The Weak Gravity Conjecture and Axion Strings

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Based on 2012.00009/hep-th w/ Ben Heidenreich, Jacob McNamara, Miguel Montero, Matthew Reece, Irene Valenzuela

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Main Takeaways

- Chern-Weil symmetries are examples of higher-form global symmetries, with currents given by wedge products of gauge field strengths
- Many familiar phenomena in QFT and string theory can be understood in terms of Chern-Weil symmetries
- In the presence of Chern-Simons terms involving multiple gauge fields, the Weak Gravity Conjectures for these gauge fields can be mixed up with one another
- This has important implications for high-energy physics

Chern-Weil Symmetries

Higher-Form Global Symmetries

Gaiotto, Kapustin, Seiberg, Willett, '14

- In QFT, local operators may carry charge under an "ordinary" global symmetry
- A continuous global symmetry typically features a conserved (d-1)-form Noether current, satisfying

$$\mathrm{d}J_{d-1} = 0$$

- Similarly, q-dimensional operators may carry charge under a "q-form" global symmetry
- A continuous q-form symmetry typically features a conserved (d-q-1)-form Noether current, satisfying

$$\mathrm{d}J_{d-q-1} = 0$$

Chern-Weil Global Symmetries

Heidenreich, McNamara, Montero, Reece, TR, Valenzuela, '20

- G gauge theory has conserved currents of the form

$$J = \mathrm{Tr}(F^k) \ := \mathrm{Tr}(\underbrace{F \wedge F ... \wedge F}_{k})$$

- Their conservation follows from $dF = 0$ (G abelian)/the Bianchi identity $dF + [A, F] = 0$ (G non-abelian)
- They lead to $(d-2k-1)$-form global symmetries
- In 4d, $\mathrm{Tr}(F^2)$ is a 4-form, so trivially conserved Nonetheless, there is a sense in which it generates a (-1)-form symmetry, as it has quantized (integral) periods. The associated charge is instanton number.

Eliminating CW Symmetries

- Since QG does not have exact global symmetries, these CW symmetries must either be *broken* or *gauged*

Broken	Gauged
Add monopoles	Add d-4 form C_{d-4}
$dF \neq 0$	$\mathcal{L} \supset C_{d-4} \wedge F \wedge F$
$\Rightarrow d(F \wedge F) \neq 0$	$\Rightarrow F \wedge F = d(\cdots)$
$\Rightarrow$ symmetry broken	$\Rightarrow$ symmetry gauged

Breaking CW Symmetries by Unification

- Consider GUT symmetry breaking in d dimensions:

$$SU(5) \to SU(3) \times SU(2) \times U(1)$$

- UV: expect one CW current, $\mathrm{Tr}\, F^2_{SU(5)}$, gauged by C_{d-4}
- IR: expect three CW currents, $\mathrm{Tr}\, F^2_{SU(3)}$, $\mathrm{Tr}\, F^2_{SU(2)}$, $\mathrm{Tr}\, F^2_{U(1)}$
- An IR theorist might over-count CW symmetries and expect more d-4 forms than actually exist. One CW symmetry will be gauged, other CW currents in IR are broken in UV by unification

Implications of Chern-Weil Symmetries

Axions and Quantum Gravity

- Axions are ubiquitous in string compactifications
- CW currents gauged by $C_{d-4} \wedge \mathrm{Tr}(F \wedge F)$ Chern-Simons terms in d > 4 dimensions, which reduce in 4d to $\theta\, \mathrm{Tr}(F \wedge F)$
- Chern-Weil perspective helps explain prevalence of axions in quantum gravity: they are needed to remove would-be global symmetries by gauging them (not just "looking under the lamppost")

The Axion Quality Problem

- Common concern about axions for solving CP problem is the axion quality problem. Misaligned contributions to potential could spoil the solution:

$$\Lambda_{\mathrm{UV}}^4 \left[e^{-S_{\mathrm{QCD}}+i\theta} + e^{-S_{\mathrm{other}}+i\theta+i\delta} + \mathrm{h.c.}\right]$$

- If $\delta \neq 0$, need $S_{\mathrm{other}} \gg S_{\mathrm{QCD}}$
- The Chern-Weil perspective ameliorates this worry: given two kinds of instantons, expect either two different axions (both symmetries gauged), or else expect some way to transform instantons into one another (one symmetry broken)
- Suggests we only need worry about gauge sectors that can be unified with QCD

The Witten Effect

- Axion gauges (-1)-form CW symmetry via $\theta F \wedge F$
- Monopoles break (-1)-form CW symmetry via $dF \neq 0$ $\Rightarrow d(F \wedge F) \neq 0$
- Seem to gauge and break same symmetry!
- Resolution: collective coordinate σ on monopole worldline, gauged current instead given by

$$J = F \wedge F - d_A\sigma \wedge J_m$$

- σ responsible for Witten effect:

$$\theta \to \theta + 2\pi \Rightarrow (n_e, n_m) \to (n_e + n_m, n_m)$$

Dissolved Charges

- In presence of $\theta F \wedge F$ coupling, electric charge can be dissolved in the monopole worldlines due to the collective coordinate σ
- In addition, string states of the "axion string" charged magnetically under θ carry electric charge under A
- More generally, consistent gauging/breaking of Chern-Weil symmetries in the presence of Chern-Simons terms implies that certain charged objects can be dissolved in others—reproduces much of the known structure of D-brane interactions in Type II string theory

Axion Strings and the Weak Gravity Conjecture

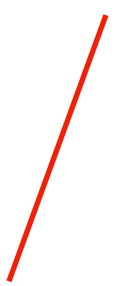
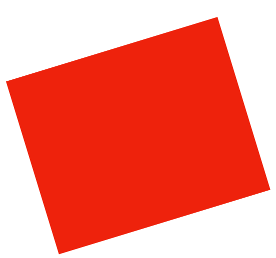
Weak Gravity Conjecture (WGC)

In any U(1) gauge theory coupled to quantum gravity, there must exist a "superextremal" state of charge q, mass m, with

$$\frac{q}{m} \geq \frac{Q}{M}\Big|_{\rm ext} \sim \frac{1}{M_{\rm Pl;d}^{(d-2)/2}}$$

Arkani-Hamed, Motl, Nicolis, Vafa, '06

The P-form WGC

	Object	dim.	Tension	WGC
	Particle	0	m	$\frac{q}{m} \gtrsim \frac{1}{M_{\mathrm{Pl;d}}^{(d-2)/2}}$
	String	1	T	$\frac{q}{T} \gtrsim \frac{1}{M_{\mathrm{Pl;d}}^{(d-2)/2}}$
	p-brane	p	T_p	$\frac{q}{T_p} \gtrsim \frac{1}{M_{\mathrm{Pl;d}}^{(d-2)/2}}$
	Instanton	-1	S	$\frac{1}{fS} \gtrsim \frac{1}{M_{\mathrm{Pl;d}}^{(d-2)/2}}$

"axion decay constant"

WGC Mixing

$$S = \int \Big[-\frac{1}{2g^2} F \wedge \star F - \frac{1}{2} f_\theta^2 d\theta \wedge \star d\theta + \frac{1}{8\pi^2} \theta F \wedge F \Big]$$

Axion WGC: $f_\theta S_{\rm inst} \lesssim M_{\rm Pl}$

WGC for strings: $T \lesssim f_\theta M_{\rm Pl}$

Further assume: $S_{\rm inst} \sim 8\pi^2/g^2$

$$\Rightarrow \; T \lesssim \frac{g^2}{8\pi^2} M_{\rm Pl}^2 \; \Rightarrow M_s \sim \sqrt{2\pi T} \lesssim g M_{\rm Pl}$$

WGC Mixing (cont.)

- Due to the $\theta F \wedge F$ coupling, the string states carry electric charge. The mass of a charge n string state satisfies

$$m_n \sim n M_s \lesssim n g M_{\rm Pl}$$

- So these states satisfy the WGC!
- If the axion is a *fundamental axion*, i.e., the core of the axion string probes the deep UV, then there will be an infinite tower of such charged string states, satisfying the Tower WGC

Exception: KK reduction

5d: $$\frac{1}{6(2\pi)^2}\int A\wedge dA\wedge dA$$

4d: $$\frac{1}{8\pi^2}\int\left[-\theta F\wedge F+\frac{\theta^2}{2\pi}H\wedge F-\frac{\theta^3}{3(2\pi)^2}H\wedge H+\cdots\right]$$

$$H = dB\,,\quad F = dA$$

No $\theta H\wedge H$ term!

But, there is a $\theta F\wedge F$ term $\Rightarrow T\sim(eM_{\mathrm{Pl}})^2$

$$e\sim e_{\mathrm{KK}}^{1/3}\ \Rightarrow M_s\sim\sqrt{2\pi T}\sim eM_{\mathrm{Pl}}\sim e_{\mathrm{KK}}^{1/3}M_{\mathrm{Pl}}$$

$$\sim M_{\mathrm{Pl};5}$$

A 5d SUGRA analog

- Consider 5d SUGRA theory with exactly two abelian gauge fields, A and B
- From cubic structure of prepotential, can show:

$$B \wedge F \wedge F \quad \Rightarrow \quad g_B \sim g_A^{-2} \Rightarrow \sqrt{T} \sim g_A$$

$$\text{No } B \wedge F \wedge F \quad \Rightarrow \quad g_B \sim g_A^{-1/2} \Rightarrow \sqrt{T} \sim g_A^{1/4}$$

$$\sim M_{\text{Pl};6}$$

- Two cases correspond to emergent string limit and decompactification limit (cf. Emergent String Conjecture Lee, Lerche, Weigand '19)

Pheno Implications

- Tower/sublattice WGC imply EFT breaks down at species bound scale:

$$\Lambda_{\mathrm{UV}} \sim e^{1/3} M_{\mathrm{Pl}}$$

- In presence of $\theta F \wedge F$ term, EFT breaks down at lower string scale

$$\Lambda_{\mathrm{UV}} \sim M_s \sim e M_{\mathrm{Pl}}$$

- This implies an incompatibility between EFTs at high energy (GUTs, high-scale inflation, etc.) and those with a tiny coupling constant (chromonatural inflation, dark radiation, etc.)

Conclusions

Main Takeaways

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Future Research

- When are $\theta F \wedge F$ couplings required for consistency of the theory?
- Aside from KK theory, are there examples without such couplings?
- Are there higher-dimensional examples of WGC-mixing in the presence of Chern-Simons terms? (work to appear with Sami Kaya)
- How does this story extend to θF_4 couplings, as appears in the 4d description of axion monodromy?
- How does this WGC-mixing fit with the notion of higher-group global symmetries? Is there a more general story to tell?