Black Hole Thermodynamics & the Weak Gravity Conjecture

Gary Shiu
University of Wisconsin-Madison
Unreasonable Effectiveness of Thermodynamics

- Thermodynamics offered insights into **quantum mechanics** before a microscopic treatment was available, e.g., UV catastrophe of blackbody radiation.
- Similarly, studies of thermodynamics of black holes have lent surprising insights into **quantum gravity**, e.g.,
  - Hawking radiation: \( T_H = \frac{\hbar c^3}{8\pi GMk_B} \)
  - Bekenstein-Hawking area law: \( S = \frac{A}{4G} \)
- Holographic principle
- It seems worthwhile to leverage this unreasonable effectiveness of thermodynamics for the **swampland program**.
WGC and Black Holes

- Extremal black holes are kinematically unstable:

\[ \exists \text{ state with } \frac{q}{m} \geq \lim_{M \to \infty} \frac{Q}{M} \text{ (ext)} \]

Mild form: WGC satisfied by large but finite mass BHs.

- Consistency with dimensional reduction, modular invariance, unitarity/causality suggests stronger version e.g. Tower WGC [Andriolo, Junghans, Noumi, GS, ‘18] and sub-Lattice WGC [Heidenreich, Reece, Rudelius, ‘16];[Montero, GS, Soler, 16].

- Tower WGC is also motivated by the distance conjecture [Ooguri, Vafa, ’06] and the emergence proposal [Heidenreich, Reece, Rudelius, ‘18];[Grimm, Palti, Valenzuela, ‘18].
Evidence from String Theory

- Evidence from string theory (e.g., heterotic string spectrum [Arkani-Hamed, Motl, Nicolis, Vafa ’06]):

- While this provides evidence for stronger forms of the WGC, we need detailed knowledge of the UV complete theory (e.g. spectrum, modular invariance, ...).

- Connection to BH instability is less clear.

- What physically goes wrong if the spectrum does not follow this pattern?

Further checks in string and F-theory constructions [Lee, Lerche, Weigand; ...]

- Figure 4. The charge $M$ of the heterotic string states of charge $Q$ approaches the $M = Q$ line from below. The yellow area denotes the allowed region.
Extremal BHs

- Leading corrections to the extremality bound, e.g. in Einstein-Maxwell theory:

  \[
  \mathcal{L} = \frac{1}{2} R - \frac{1}{4} F^2 + \frac{\alpha_1}{4} (F^2)^2 + \frac{\alpha_2}{4} (FF)^2 + \frac{\alpha_3}{2} FFW
  \]

  \[
  \sqrt{2(Q^2 + P^2)} \leq M \left[ 1 + \frac{32\pi^2}{5(Q^2 + P^2)^3} \left[ 2\alpha_1(Q^2 - P^2)^2 + 2\alpha_2(2QP)^2 - \alpha_3(Q^4 - P^4) \right] \right]
  \]

- The leading corrections increase Q/M for extremal electric heterotic BHs [Yats, Motl, Padi, ‘06].

- This behavior (A) was shown [Hamada, Noumi, GS, ‘18] to follow from gravitational positivity bounds (unitarity, causality) under the assumption of Regge boundedness & gravity subdominance.

- RG running to deep IR [Charles, ‘19];[Arkani-Hamed et al, ‘21]: log running from massless fields eventually dominates if the BH is exponentially large, \( r_H \gtrsim 10^{4000} H_{\text{today}}^{-1} \).
From Black Holes to Strings

• String theory suggests a tower of superextremal states hugging extremality from below:

- The correspondence principle \cite{Susskind, '93};\cite{Horowitz, Polchinski '96}: $S_{\text{String}} = O(1) \, S_{\text{BH}}$.

- Extremal BHs with near horizon BTZ geometry: matching of anomalies ensures $S_{\text{String}} = S_{\text{BH}}$, superextremal states stay superextremal upon turning on $g_s$ \cite{Aalsma, Cole, GS, '19}.

Turn on a small string coupling

\[ g_c \sim N^{-1/4} \ll 1 \]

The excited string states turned into BHs
Charge Convexity Conjecture

- Another way to formulate the WGC is the existence of a self-repulsive state [Arkani-Hamed, Motl, Nicolis, Vafa ’06]. It was emphasized by [Palti, ’17] that the formulations differ in the presence of scalars, and further studied in [Lust, Palti, ‘17]; [Lee, Lerche, Weigand, ’18]; [Heidenreich, Reece, Rudelius, ’19], ....

- This weakly coupled gravitational picture has motivated the Charge Convexity Conjecture:

  Abelian Convex Charge Conjecture: Consider any CFT with a U(1) global symmetry. Denote by \( \Delta(q) \) the dimension of the lowest dimension operator of charge \( q \). Then this must satisfy a convex-like constraint

  \[
  \Delta(n_1q_0 + n_2q_0) \geq \Delta(n_1q_0) + \Delta(n_2q_0),
  \]

  \[(1.1)\]

  for any positive integers \( n_1, n_2 \), for some \( q_0 \) of order one.

Conjectured to hold for all CFTs, not only holographic ones.
Charge Convexity Conjecture

- For holographic CFTs, evidence of the CCC came mostly from studying the large q limit:

\[ \Delta(q) = Aq^{d/(d-1)} + \ldots \]

which coincides with the (classical) extremality bound for large AdS BHs: \( M_{\text{ext}} \sim Q_{\text{ext}}^{d/(d-1)} \)

- However, there is a regime of \( q \) where \( q \) is large compared with the central charge \( c \) but not as large as that correspond to AdS black holes: flat space BHs have classically \( M_{\text{ext}} \sim Q_{\text{ext}} \).

\[ \Delta_{\text{AdS BH}} \gg \Delta_{\text{flat space BH}} \sim \mathcal{O}(c) \]

- **Higher derivative corrections turn the extremality curve concave.** If \( \exists \) a multi-particle state with the same charge but smaller mass, the CCC is a priori not violated.

- Why should the CCC impose an opposite condition on such a multi-particle state as that on a healthy BH?
Black Hole Extremality

Figure 3: Extremal curves of two-derivative, charged black holes (in units where $\text{AdS} = \text{R}_{\text{dS}} = 1$). For flat space the curve is linear, $M = p^2 Q$, while for AdS the curve is convex, eventually growing as $M \sim Q^3/2$. For de Sitter we have the "shark fin" shape; the top edge are Nariai black holes and the lower edge is concave.

2 Thoughts

For charged black holes in the two-derivative theory the extremal curve bends away from the linear $Q \sim M$ line in a way that depends on the sign of the cosmological constant: see Figure 3.

Asymptotics

<table>
<thead>
<tr>
<th>Asymptotics</th>
<th>Extremal curve</th>
<th>Concave/convex?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat space: $\Lambda = 0$</td>
<td>$M^2_{\text{ext}} = 2Q^2$</td>
<td>$\frac{\partial^2 H}{\partial Q^2}</td>
</tr>
<tr>
<td>AdS: $\Lambda = \frac{3}{2R} &gt; 0$</td>
<td>$M^2_{\text{ext}} = \frac{4}{9}Q^2 + \frac{32\pi^2 \mathcal{R}^2}{27} \left(1 + \frac{3Q^2}{8\pi^2 \mathcal{R}^3} \right)^{3/2} - 1$</td>
<td>$\frac{\partial^2 H}{\partial Q^2}</td>
</tr>
<tr>
<td>de Sitter: $\Lambda = -\frac{3}{2R} &lt; 0$</td>
<td>$M^2_{\text{ext}} = \frac{4}{9}Q^2 - \frac{32\pi^2 \mathcal{R}^2}{27} \left(1 - \frac{3Q^2}{8\pi^2 \mathcal{R}^3} \right)^{3/2} - 1$</td>
<td>$\frac{\partial^2 H}{\partial Q^2}</td>
</tr>
</tbody>
</table>

Table 1: The expressions for de Sitter correspond to the "maximal charge" branch of the extremal, $T = 0$ curve of Figure 3, rather than the Nariai branch.

The expectation from WGC in flat space is that $M_{\text{ext}}$ vs. $Q$ shifts "downward" so that $M_{\text{ext}}(Q)$ is a concave function of $Q$. This phrasing cannot immediately extend to (A)dS, since the leading-order extremal curve in AdS is already convex and the leading-order extremal curve in dS is far from the $M \sim Q$ line.

Let’s explore different statements of stability and see how they extend to $\Lambda \neq 0$.
Monotonicity Continues

• We have evidence that this monotonicity continues from flat space BHs to large AdS BHs [Loges, Noumi, GS, work in progress]. This result if universal has consequences on $1/q$ corrections to $\Delta(q) \geq O(c)$ operators in the dual CFT (to be formulated more precisely).

\[ M = Q \]

This monotonicity implies a **Tower WGC**

• Here, we present evidence for this monotonic behavior (on the gravity side) based on earlier works on the thermodynamics of (axio)-dilatonic Einstein-Maxwell BHs [Loges, Noumi, GS, '19]; [Loges, Noumi, GS, '20] and BTZ black holes [Aalsma, Cole, Loges, GS, '20].
BH Thermodynamics and WGC
Corrections to Extremality

- Higher order corrections change the BH solution and the extremality bound. Finding corrected solutions is often intractable (especially with additional scalars).

- Thermodynamic approach sidesteps this difficulty:

\[ Z = \text{Tr} \left( e^{-\beta (H - \Phi Q)} \right) = \sum_{\text{saddles}} e^{-I_E} \equiv e^{-\beta G(T,\Phi,P)} \]

\[ G = M - TS - Q\Phi \quad dG = -SdT - Qd\Phi + \Psi dP \]

- Evaluating free energy \( G \) to first order requires only the uncorrected solution:

\[ I_E = I_E^{(0)} + \alpha I_E^{(i)} + \mathcal{O}(\alpha^2) \]

\[ I_E[\phi^{(0)} + \alpha \delta \phi] = I_E^{(0)}[\phi^{(0)} + \alpha \delta \phi] + \alpha_i I_E^{(i)}[\phi^{(0)} + \alpha \delta \phi] + \mathcal{O}(\alpha^2) = I_E^{(0)}[\phi^{(0)}] + \alpha_i I_E^{(i)}[\phi^{(0)}] + \mathcal{O}(\alpha^2) \]

- Subtleties for boundary terms & counter-terms [Reall Santos '19]
Ensembles

- WGC naturally phrased at fixed Q, P and $T \to 0$.
- Correlated with $\Delta S|_{z=1} > 0$ (microcanonical ensemble) [Hamada, Noumi, GS, '18];[Goon, Penco, '19]
Einstein-Maxwell-Dilaton

- Given the importance of scalars in formulating the WGC, consider:

\[ \mathcal{L} = \frac{1}{2} R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{4} e^{-2\lambda \phi} F^2 + \mathcal{O}(\alpha) \text{ corrections} \]

- Zeroth-order dyonic BH solutions (\( \lambda^2 = 1/2 \)):

\[ ds^2 = -\frac{r(r - \xi)}{(r + P_e)(r + P_m)} dt^2 + \frac{(r + P_e)(r + P_m)}{r(r - \xi)} dr^2 + (r + P_e)(r + P_m) d\Omega^2_2 \]

\[ F = \frac{Q}{4\pi (r + P_e)^2} dt \wedge dr + \frac{P}{4\pi} d(\cos \theta) \wedge d\phi \]

\[ e^{-2\lambda \phi} = \frac{r + P_e}{r + P_m} \quad \rightarrow \quad Q^2 = (4\pi)^2 P_e (P_e + \xi), \quad P^2 = (4\pi)^2 P_m (P_m + \xi) \]

- Horizons: \( r = 0, \xi \) \quad Extremality: \( \xi \rightarrow 0^+ \)
Scalar WGC

- Uncorrected solution satisfies the extremality bound:

\[ M^2 = (4\pi \xi)^2 + 2 \left( Q^2 + P^2 - Q_\phi^2 \right) \geq 2 \left( Q^2 + P^2 - Q_\phi^2 \right) \]

- Long range scalar force: \( e^{-2\lambda \phi} \sim 1 + \frac{Q_\phi}{r} + \ldots \)

- \( Q_\phi \propto Q - P \) is not an independent parameter (no scalar hair), in particular \( M^2 \geq 0 \)

- Scalar WGC is satisfied by BHs if the corrections make:

\[ M^2 + 2Q_\phi^2 < 2 \left( Q^2 + P^2 \right) \]
Thermodynamics (Grand Canonical Ensemble)

- From $I_E$, it is most direct to obtain the thermodynamic functions in the GCE:

\[
G(T, \Phi, P) = \frac{1 - \Phi^2}{2T} + \frac{P^2T}{2(1 - \Phi^2)} + \mathcal{O}(\alpha) \text{ corrections}
\]

\[
S(T, \Phi, P) = \frac{1 - \Phi^2}{2T^2} - \frac{P^2}{2(1 - \Phi^2)} + \mathcal{O}(\alpha) \text{ corrections}
\]

\[
Q(T, \Phi, P) = \frac{\Phi}{T} - \frac{P^2\Phi T}{2(1 - \Phi^2)^2} + \mathcal{O}(\alpha) \text{ corrections}
\]

\[
\Psi(T, \Phi, P) = \frac{PT}{1 - \Phi^2} + \mathcal{O}(\alpha) \text{ corrections}
\]

\[
M(T, \Phi, P) = \frac{1}{T} - \frac{P^2\Phi^2 T}{(1 - \Phi^2)^2} + \mathcal{O}(\alpha) \text{ corrections}
\]

Extremality:

\[
T \to 0, \quad \Phi^2 \to 1, \quad \frac{T}{1 - \Phi^2} \text{ fixed}
\]
Thermodynamics (Canonical Ensemble)

• The WGC is most naturally phrased at fixed $Q, P$ and $T \to 0$ (hence in CE):

\[
G(T, Q, P) = P \left[ 1 + \frac{1}{8} Q^2 T^2 + \ldots \right] + \mathcal{O}(\alpha) \text{ corrections}
\]

\[
S(T, Q, P) = \frac{QP}{2} \left[ 1 + \frac{1}{2} (Q + P)T + \ldots \right] + \mathcal{O}(\alpha) \text{ corrections}
\]

\[
\Phi(T, Q, P) = \left[ 1 - \frac{1}{2} PT + \ldots \right] + \mathcal{O}(\alpha) \text{ corrections}
\]

\[
\Psi(T, \Phi, P) = \left[ 1 - \frac{1}{2} QT + \ldots \right] + \mathcal{O}(\alpha) \text{ corrections}
\]

\[
M(T, Q, P) = (Q + P) \left[ 1 + \frac{1}{8} QPT^2 + \ldots \right] + \mathcal{O}(\alpha) \text{ corrections}
\]

Extremality:

\[ T \to 0 \]
Leading Corrections

- Seven independent 4-derivative operators:

\[ \alpha_i I_i \equiv \int d^4 x \sqrt{-g} \left[ \frac{\alpha_1}{4} e^{-6\lambda \phi} (F^2)^2 + \frac{\alpha_2}{4} e^{-6\lambda \phi} (\tilde{F}^2)^2 + \frac{\alpha_3}{2} e^{-4\lambda \phi} (FFW) + \frac{\alpha_4}{2} e^{-2\lambda \phi} (R_{GB}) + \frac{\alpha_5}{4} e^{-2\lambda \phi} (\partial \phi)^4 + \frac{\alpha_6}{4} e^{-4\lambda \phi} (\partial \phi)^2 (F^2) + \frac{\alpha_7}{4} e^{-4\lambda \phi} (\partial \phi \partial \phi FF) \right] \]

- We have computed all thermodynamic quantities to linear order in \( \alpha_i \) [Loges, Noumi, GS, ’19], e.g. for the \( F^4 \) operator:

\[ M(T, Q, P) = (Q + P) - \frac{32\pi^2 \alpha_i}{5QP} \frac{(1 - \zeta)(8 + 103\zeta - 137\zeta^2 - 37\zeta^3 + 3\zeta^4) + 60\zeta(1 - 2\zeta^2) \log \zeta}{6(1 + \zeta)(1 - \zeta)^5} \]

with \( \zeta = P/Q \).
A Positivity Puzzle

Figure 3: The functions $M_i(\zeta)$, with solid and dashed lines indicating positive and negative values respectively. Only $M_2$ is nonzero for $\zeta \neq 1$.

The magnetic limit here should not be taken too seriously, since the extremal limit is not captured by the expansions of section 4.1 and there is no reason to expect that the extremal and $\zeta \rightarrow 1$ limits commute.

For the entropy corrections, inverting $M_i = (Q^e + Q^m)$ gives

$$z_{ext} = 1 + \frac{32\pi^2}{5QP} \alpha_i M_i(\zeta)$$

Gravitational positivity bound: $\alpha_{1,2,5,7} > 0$
Staying Positive

• Under the same assumption of Regge boundedness and gravity subdominance (e.g. integrating out scalars or UV completions with open string Regge tower), $z_{ext} > 1$.

• Further non-trivial checks [Loges, Noumi, GS, ’20]: including axion but with symmetries in action
  • $SL(2,\mathbb{R})$ (broken to $SL(2,\mathbb{Z})$ by non-perturbative effects): positivity bounds $\Rightarrow z_{ext} > 1$
  • $O(d, d, \mathbb{R})$: gravitational 4-derivative terms are not subdominant, but WGC follows from NEC.

• $N \geq 2$ SUSY: puzzling term which give $\Delta z_{ext} < 0$ are necessary to ensure that the extremality bound is uncorrected for BPS states.

• For healthy theories, the leading corrections shift the extremality bound positively, making BHs a WGC state.
Covariant Formulation of WGC
A New Spin on the WGC

- We reformulate the WGC as a **covariant integrated condition**:

  \[
  \int \frac{d^{d-1}x}{\Sigma} \sqrt{h} \, \delta T^{\text{eff}}_{ab} \, \xi^a n^b \leq 0
  \]

  derived using the **covariant phase formalism** of Iyer-Wald.

- Construct a Hamiltonian generating a diffeomorphism parametrized by \( \xi \) and gauge transformation \( A \to A + d\lambda \).

- The Hamiltonian obeys a conservation law on-shell. Off-shell variation gives:

  \[
  \left( \int_{S_{\infty}^{d-2}} - \int_{S_{\text{hor}}^{d-2}} \right) \delta H = \int_{\Sigma} \frac{d^{d-1}x}{\sqrt{h}} \, \delta T^{\text{eff}}_{ab} \, n^a \xi^b
  \]

  **Asymptotic charges**  **correction to the horizon**

[Aalsma, Cole, Loges, GS, '20]
A New Spin on the WGC

- **Advantages:** No need to solve the corrected Einstein equations to derive corrections to the extremality bound; Valid for any corrections, not just higher derivative corrections

\[
\left( \int_{S_\infty^{d-2}} - \int_{S_\text{hor}^{d-2}} \right) \delta H = \int_{\Sigma} d^{d-1}x \sqrt{\gamma} \delta T_{ab} \nabla^a \nabla^b
\]

- For charged 4d BH and BTZ BHs respectively, the asymptotic charges are:

\[
\partial_t \leftrightarrow M, \quad \partial_\phi \leftrightarrow J, \quad \lambda \leftrightarrow Q
\]

- Our covariant energy condition amounts to evaluating:

\[
\delta M - \Phi \delta Q + \frac{r_+}{2G_4} \delta f(r_+) = \int_{\Sigma} d^3x \sqrt{\gamma} \delta T_{ab}^{\text{eff}} n^a \xi^b, \quad \xi = \partial_t
\]

\[
\delta M - \Omega \delta J + \frac{1}{8G_3} \delta N(r_+)^2 = \int_{\Sigma} d^2x \sqrt{\gamma} \delta T_{ab}^{\text{eff}} n^a \xi^b, \quad \xi = \partial_t - \Omega \partial_\phi
\]
Spinning WGC
Spinning WGC

- WGC is a statement that charged extremal BHs are unstable.
- Q: Is there an analogous statement for rotating BHs?
- A: Probably not - heuristic motivation is gone (Penrose process).
- However, for BTZ BHs, there is a spinning WG Theorem:

  \[
  \text{Extremal BTZ BHs satisfy } \frac{J}{M} \geq \lim_{M \to \infty} \frac{J}{M}_{\text{ext}}
  \]

- Even though gravity is not dynamical in 3d, BTZ geometry is distinguished as it appears as near-horizon limit of many stringy BHs.

[Aalsma, Cole, Loges, GS, '20]
Corrections to BTZ

- Consider the 3d action on an AdS\(3\) background perturbed by the leading 4-derivative operators (purely gravitational):

\[
\int d^3x \sqrt{-g} \left[ \frac{1}{16\pi G_3} \left( R + \frac{2}{\ell^2} \right) + \alpha_1 \ell R^2 + \alpha_2 \ell R_{ab} R^{ab} \right]
\]

- Directly compute the shift in BH horizon (covariant formulation):

\[
\delta T_{ab} = - \frac{4(3\alpha_1 + \alpha_2)}{\ell^3} g_{ab} + \mathcal{O}(\alpha^2) ; \quad \int \Sigma d^2x \sqrt{h} \delta T_{ab} h^{a \xi} \xi^b = - \frac{4\pi r^2}{\ell^3} (3\alpha_1 + \alpha_2)
\]

- Redefinition of the AdS length: \(\ell' = \ell - 42\pi G_3 (3\alpha_1 + \alpha_2)\)

\[
c' = \frac{3\ell'}{2G_3} = \frac{3\ell}{2G_3} \left[ 1 - \frac{48\pi G_3 (3\alpha_1 + \alpha_2)}{\ell} \right] \quad \text{and} \quad \frac{|J_3|}{\ell M_3} \leq 1 + \frac{48\pi G_3 (3\alpha_1 + \alpha_2)}{\ell}
\]

[Aalsma, Cole, Loges, GS, '20]
Holographic RG Argument

\[ \text{AdS}_{UV}(\ell) \xrightarrow{+\delta \varphi} \text{AdS}_{IR}(\ell') \xrightarrow{\text{NEC}: \ell > \ell'} \text{AdS}_{IR}(\ell) + \text{h.d.} \]

- Not necessary to assume that the UV CFT is dual to pure Einstein gravity (though convenient because the central charge takes the Brown-Henneaux form).

- As long as the NEC is satisfied in the bulk, the c-theorem implies a decrease in central charge in the IR and an increase in extremality bound.
Charged WGC

• Many charged extremal solutions have a near horizon BTZ geometry; in fact one often uses this to compute their entropy.

• Given the entropy-extremality relation [Hamada, Noumi, GS ‘18], can we use the spinning WGC to infer the charged WGC?

• An example is the boosted 5D black string which has an M-theory origin as the intersection of three M5-branes:
Extremality and Entropy

- The boosted 5D black string is described by the 5D action:

\[
I = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left( R - \frac{3}{4} F_{MN} F^{MN} + \alpha_1 Q^2 F_{MN} F^{MN} F_{OP} F^{OP} + \alpha_2 Q^2 F_{MN} F_{OP} W^{MNOP} + \alpha_3 Q^2 E_5 \right)
\]

\( W_{MNOP} \) is the Weyl tensor and \( E_5 = R_{MNOP} R^{MNOP} - 4 R_{MN} R^{MN} + R^2 \)

### Table 1

<table>
<thead>
<tr>
<th>Region</th>
<th>( T = 0 )</th>
<th>( z = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( BTZ \times S^2 )</td>
<td>( z = 1 + \frac{8\alpha_1 + 3\alpha_2 - 12\alpha_3}{2} )</td>
<td>( z = 1 )</td>
</tr>
<tr>
<td></td>
<td>( S = 2\pi Q \sqrt{\frac{M_5}{G_5}} \left( 1 - \frac{8\alpha_1 + 3\alpha_2 - 12\alpha_3}{2} \right) )</td>
<td>( S = 2\pi Q \sqrt{\frac{M_5}{G_5}} \left( 1 + \sqrt{\frac{8\alpha_1 + 3\alpha_2 - 12\alpha_3}{2}} \right) )</td>
</tr>
<tr>
<td></td>
<td>( = \frac{\pi Q^2}{G_4} \left( 1 - \frac{8\alpha_1 + 3\alpha_2 - 12\alpha_3}{2} \right) )</td>
<td>( = \frac{\pi Q^2}{G_4} \left( 1 + \sqrt{\frac{8\alpha_1 + 3\alpha_2 - 12\alpha_3}{2}} \right) )</td>
</tr>
</tbody>
</table>

### Table 2

<table>
<thead>
<tr>
<th>Region</th>
<th>( T = 0 )</th>
<th>( z = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 4D )</td>
<td>( z = 1 + \frac{2\alpha_1 + \alpha_2}{10} )</td>
<td>( z = 1 )</td>
</tr>
<tr>
<td></td>
<td>( S = \frac{\pi Q^2}{G_4} \left( 1 - 4\alpha_1 + 4\alpha_3 \right) )</td>
<td>( S = \frac{\pi Q^2}{G_4} \left( 1 - \frac{8\alpha_1 + 3\alpha_2 - 12\alpha_3}{2} \right) )</td>
</tr>
<tr>
<td></td>
<td>( = \frac{\pi Q^2}{G_4} \left( 1 + \frac{8\alpha_1 + 7\alpha_2 + 6\alpha_3}{40} \right) )</td>
<td>( = \frac{\pi Q^2}{G_4} \left( 1 + \sqrt{\frac{8\alpha_1 + 7\alpha_2 + 6\alpha_3}{10}} \right) )</td>
</tr>
</tbody>
</table>

[Refs: Aalsma, Cole, Loges, GS, ’20]
The entropy of the BTZ and 4d charged BH agree at zero temperature, but the extremality bounds do not.

The entropy-extremality relation [Hamada, Noumi, GS ’18] is between the extremality bound (fixed $Q/J, T$) & the microcanonical entropy (fixed $Q/J, M$).

The extremality bounds for the spinning WGC and charged WGC do not line up; rather they together strengthen the WGC in 5D:

---

**Total Landscaping**

[Aalsma, Cole, Loges, GS, ’20]
Summary
Summary

- Thermodynamics of BHs has lent insights into quantum gravity. Worthwhile to leverage this effectiveness for swampland criteria such as WGC, RFC, CCC, …

- We provided evidence that the extremality curve ($M$ vs $Q$) for BHs approaches the classical extremality bound monotonically from below: (axio)-dilatonic Einstein-Maxwell BHs, BTZ BHs.

- Evidence that this monotonicity behavior continues from the flat space BH regime to the large AdS BH regime [Loges, Noumi, GS, work in progress]. If universal, this behavior has interesting consequences for $1/q$ corrections to $\Delta(q) \geq \mathcal{O}(c)$ operators in dual CFT.

- Covariant formulation of WGC in terms of effective stress tensor (Iyer-Wald).

- Spinning WGC via c-theorem and Total Landscaping Principle.