The tadpole conjecture at large complex-structure

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Geometry, Strings and the Swampland

Ringberg Castle — 10.11.2021

This talk is based on ::

- *The tadpole conjecture at large complex-structure*
  E. Plauschinn
  arXiv:2109.00029

- *Moduli Stabilization in Asymptotic Flux Compactifications*
  T. Grimm, D. van de Heisteeg, E. Plauschinn
  arXiv:2110.05511

motivation

Some motivation.

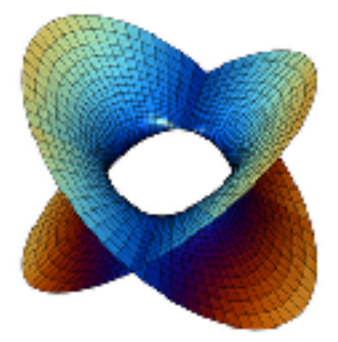
motivation — the landscape

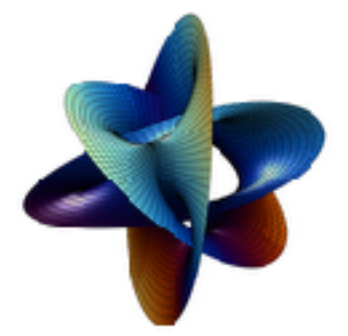
Compactifications of string theory give rise to an abundance of lower-dimensional theories — the **string theory landscape**.

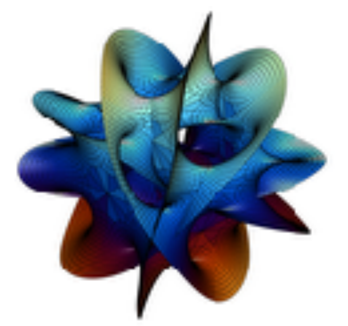
Famous estimates for its **size** are 10^{500}, 10^{930}, 10^{1500}, 10^{272000}.

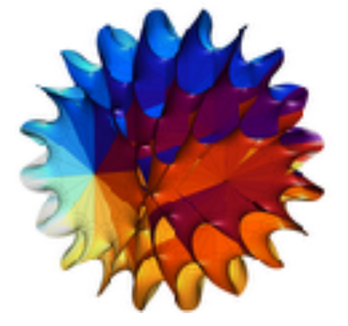
Bousso, Polchinski — 2000
Schellekens — 2016
Lerche, Lüst, Schellekens — 1987
Taylor, Wang — 2015

But one is often only interested in four-dimensional theories with no or few **massless scalar fields**.









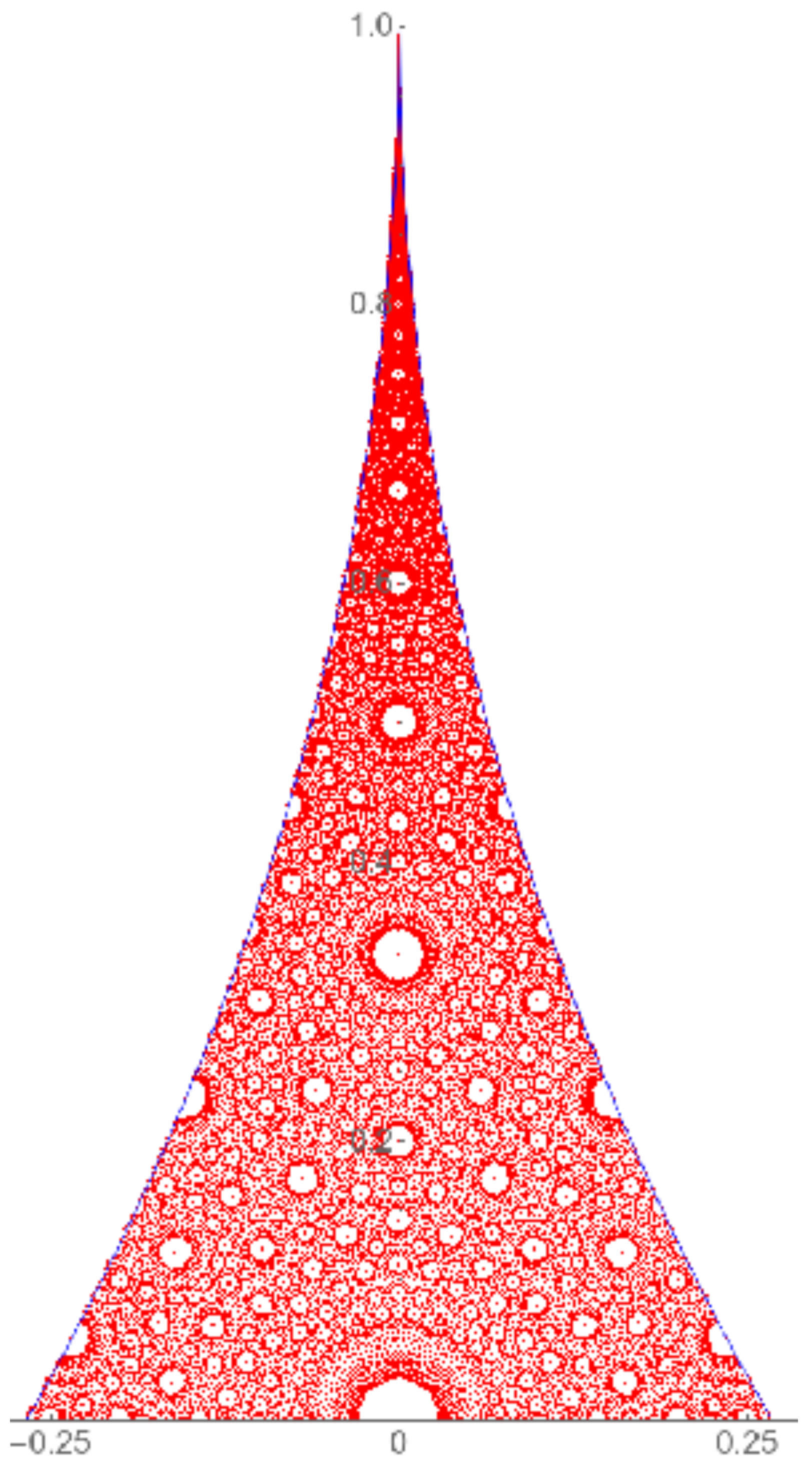
motivation — moduli stabilization

In type IIB orientifold compactifications on Calabi-Yau three-folds, fluxes generate a mass-term for the complex-structure and axio-dilaton moduli.

An underlying assumption of the KKLT and Large-Volume scenarios is that all of these moduli can be stabilized in a suitable regime.

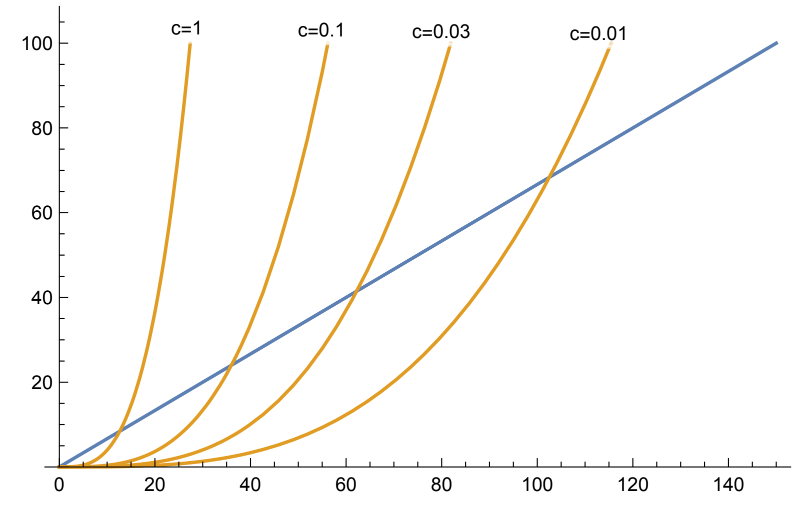
This assumption can fail.

Bena, Dudas, Graña, Lüst — 2018
Betzler, EP — 2019
Braun, Valandro — 2020



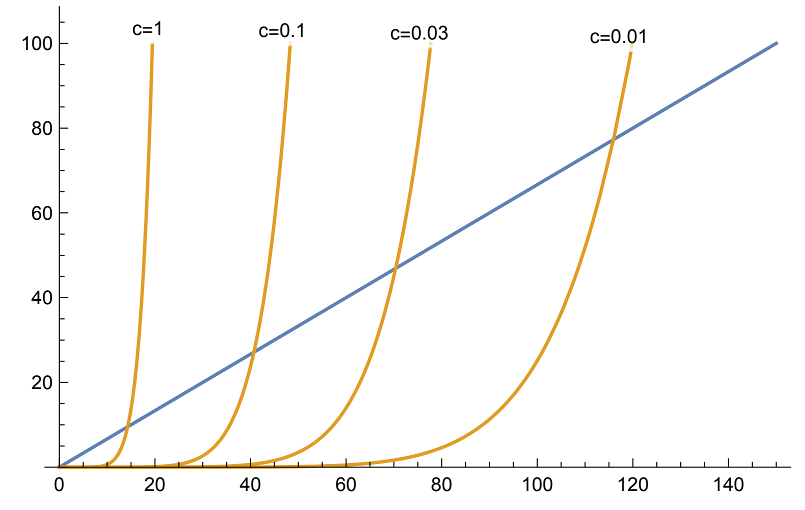
The tadpole conjecture states that for a large number of complex-structure moduli, not all of them can be stabilized by fluxes.

Bena, Blåbäck, Graña, Lüst — 2020



Goal for this talk ::

- Explain why it is difficult to prove this conjecture,
- show that in the large complex-structure limit generically the conjecture is satisfied, and
- discuss moduli stabilization in asymptotic regions.



outline

1. motivation
2. the tadpole conjecture
3. boundary behaviour
4. large complex-structure
5. summary
6. asymptotic hodge theory
7. summary

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Explanation of the tadpole conjecture.

D-branes and O-planes are charged under Ramond-Ramond gauge potentials and therefore contribute to Bianchi identities. The integrated expressions are the tadpole cancellation conditions.

In F-theory, the D3-brane tadpole equation reads (with χ the Euler number of the four-fold)

$$N_{\mathrm{D3}} + \frac{N_{\mathrm{flux}}}{2} = \frac{\chi}{24} \, .$$

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$$N_{\mathrm{D3}} + \frac{N_{\mathrm{flux}}}{2} = \frac{\chi}{24} \qquad \xrightarrow{\quad h^{3,1}\gg 1 \quad} \qquad \frac{N_{\mathrm{flux}}}{2} \leq \frac{h^{3,1}}{4}\,.$$

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The tadpole conjecture states that in the large $h^{3,1}$-limit, the flux number satisfies

$$\frac{N_{\rm flux}}{2} > \frac{h^{3,1}}{3} \,.$$

Bena, Blåbäck, Graña, Lüst — 2020

Implications ::

- If true, the tadpole conjecture implies that the landscape of theories with no massless scalar fields is smaller than expected.

Comments ::

- The conjecture also applies to type IIB orientifold compactifications.

- In a number of examples the tadpole conjecture is satisfied, even for

$$\frac{N_{\text{flux}}}{2} > 0.44 \times n_{\text{mod}} \, .$$

Bena, Blåbäck, Graña, Lüst — 2020 & 2021

- A scenario which violates the tadpole conjecture has been proposed by Marchesano, Prieto & Wiesner.

Marchesano, Prieto, Wiesner — 2021

see also :: Lüst — 2021
Grimm, v.d. Heisteeg, EP — 2021

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Estimating the behaviour of N_{flux} when approaching a boundary.

Consider type IIB orientifold compactification on Calabi-Yau three-folds with NS-NS and R-R fluxes ::

- The global minimum of the scalar potential corresponds to the self-duality condition

$$\star G_3 = i G_3 \,, \qquad\qquad G_3 = F_3 - \tau H_3 \,, \quad \tau = c + i s \,.$$

- The Hodge-star matrix *M* depends on the complex-structure moduli, and the above relation reads in matrix notation

$$\mathsf{F}_3 = \big[\eta \mathcal{M} s + \mathbb{1} c\big]\, \mathsf{H}_3 \,, \qquad\qquad \eta = \begin{pmatrix} 0 & +\mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}.$$

- The flux number is defined and satisfies

$$N_{\text{flux}} = \mathsf{F}_3^T \,\eta\, \mathsf{H}_3 \geq 0 \,.$$

boundary behaviour — bloch-messiah decomposition

The Hodge-star matrix *M* is a real, symmetric, symplectic matrix. It can therefore be decomposed as

$$\mathcal{M} = U^T \Sigma U \,,$$

$$\Sigma = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{pmatrix} ,$$

$$\lambda = \mathrm{diag}\big(\text{eigenvalues } \mathcal{M}\big) \,,$$

$$U \in \mathrm{Sp}\big(2h^{2,1}+2, \mathbb{R}\big) \cap \mathrm{O}(2h^{2,1}+2, \mathbb{R}) \,.$$

The self-duality condition and the flux number (at the minimum) can then be expressed as

$$\widetilde{\mathsf{F}}_3 = \begin{pmatrix} \mathbb{1}\, c & \lambda^{-1} s \\ -\lambda s & \mathbb{1}\, c \end{pmatrix} \widetilde{\mathsf{H}}_3 \,,$$

$$\widetilde{\mathsf{H}}_3 = U\, \mathsf{H}_3 \,, \qquad \widetilde{\mathsf{F}}_3 = U\, \mathsf{F}_3 \,,$$

$$N_{\text{flux}} = s \sum_I \Big[\lambda_I \left(\tilde{h}^I\right)^2 + \tfrac{1}{\lambda_I} \left(\tilde{h}_I\right)^2 \Big] \,,$$

$$\widetilde{\mathsf{H}}_3 = \begin{pmatrix} \tilde{h}^I \\ \tilde{h}_I \end{pmatrix} .$$

The flux number (at the minimum) can be expressed in the following way

$$N_{\text{flux}} = s \sum_I \Big[\lambda_I \big(\tilde{h}^I\big)^2 + \tfrac{1}{\lambda_I} \big(\tilde{h}_I\big)^2 \Big] .$$

Boundary behaviour of the flux number (for $\tilde{h}^I \neq 0$) ::

- At a boundary in complex-structure moduli space, the matrix *M* is expected to degenerate. This implies

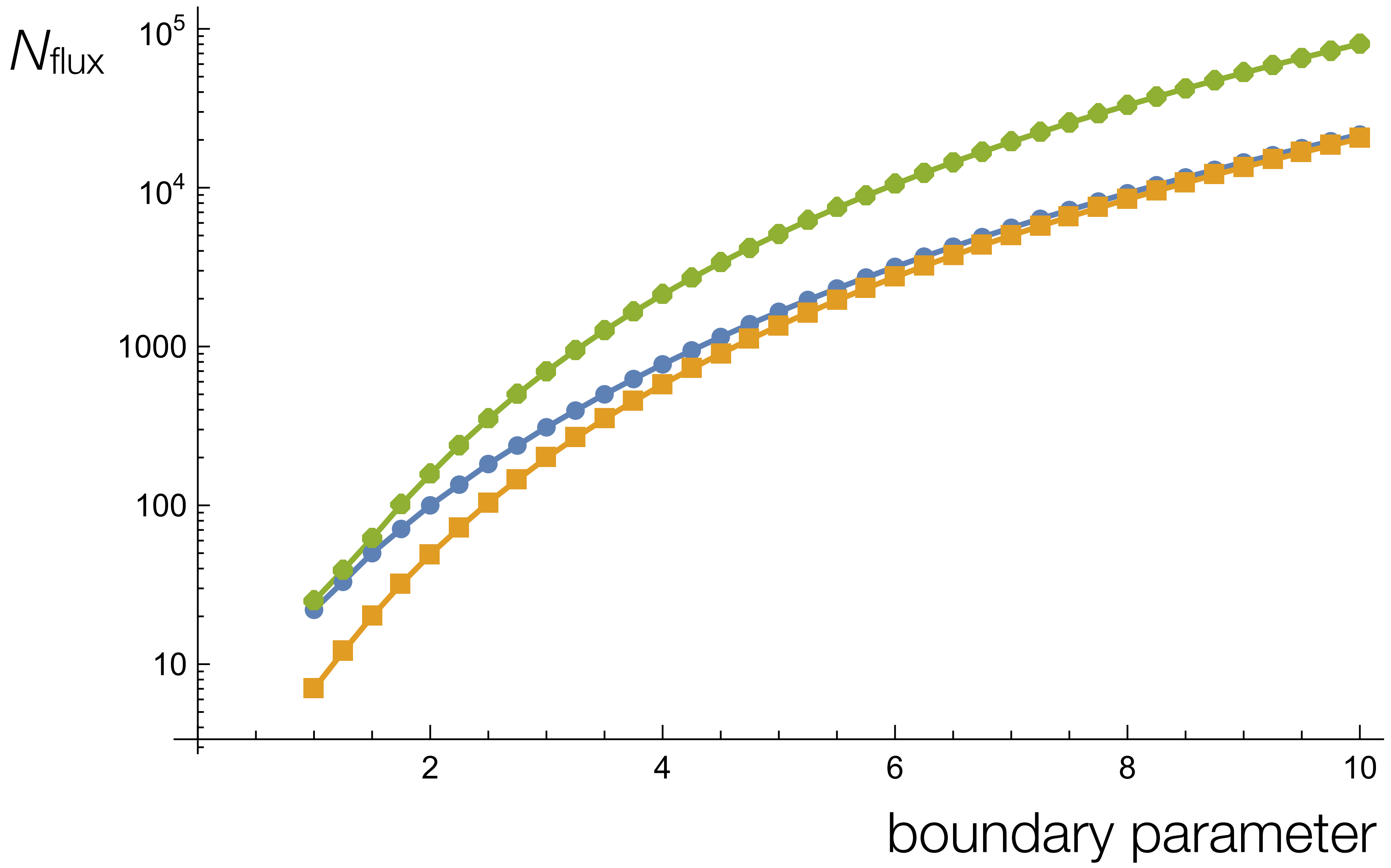
$$\lambda_I \to \infty \qquad \longrightarrow \qquad N_{\text{flux}} \to \infty \, .$$

- For the boundary in axio-dilaton moduli space one finds

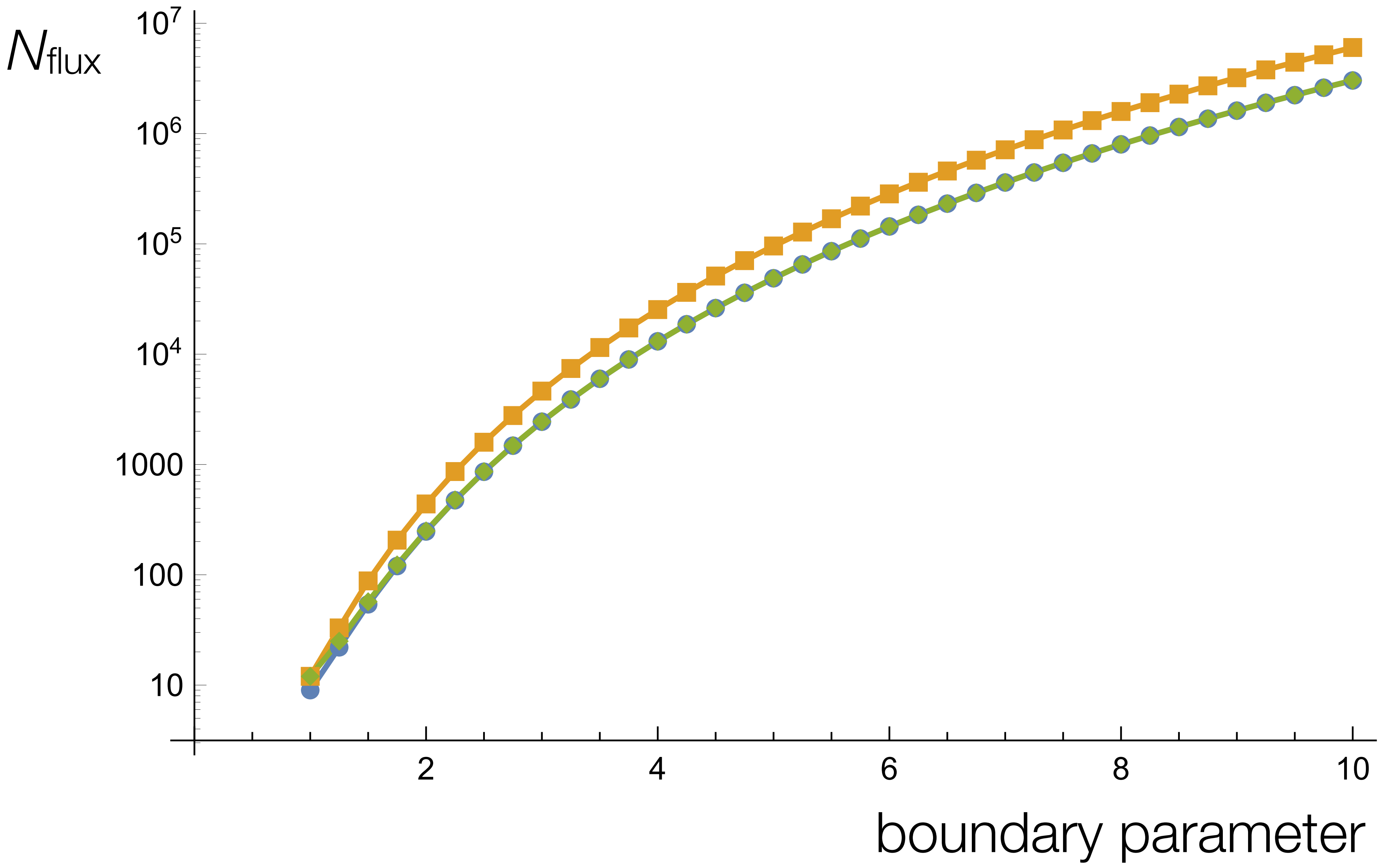
$$s \to \infty \qquad \longrightarrow \qquad N_{\text{flux}} \to \infty \, .$$

boundary behaviour — growth of flux number

For simple examples and generic configurations, the flux number grows significantly near a boundary ::



$h^{2,1}$=2 (LCS)



$h^{2,1}$=3 (LCS)

Grimm, v.d. Heisteeg, EP — 2021

boundary behaviour — discussion

Summary ::

- Near a boundary in moduli space, the flux number N_{flux} generically diverges.
  (The behaviour in non-generic situations is not derived here.)

Implications ::

- The smallest values for N_{flux} will be found in the interior of moduli space.
- It can thus be challenging to prove the tadpole conjecture or to find computationally-controlled counter-examples.

Comment ::

- The same behaviour is found using asymptotic Hodge theory.

Grimm — 2020
Grimm, v.d. Heisteeg, EP — 2021

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The tadpole conjecture in the large complex-structure regime.

Consider the large complex-structure regime, for which the moduli space is described by the prepotential

$$\mathcal{F} = -\frac{1}{3!}\,\frac{\kappa_{ijk}\,X^i X^j X^k}{X^0}\,,$$

$$z^i = X^i/X^0 = u^i + i\,v^i\,,$$
$$i,j,k = 1,\ldots,h^{2,1}\,.$$

Using the self-duality condition, the flux number (at the minimum) can be expressed as

$$\begin{aligned} N_{\text{flux}} = \;& s\left[\,\tfrac{\kappa}{6}\,(h^0)^2 + \tfrac{2}{3}\,\kappa\,(h^i - u^i h^0)\,G_{ij}\,(h^j - u^j h^0)\,\right] \\ &+\frac{1}{s}\left[\,\tfrac{\kappa}{6}\,(\mathsf{f}^0)^2 + \tfrac{2}{3}\,\kappa\,(\mathsf{f}^i - u^i\mathsf{f}^0)\;G_{ij}\,(\mathsf{f}^j - u^j\mathsf{f}^0)\,\right], \end{aligned}$$

$G_{i\bar{j}} =$ Kähler metric,

$\kappa = \kappa_{ijk}v^i v^j v^K\,.$

Using statistical results by Demirtas et al. for κ_{ijk}, the scaling of the flux number can be estimated as

$$N_{\text{flux}} \gtrsim \begin{cases} (h^{2,1})^{+1/2}\,\|v\| & \text{for } h^0 = f^0 = 0\,, \\ (h^{2,1})^{-1/2}\,\|v\|^3 & \text{else}\,. \end{cases}$$

Demirtas, Long, McAllister, Stillman — 2018
EP — 2021

large complex-structure — complex-structure cone

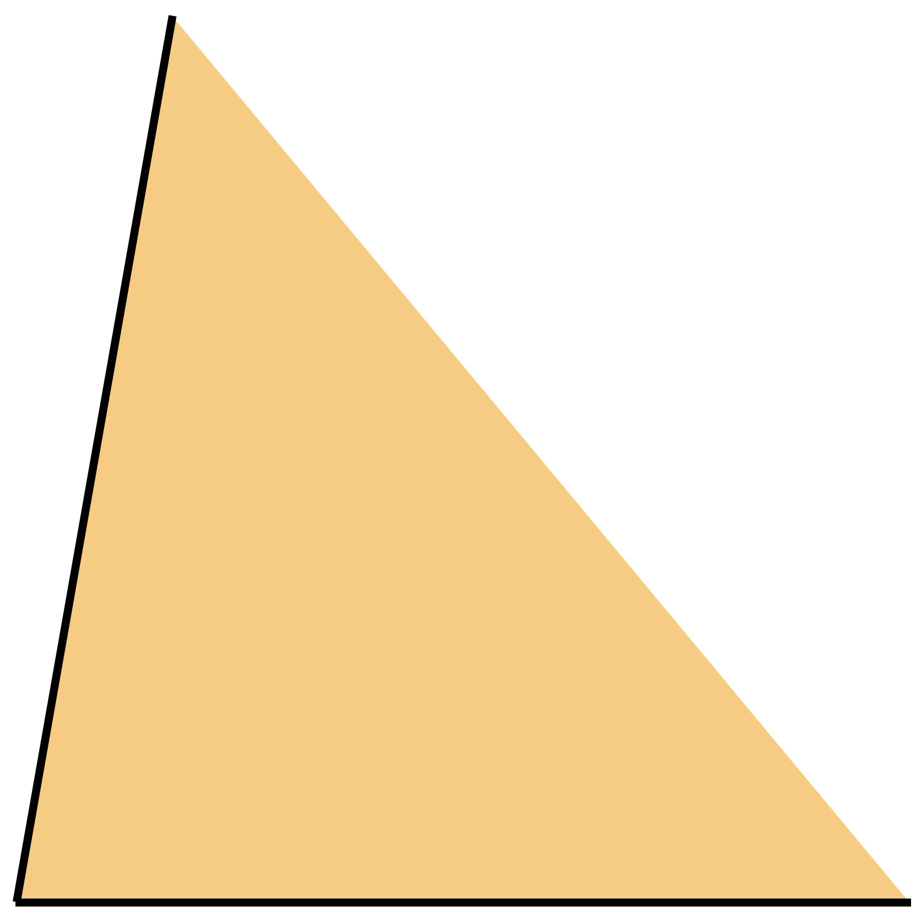
At large complex-structure, the complex-structure moduli space is mirror-dual to the Kähler moduli space

$$\begin{tikzcd} \boxed{\begin{array}{c}\text{complex-structure}\\ \text{moduli space}\end{array}} \arrow[rr, leftrightarrow, "\text{mirror symmetry}"] \arrow[d] & & \boxed{\begin{array}{c}\text{Kähler}\\ \text{moduli space}\end{array}} \arrow[d] \\ \boxed{\begin{array}{c}\text{bounded by}\\ \text{"complex-structure cone"}\end{array}} & & \boxed{\begin{array}{c}\text{bounded by}\\ \text{Kähler cone}\end{array}} \end{tikzcd}$$

Statistical data on the Kähler cone has been determined by Demirtas et al. for the Kreuzer-Skarke list.

Demirtas, Long, McAllister, Stillman — 2018

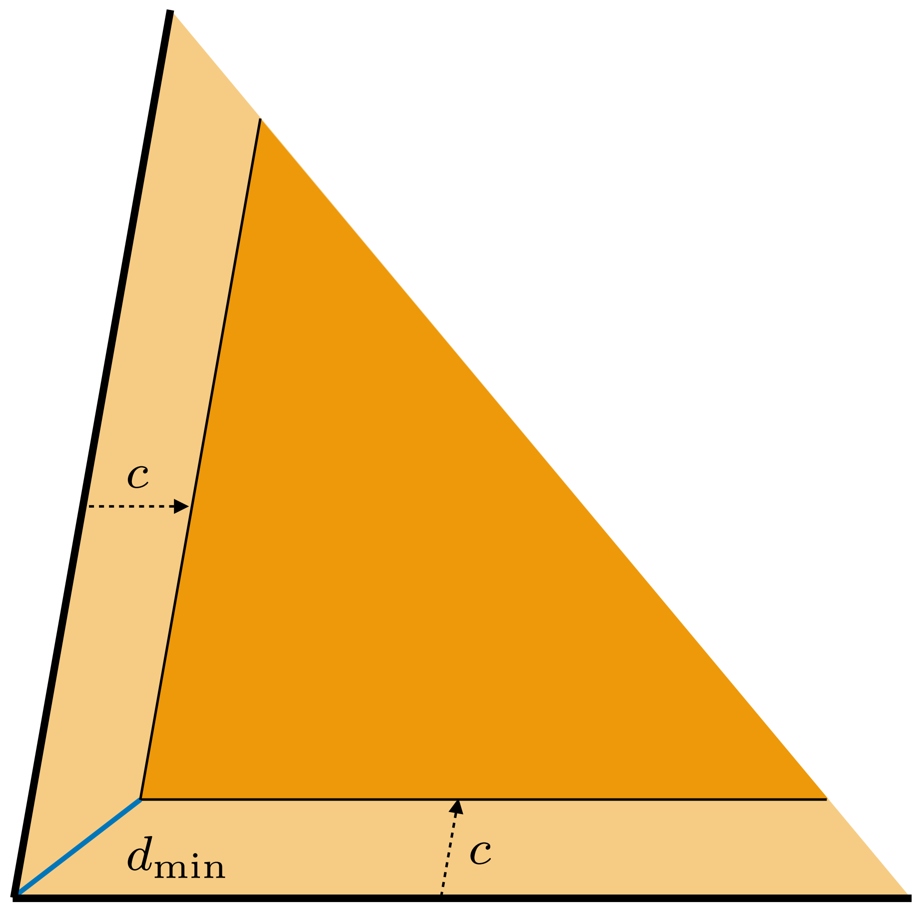
The Kähler cone contains all Kähler forms such that curves, divisors and the CY_3 itself have positive volume.



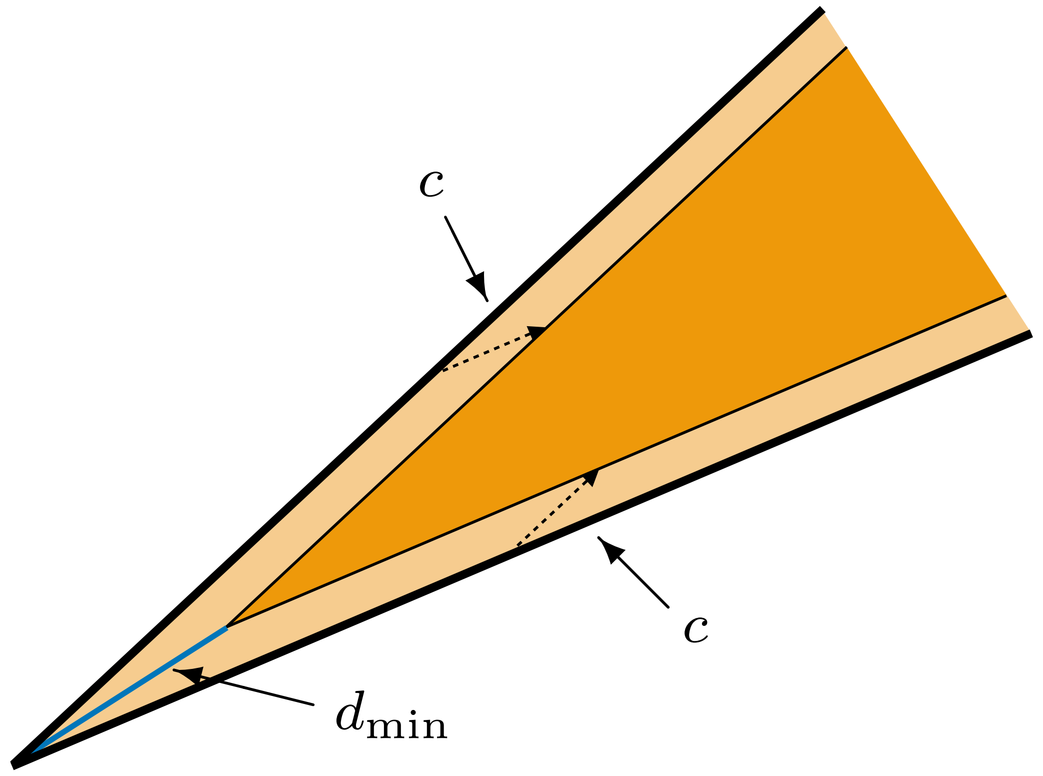
Demirtas, Long, McAllister, Stillman — 2018

The Kähler cone contains all Kähler forms such that curves, divisors and the CY_3 itself have positive volume.

The stretched Kähler cone contains all Kähler forms such that all volumes are larger than a constant c.



wide Kähler cone



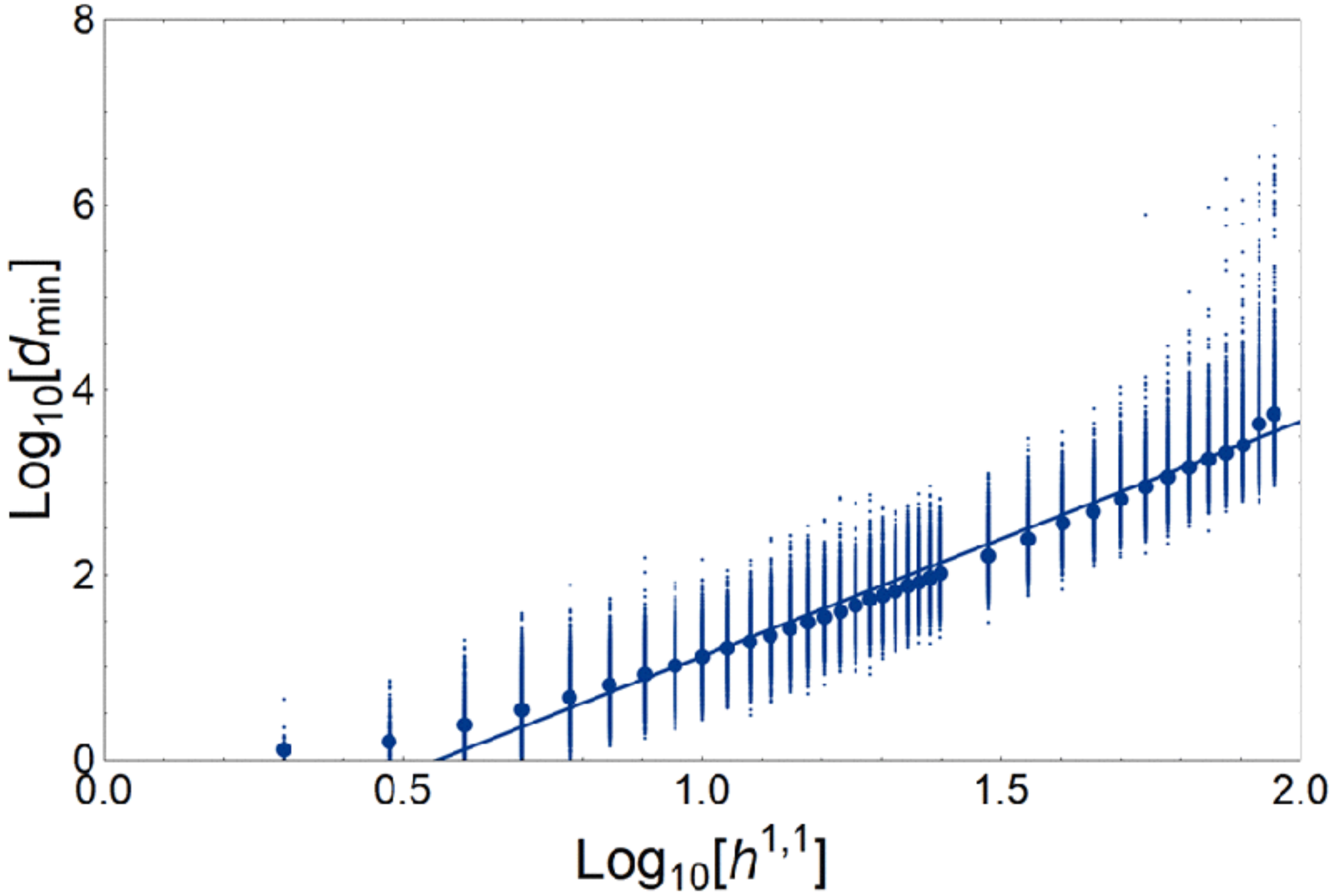
narrow Kähler cone

Demirtas, Long, McAllister, Stillman — 2018

For the Kreuzer-Skarke list, one finds that the Kähler cone becomes narrower for larger dimensions $h^{1,1}$.

The narrowness can be encoded in the distance between the ordinary and stretched Kähler cones

$$d_{\text{min}}[c] \simeq 10^{-1.4}\,(h^{1,1})^{2.5}\,c\,.$$



Demirtas, Long, McAllister, Stillman — 2018

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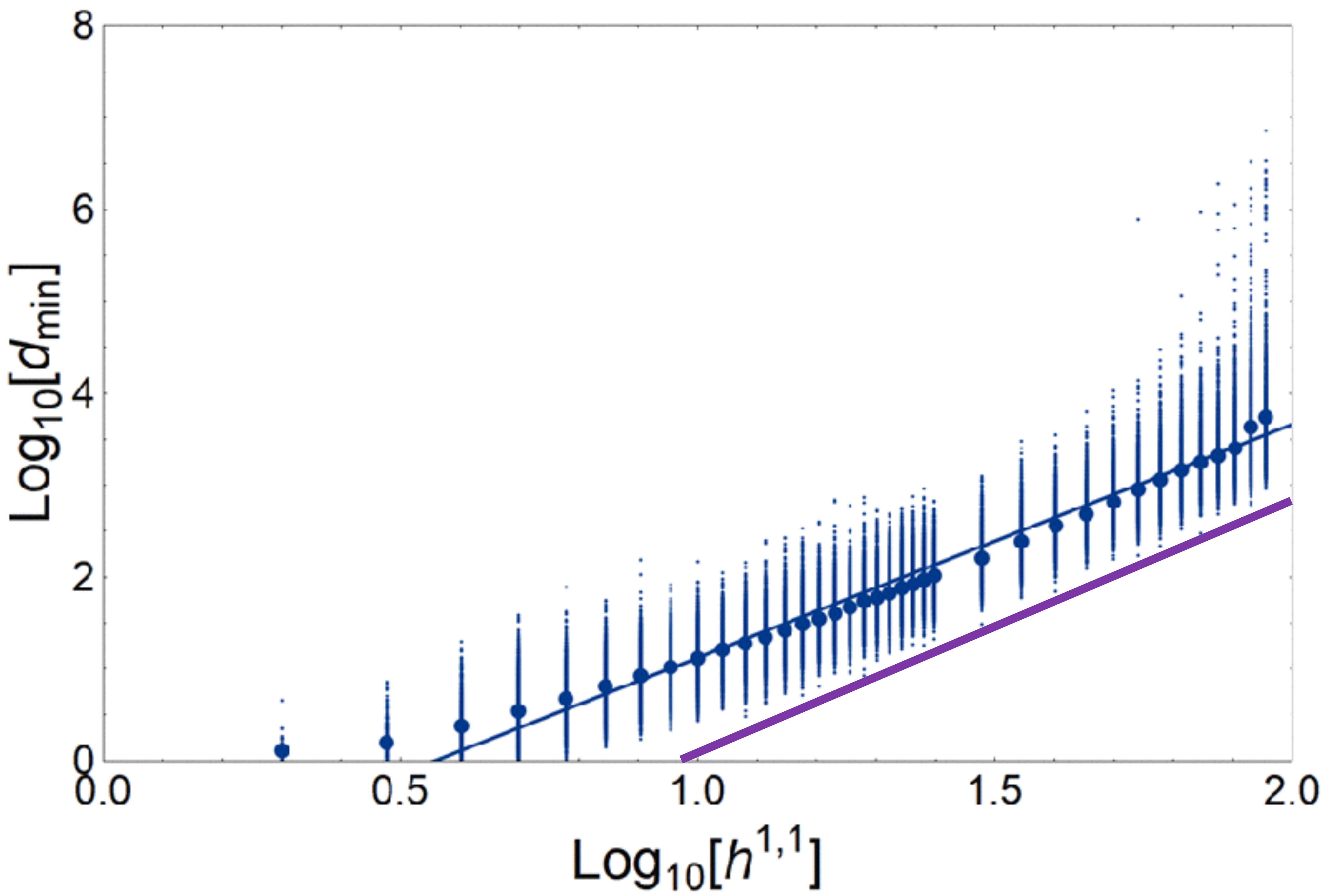
$$d_{\mathsf{min}}[c] \simeq 10^{-1.4}\,(h^{1,1})^{2.5}\,c\,.$$

A lower bound can be estimated from the data as

$$d_{\min}[c] \gtrsim 10^{-2.6}\,(h^{1,1})^{2.7}\,c\,.$$



Demirtas, Long, McAllister, Stillman — 2018

The result from above can now be combined in the following way (here $\gamma = 1, 3$) ::

$$\boxed{N_{\text{flux}} \gtrsim (h^{2,1})^{1-\frac{\gamma}{2}} \, \|v\|^{\gamma}}$$

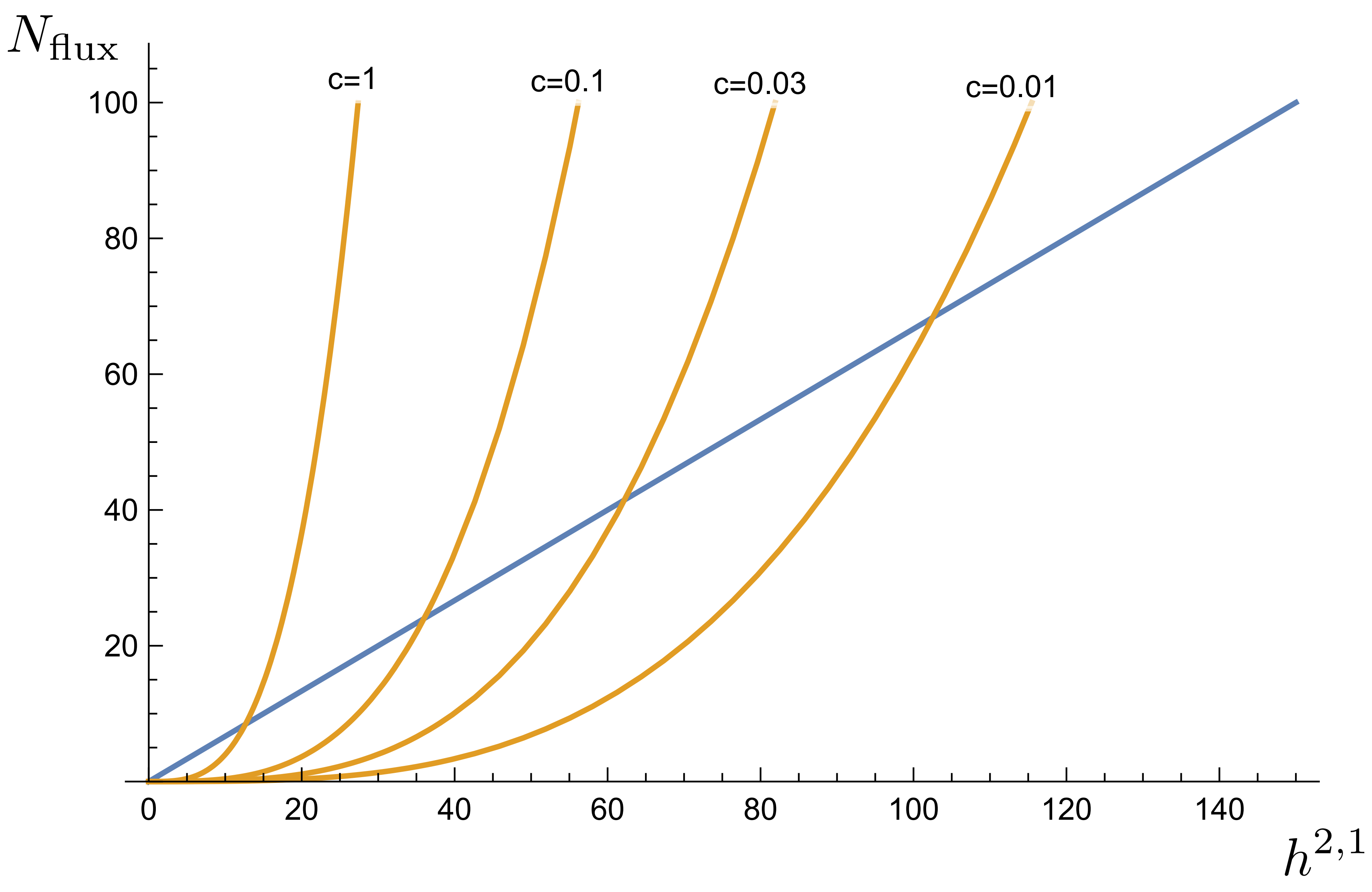
$$\boxed{\|v\| \geq d_{\min} \gtrsim 10^{-2.6} \, (h^{2,1})^{2.7} \, c}$$

$$\boxed{N_{\text{flux}}[c] \gtrsim (h^{2,1})^{1-\frac{\gamma}{2}} \, \left[10^{-2.6} \, (h^{2,1})^{2.7} \, c\right]^{\gamma}}$$

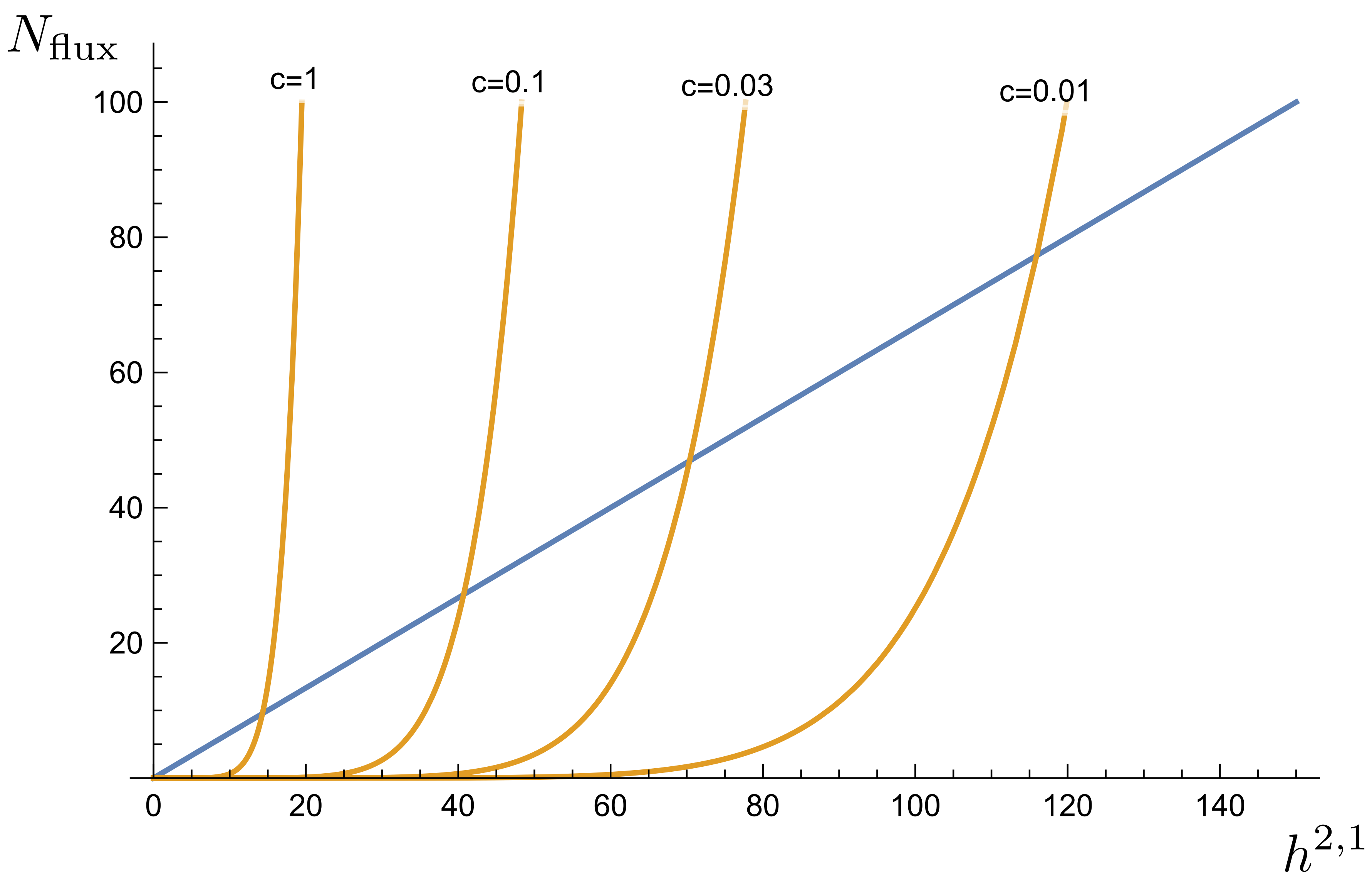
The constant *c* parametrizes minimal volumes of cycles on the mirror-dual side. For a well-controlled large complex-structure limit, *c* should not be much smaller than one.

large complex-structure — comparison

Even for small values of c, the flux number in the large complex-structure regime (orange curves) quickly exceeds the linear bound of the tadpole conjecture (blue curves).



$\gamma = 1$



$\gamma = 3$

Summary ::

- For generic flux choices and in the large complex-structure limit, the tadpole conjecture is satisfied.
- A smaller flux number, violating the conjecture, may be found outside of this regime or for non-generic flux choices.

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Summary ::

- The tadpole conjecture states that for large $h^{2,1}$, not all moduli can be stabilized.
- When approaching a boundary in moduli space, the flux number diverges.
- In the large complex-structure regime, the tadpole conjecture is satisfied for generic flux choices.

Implications ::

- If true, the tadpole conjecture implies that the landscape of theories with no or few massless scalar fields is smaller than expected.
- A common assumption in the KKLT and Large Volume scenarios may not be satisfied.