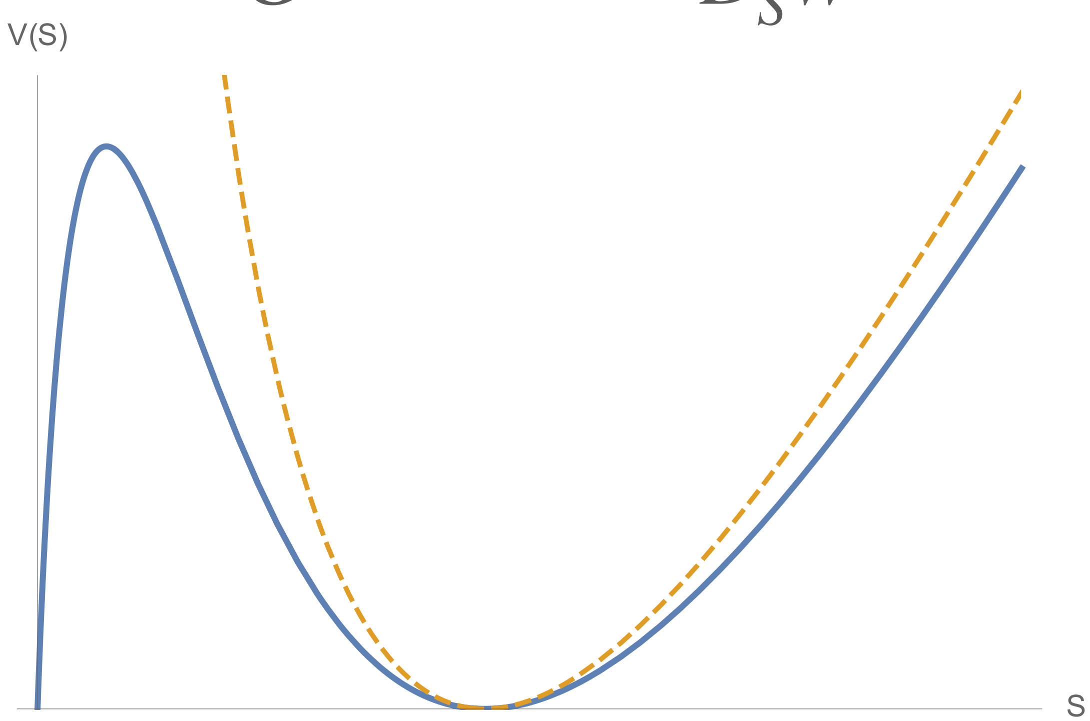
Here: back-reaction of anti-brane on complex structure stabilization

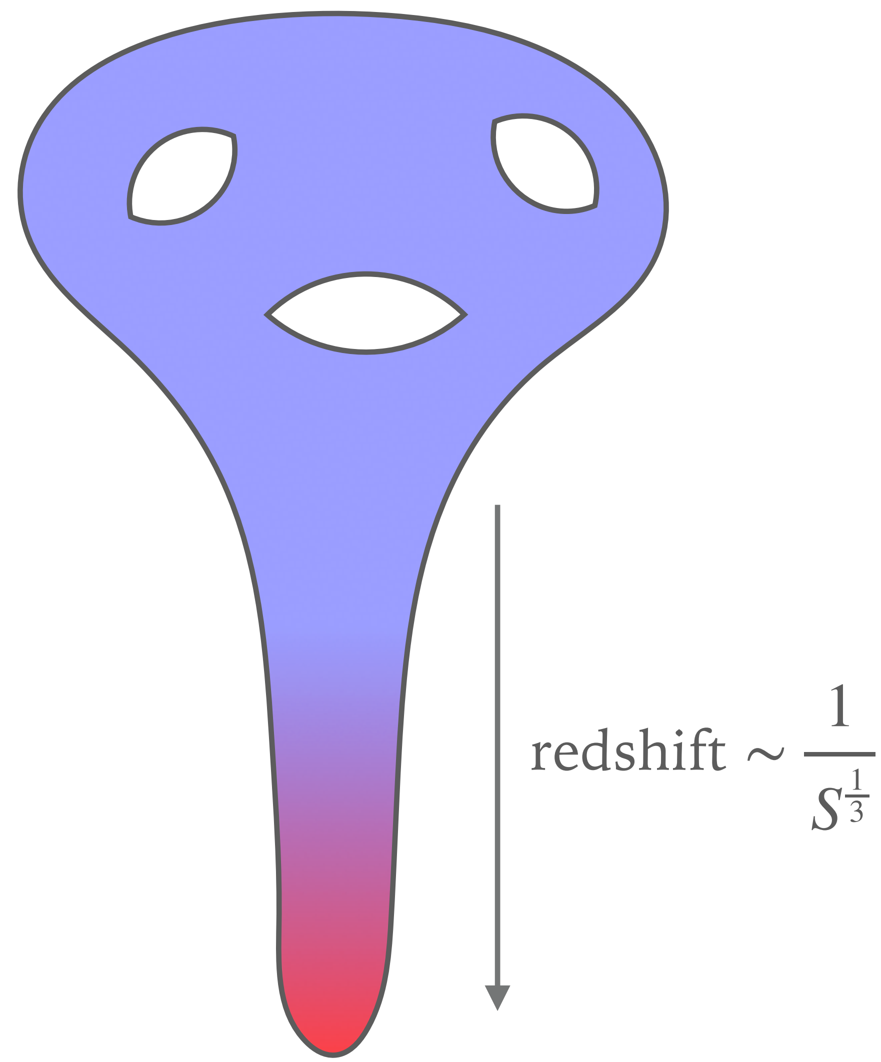
UPLIFTING RUNAWAYS

[Bena, Dudas, Graña, SL '18]

- fluxes (M, K) along the three-cycling of the deformed conifold generate a potential for the CS modulus S:
  [Douglas, Shelton, Torroba, '07, '08]

$$V_{KS} \sim \underbrace{\frac{\left|S\right|^{4/3}}{g_s(\alpha' M)^2}}_{G^{S\bar{S}}} \underbrace{\left|\frac{M}{2\pi i}\log\frac{\Lambda_0^3}{S} + i\frac{K}{g_s}\right|^2}_{D_S W}$$



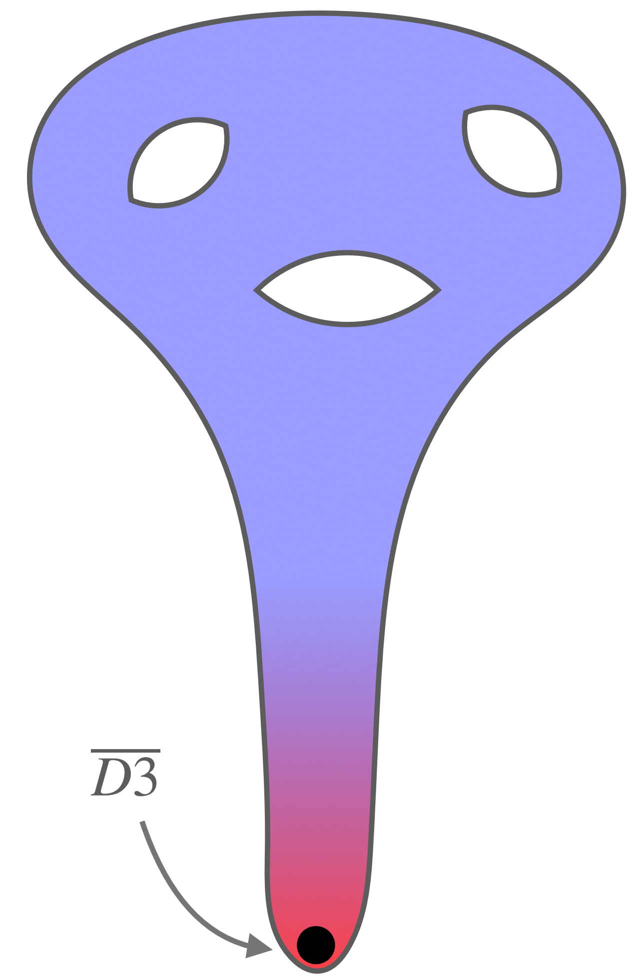


(see also [Blumenhagen, Kläwer, Schlechter '19], [Bena, Buchel, SL '19], [Randall '19])

UPLIFTING RUNAWAYS

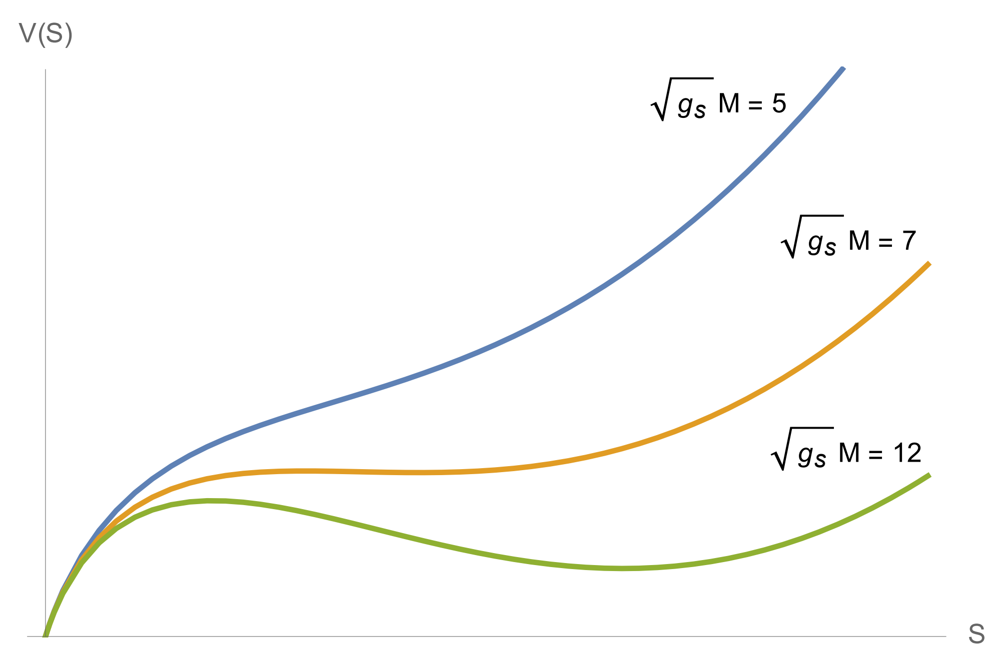
[Bena, Dudas, Graña, SL '18]

- addition of an anti-brane:
  contributes to the potential:



$$V_{\overline{D3}} \sim 2e^{4A}T_{D3} \sim \frac{|S|^{4/3}}{g_s(\alpha' M)^2}$$

- combined potential:



- runaway to infinite throat ($S \to \infty$ = KT-solution) unless:

$$g_s M^2 > \gamma_{\overline{D3}}^2 N_{\overline{D3}}\,, \qquad \gamma_{\overline{D3}} \approx 6.8$$

THE WARPED CONIFOLD POTENTIAL

A CLOSER LOOK AT THE CONIFOLD POTENTIAL

Potentials from fluxes:

- Moduli of Calabi-Yau compactifications:
  - complex structure (volumes of three-cycles)
  - Kähler (volumes of two/four-cycles)
- magnetic flux on a cycle:

$$\int_C F \in \mathbb{Z}_{\neq 0} \qquad \rightarrow \text{ stabilize the corresponding modulus}$$

- IIB: three-form fluxes G_3 $\rightarrow$ stabilize complex structure

$$V \sim \int_{CY} e^{4A} G_3 \wedge (\star \overline{G}_3 + i\overline{G}_3)$$

[Shiu, Torroba, Underwood, Douglas '08]

A CLOSER LOOK AT THE CONIFOLD POTENTIAL

- Flux potential as a (no-scale) F-term potential:

$$V \sim \int_{CY} e^{4A} G_3 \wedge (\star \overline{G}_3 + i \overline{G}_3) \sim e^K G^{I\bar{J}}\, D_I W\, \overline{D}_{\bar{J}} \overline{W}$$

- Expand G_3 in terms of harmonic forms:

$$W = \int_{CY} G_3 \wedge \Omega \qquad\qquad K = -\log \int_{CY} \Omega \wedge \overline{\Omega}$$

[Gukov, Vafa, Witten ’99]

- Introduction of a non-trivial warp factor ($ds_4^2 \to e^{2A} ds_4^2$)

$$W\!: \text{ unchanged (holomorphic)}$$

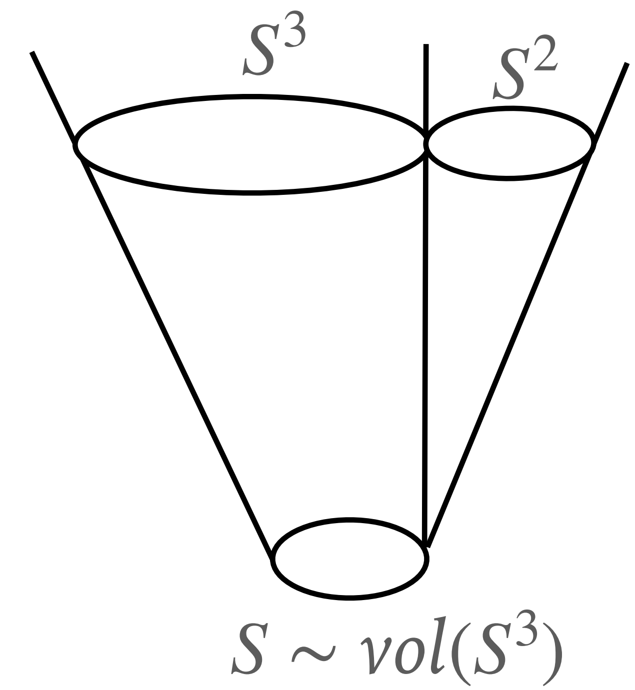
$$K = -\log \int_{CY} e^{-4A}\, \Omega \wedge \overline{\Omega}$$

[DeWolfe, Giddings ’02]

A CLOSER LOOK AT THE CONIFOLD POTENTIAL

- deformed conifold:

$$\sum_{i=1}^{4} z_i^2 = S$$



$\rightarrow$ S = complex structure modulus

- Superpotential:

$$W \sim \int G_3 \wedge \Omega = \frac{M}{2\pi i} S\left(\log \frac{\Lambda_0^3}{S} + 1\right) + \frac{i}{g_s} KS$$

- Kähler potential requires knowledge of warp factor: Klebanov-Strassler solution:

$$e^{-4A(\tau)} \sim \frac{g_s(\alpha' M)^2}{|S|^{\frac{4}{3}}} I(\tau)$$

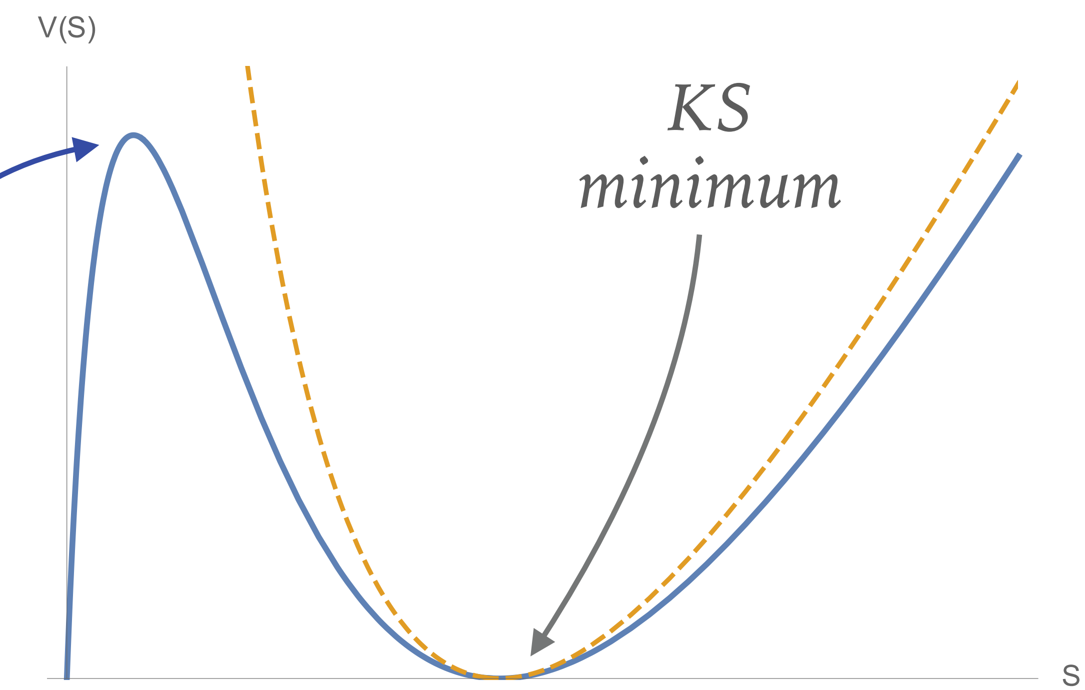
A CLOSER LOOK AT THE CONIFOLD POTENTIAL

- Kähler metric: [Douglas, Shelton, Torroba, '07, '08]

$$G_{S\bar{S}} = \partial_S \partial_{\bar{S}} K \sim \int e^{-4A} \chi_S \wedge \bar{\chi}_{\bar{S}} \approx e^{-4A(\tau=0)} \sim \frac{g_s(\alpha' M)^2}{|S|^{\frac{4}{3}}}$$

- Anti-brane instability:
  Requires knowledge of the (off-shell) potential away from the minimum!

can we use the KS warp factor here?!



EFFECTIVE POTENTIALS FORM A 5D TRUNCATION

EFFECTIVE POTENTIALS FROM 5D "FAKE" SUPERGRAVITY

[Papadopoulos, Tseytlin '00], [Berg, Haack, Mück '05, '06]

- instead of 10D IIB supergravity
  consider 5D "fake" supergravity:

$$S = \int d^5x\sqrt{g}\left[-\frac{1}{4}R + \frac{1}{2}G_{ab}\partial_A\phi^a\partial^A\phi^b + V(\phi)\right]$$

with superpotential: $$V(\phi) = \frac{1}{2}G^{ab}W_aW_b - \frac{4}{3}W^2$$

- allows for "domain wall" solutions:

$$ds^2 = e^{2A(\tau)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2f(\tau)}d\tau^2 \qquad \dot{A} = -\frac{2}{3}e^f W$$

$$\phi^a = \phi^a(\tau) \qquad \dot{\phi}^a = e^f G^{ab}W_b$$

- related to Klebanov-Strassler geometry as a consistent truncation exploiting $SU(2)\times SU(2)\times \mathbb{Z}_2$ symmetry

EFFECTIVE POTENTIALS FROM 5D "FAKE" SUPERGRAVITY

- introduce dependence on the 4D coordinates according to

$$\phi^\alpha(\tau) \to \phi^\alpha\left[\tau, u^I(x^\mu)\right]$$

$$ds^2 \to e^{2A\left(\tau,\, u^I(x)\right)} g_{\mu\nu}(x) dx^\mu dx^\nu + \left[e^{f\left(\tau,\, u^I(x)\right)} d\tau + K_\mu(\tau, x) dx^\mu\right]^2$$

- inserting in the 5D action gives: (assuming a suitable UV cutoff)

$u^I(x^\mu)$: 4D fields

$$S = \frac{1}{2}\int d^4x \sqrt{g^{(4)}} \int d\tau \Bigg[e^{2A+f}\left(-\frac{1}{2}R^{(4)} + \partial_i u^I \partial^i u^J \left(3\partial_I A \partial_J A + 3\partial_I A \partial_J f - g_{ab}\partial_I \phi^a \partial_J \phi^b\right)\right)$$

$$\underbrace{+e^{4A+f}\left(6(D_\tau A)^2 - g_{ab} D_\tau \phi^a D_\tau \phi^b - 2V(\phi)\right)}_{\text{effective 4D potential?!}}\Bigg]$$

effective 4D potential?!

EFFECTIVE POTENTIALS FROM 5D "FAKE" SUPERGRAVITY

- Need to impose 4D Einstein equations as a constraint:

$$R^{(4)} - 6\,\Box\, u^I D_I A - 2\partial_\mu u^I \partial^\mu u^J \Big[3D_I D_J A + 3D_I A D_J A + g_{ab} D_I \phi^a D_J \phi^b\Big]$$

$$-2e^{2A}\left[6(D_\tau A)^2 - g_{ab} D_\tau \phi^a D_\tau \phi^b + 2V(\phi)\right] = 0$$

- Puzzle: warp factor A depends on 4D momenta and curvature!

But: A needed for computation of potential?!

- Potential: $V = -\mathscr{L}\Big|_{\partial_\mu u = 0}$

$\rightarrow$ solve constraint at $R^{(4)} = C,\ \partial_\mu u = \nabla^i \partial_i u = 0$

EFFECTIVE POTENTIALS FROM 5D "FAKE" SUPERGRAVITY

[Douglas '09]

- Constraint on the warp factor (at vanishing 4D momenta):

$$0 = \frac{\delta S}{\delta A} = 3D_\tau^2 A + 6(D_\tau A)^2 + g_{ab} D_\tau \phi^a D_\tau \phi^b + 2V(\phi) - 3e^{-2A} C$$

- Normalization: Constant 4D Planck mass:
  (can also be seen from 5D eom)

$$0 = \delta\, G_N = \frac{V_W}{M_5^3} = \frac{1}{M_5^3} \int d\tau\, e^{2A+f}$$

- Re-insert into action: Resulting potential:

$$V = \frac{V_W}{M_5^3} C$$

APPLICATION TO DEFORMED CONIFOLD

- Consider a GKP like setup with strongly warped region
- Strongly warped region: Klebanov-Strassler:
  space-time independent solution of fake SUGRA:

$$\phi^a_{KS}(\tau)\,, \quad A_{KS}(\tau)$$

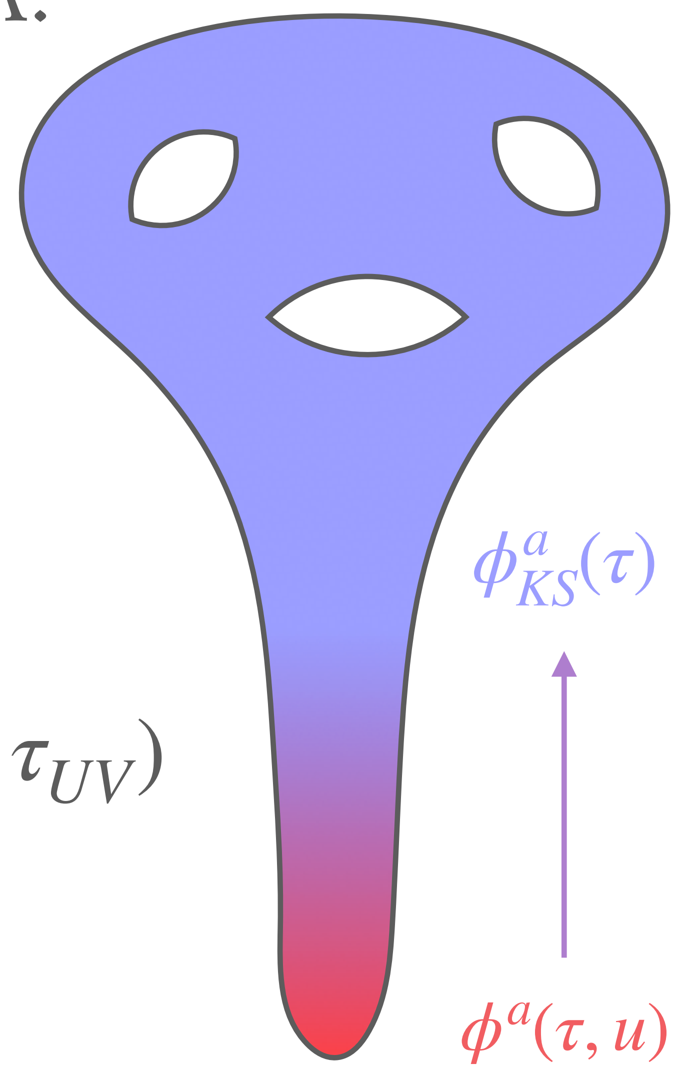
- consider a one-parameter family of field configurations

$$\phi^a(\tau, u)$$

  such that in the UV: $\phi^a(\tau \to \tau_{UV}, u) = \phi^a_{KS}(\tau \to \tau_{UV})$

- use the constraint to determine $A(\tau, u)$

$$\to \text{potential } V(u)$$



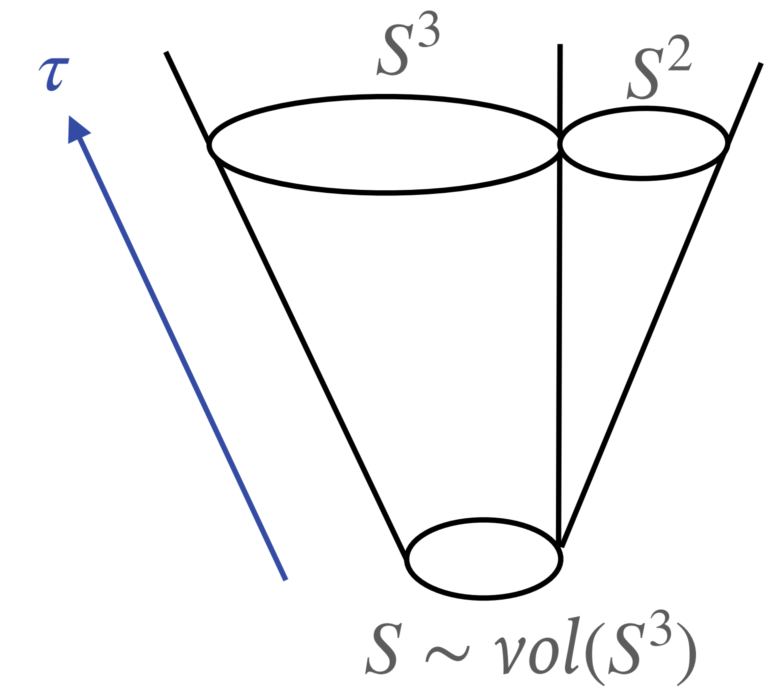
COMPLEX STRUCTURE OF THE DEFORMED CONIFOLD

COMPLEX STRUCTURE OF THE DEFORMED CONIFOLD

- Deformed conifold in $\mathbb{C}^4$:

$$\sum_{i=1}^{4} z_i^2 = S$$

S = complex structure modulus



- metric on the deformed conifold:

$$ds^2_{DC} = \frac{S^{2/3}}{2} K(\tau) \left[\frac{1}{3K^3(\tau)} \left[d\tau^2 + (g^5)^2 \right] + \cosh^2\left(\frac{\tau}{2}\right) \left[(g^3)^2 + (g^4)^2 \right] + \sinh^2\left(\frac{\tau}{2}\right) \left[(g^1)^2 + (g^2)^2 \right] \right]$$

complex structure just a conformal factor?!

Answer: did not fix gauge (coordinates) yet!

COMPLEX STRUCTURE OF THE DEFORMED CONIFOLD

- First: understand gauge fixing without warping:

$$ds_{10}^2 = ds_4^2 + ds_{DC}^2$$

- Gauge fixing of Calabi-Yau deformations:

$$g_{ij} \to g_{ij} + \delta g_{ij}$$

[Candelas, de la Ossa '91]

$\Rightarrow$ $g^{ij}\delta g_{ij} = 0$ *(traceless)* $\nabla^i \delta g_{ij} = 0$ *(harmonic)*

(will get modified in the presence of warping!)

[Giddings, Maharana '05],
[Shiu et al. '08],
[Douglas, Torroba '08]

- Deformed conifold:

$$\delta g_{ij} = \partial_S g_{ij} \sim \frac{1}{S} g_{ij}$$

harmonic but not traceless!

COMPLEX STRUCTURE OF THE DEFORMED CONIFOLD

- Add compensating diffeomorphism:

$$\delta g_{ij} = \partial_S g_{ij} + 2\,\nabla_{(i}\eta_{j)}$$

Ansatz:

$$\eta = \left(\eta^\tau(\tau),0,0,0,0,0\right)$$

Solution:

$$\eta^\tau(\tau) = -\frac{1}{2S}\frac{\sinh(2\tau) - 2\tau}{\sinh^2\tau}$$

- Interpretation:

Replace τ with “new” S-dependent radial variable: $\tau \to T(\tau, S)$

$$\frac{dT}{dS} = \eta^\tau\big(T(\tau,S),S\big) \longrightarrow$$

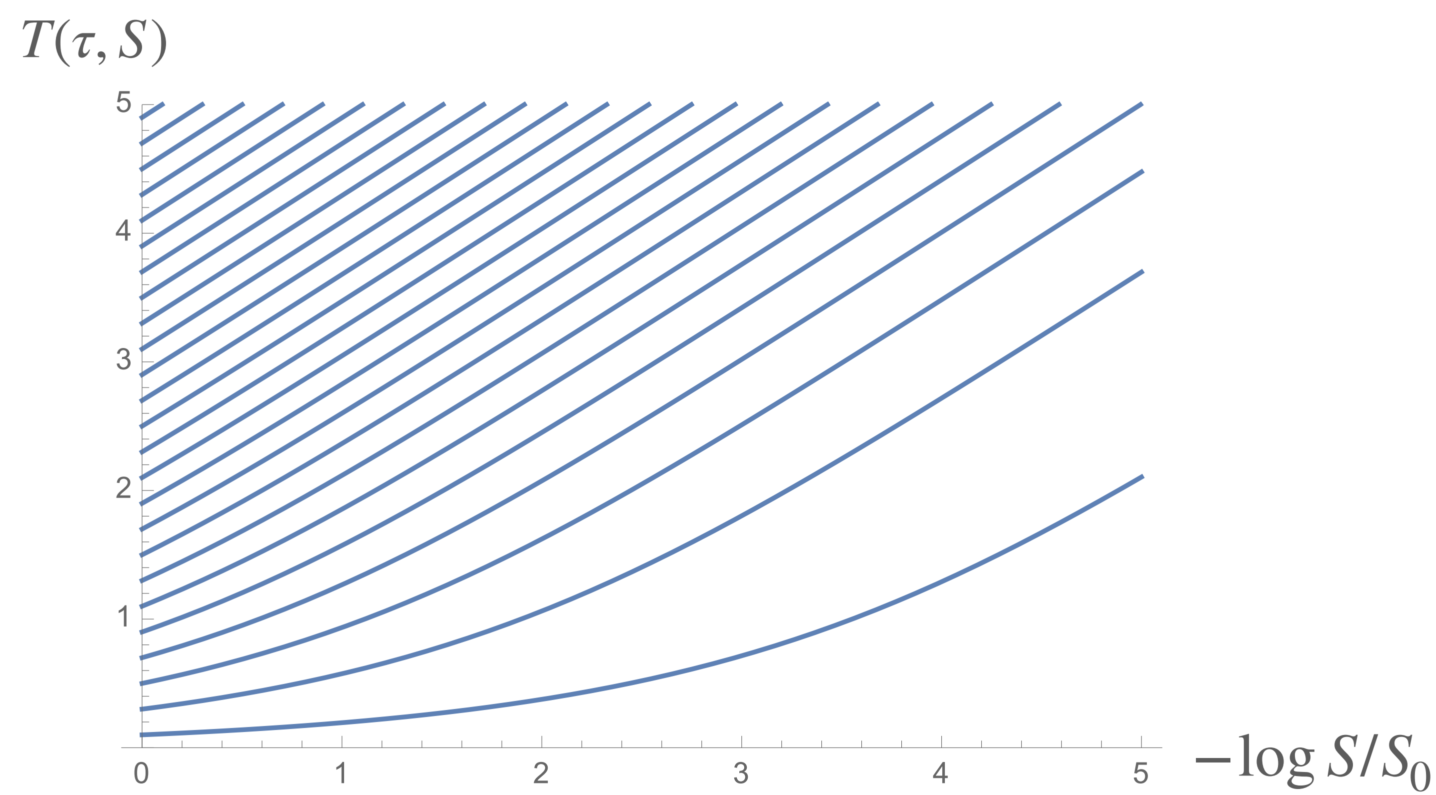
Analytic solution:

$$T(\tau,S) = F\left[F^{-1}(\tau) - \tfrac{1}{4}\log\tfrac{S}{S_0}\right]$$

with $F(x) = \dfrac{1}{2}\log\left[\sinh(2x) - 2x\right]$

COMPLEX STRUCTURE OF THE DEFORMED CONIFOLD

- The radial coordinate as a function of S:



- UV behavior ($\tau \to \infty$): $T(\tau, S) \to \tau - \log S/S_0$
- Compare with UV expansion of the metric:

$$ds^2_{DC}(\tau \to \infty) \to S^{\frac{2}{3}} e^{2T/3} \left(\frac{1}{9} dT^2 + \frac{1}{6} ds^2_{T^{1,1}} \right) = S_0^2 e^{2\tau/3} \left(\frac{1}{9} d\tau^2 + \frac{1}{6} ds^2_{T^{1,1}} \right)$$

Deformation acts only in the IR!

A POTENTIAL FOR THE WARPED DEFORMED CONIFOLD

A POTENTIAL FOR THE WARPED DEFORMED CONIFOLD

- In the $SU(2) \times SU(2) \times \mathbb{Z}_2$ truncation:
  Choose a one-parameter family of fields $\phi^a(\tau, S)$ such that
  - metric: deformed conifold up to diffeomorphism $T(\tau, S)$
  - fluxes + dilaton: given by KS solution (unchanged)

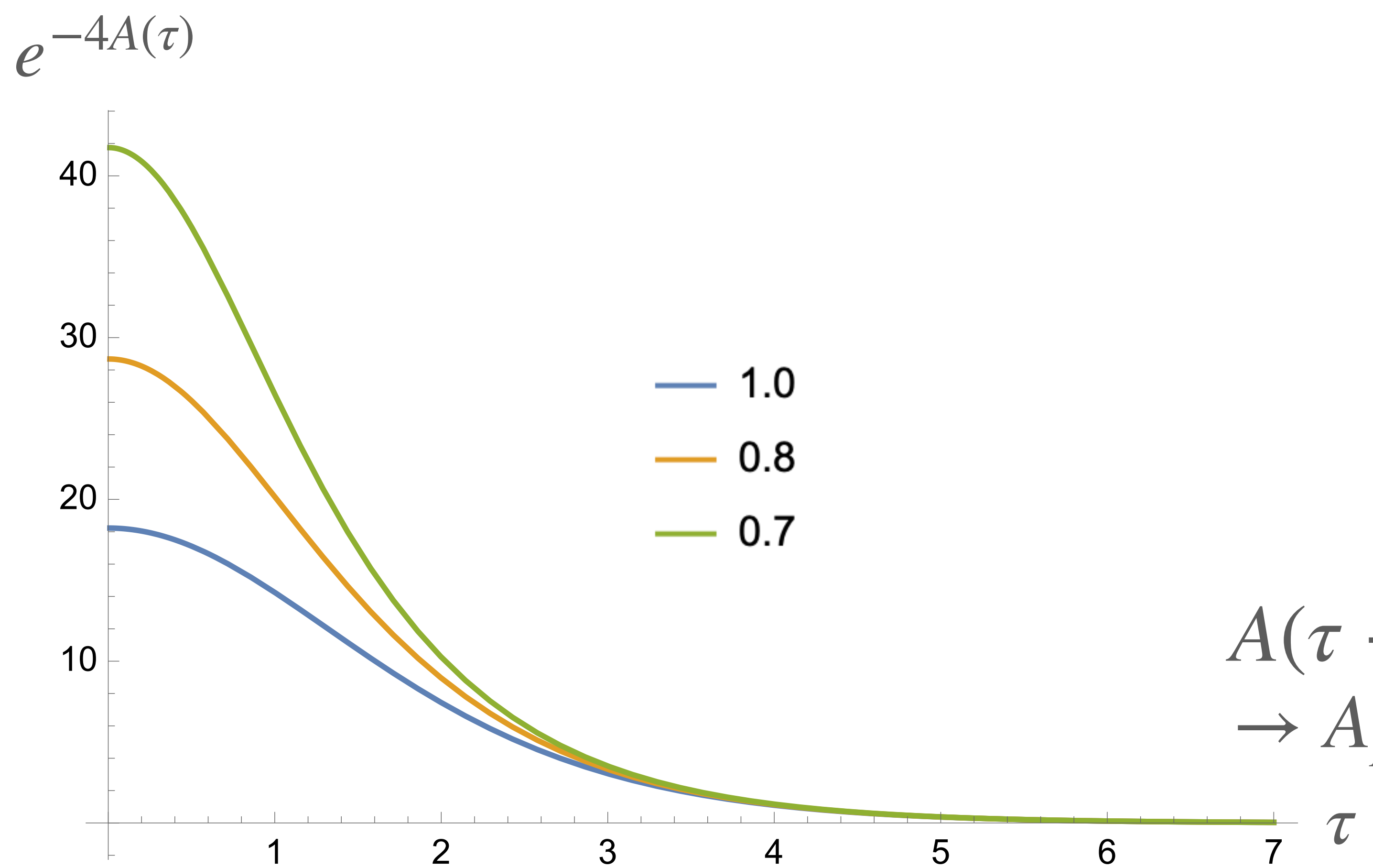
- Impose $$V_W = \int d\tau \, e^{2A+f} = \text{const}$$

  $\rightarrow$ fixes $A(\tau, S)$ in terms of $T(\tau, S)$

- use the constraint to determine $T(\tau, S)$ and hence $V(S)$

A POTENTIAL FOR THE WARPED DEFORMED CONIFOLD

➤ Numerical evaluation of the constraint yields the following warp factor:



$$A(\tau \to \tau_{UV}) \to A_{KS}(\tau_{UV})$$

➤ In the IR:

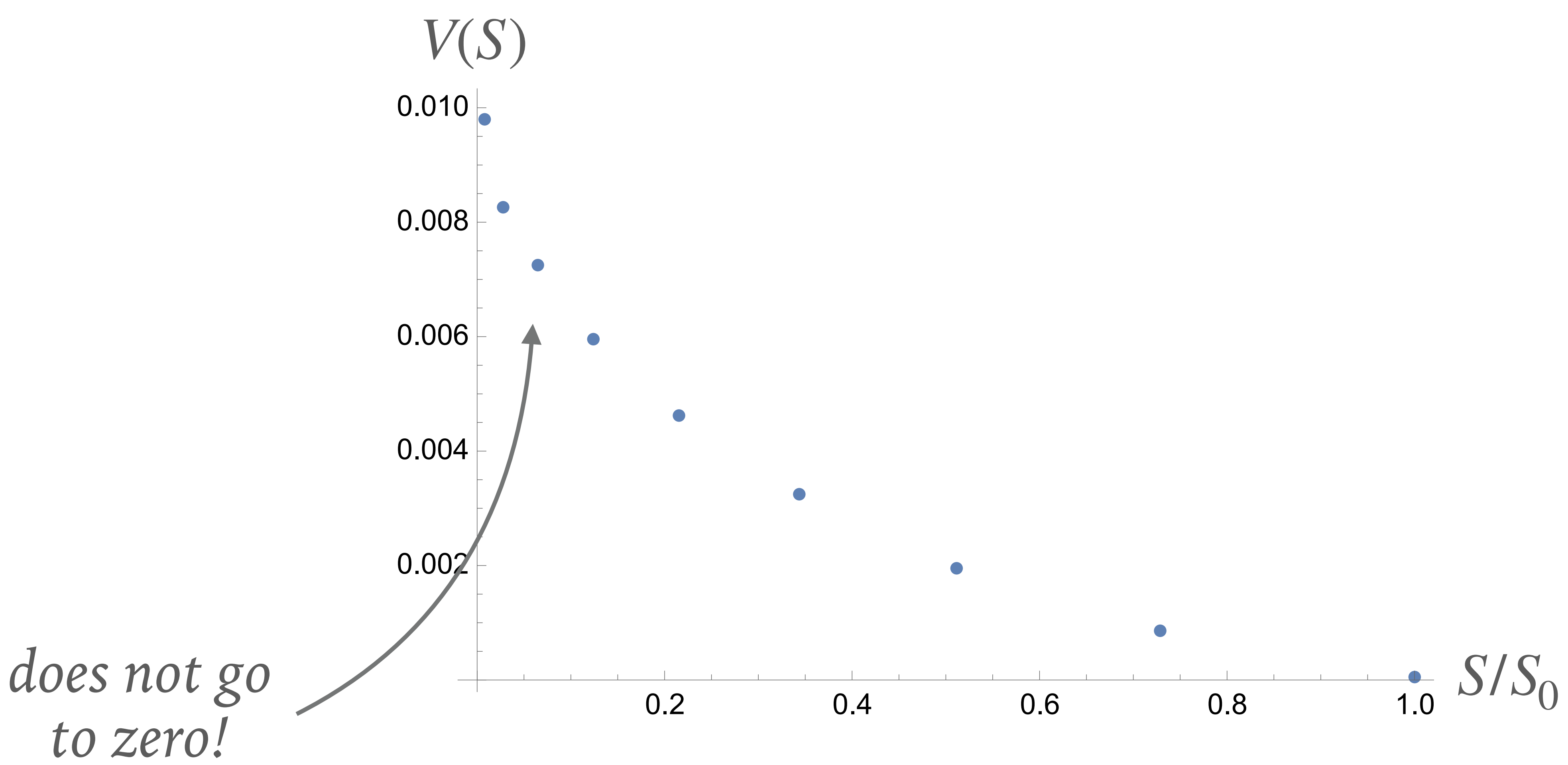
$$e^{-4A(\tau=0)} \sim \frac{1}{S}$$

(on-shell) KS warp factor:

$$e^{-4A(\tau=0)} \sim \frac{1}{S^{4/3}}$$

A POTENTIAL FOR THE WARPED DEFORMED CONIFOLD

➤ Resulting potential:



$\rightarrow$ no antibrane instability?!

Caveat: Does not take into account possible deformations of G_3 and ϕ.

CONCLUSIONS

That’s it, that’s all I wanted to say.
But before I can wish you a magnificent day,
A few more thoughts I have to convey:

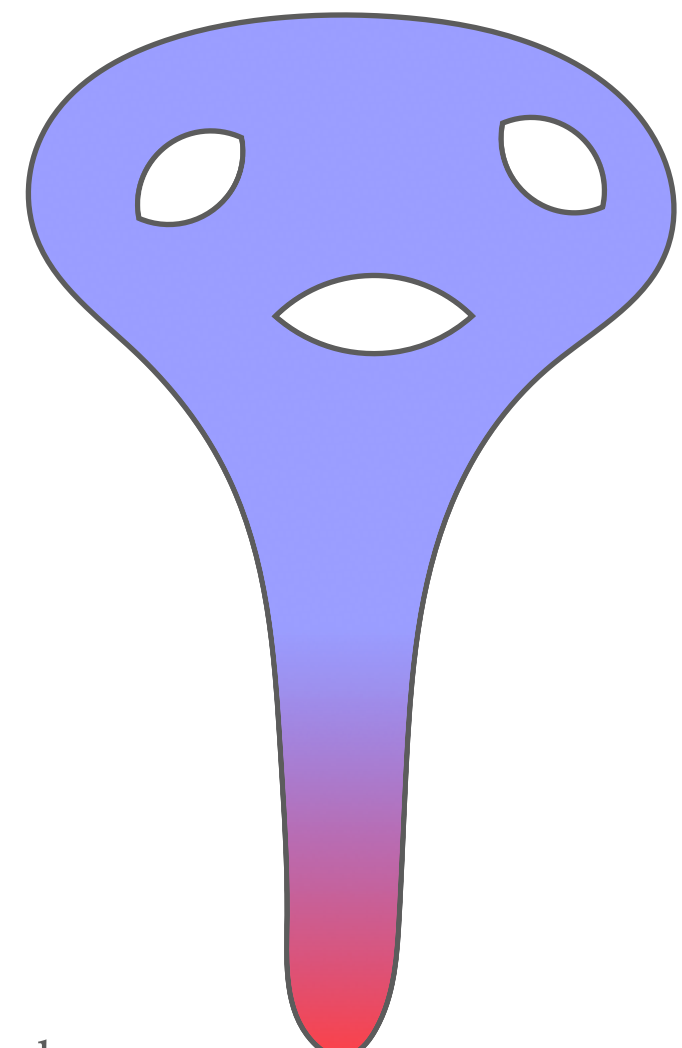
We used a potential
But it’s not evidential:

Did we do it correctly?
Have we been to directly?
Did we use all the fields?
Do we know what G_3 maybe yields?

We found a potential
And it’s very essential:

We must not forget
constraints are a threat
for systems with warping
we can’t not enforce them!

So let me tell you endmost:
My geometry looks maybe a bit like the Ringberg castle’s ghost…



THANK YOU!