Gravitational Solitons and Completeness

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J. M., "Gravitational Solitons and Completeness," [2108.02228].

B. Heidenreich, J. M., M. Montero, M. Reece, T. Rudelius, and I. Valenzuela, "Non-Invertible Global Symmetries and Completeness of the Spectrum," [2104.07036],

See also T. Rudelius and S.-H. Shao [2006.10052].

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History

Swampland folklore:

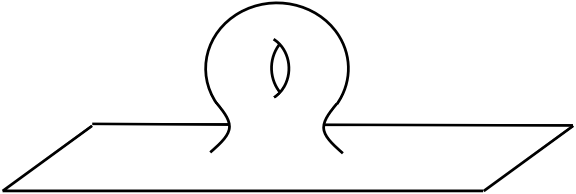
$$\textbf{No Global Symmetries} \overset{?}{\Longrightarrow} \textbf{Completeness Hypothesis}$$

Some (abridged) history:

- **2003:** Polchinski proposes completeness hypothesis.
- **2011:** Banks and Seiberg prove folklore for abelian groups.
- **2019:** Harlow and Ooguri give counterexample.
- **2020:** Rudelius and Shao prove folklore for finite groups, provided we broaden our notion of symmetry.
- **2021:** Heidenreich, M., Montero, Reece, Rudelius, and Valenzuela show the same is true for all compact groups.

Today

Our Goal: Include *gravitational solitons* in the story.



Result: With gravitational solitons, folklore is true without modification.

Warning: Talk is pure kinematics, ignoring all dynamics (stability, confinement, etc.).

Review

Completeness and Flux Strings

Why would we believe the folklore in the first place?

Basic Idea: Charged particles allow flux strings to break, avoiding conserved string charges.

$$\textbf{Completeness} \iff \textbf{All Flux Strings Breakable}$$

Incompleteness $\implies$ conservation law: some flux string cannot break.

Does this imply a global symmetry?

Conservation Laws and Global Symmetries

Lesson from topological order: not all conservation laws come from a symmetry group [see Kitaev '05].

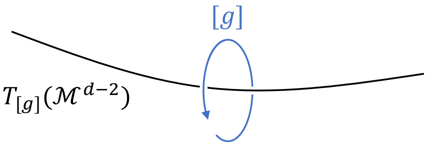
However, conservation laws *are* always associated with topological operators, which measure the conserved charges.

Definition: A *non-invertible global symmetry* is a collection of topological operators. All operators invertible $\implies$ group-like symmetry.

New Posibility: Flux strings protected by non-invertible symmetry.

Gukov-Witten Operators

Flux strings are measured by *Gukov-Witten operators* $T_{[g]}$.



In pure gauge theory,

$$T_{[g]} \text{ is topological } \iff g \in \ker(\mathrm{Adj}) = Z_G(G_0).$$

More generally, $T_{[g]}$ is topological when it is transparent to all local excitations.

Non-Invertible Symmetry of Gauge Theory

The maximal group-like sub-symmetry is the center symmetry. If

$$Z(G) \subsetneq Z_G(G_0),$$

then there can be a non-invertible symmetry (only possible for G disconnected and nonabelian).

Example: Consider

$$O(2) = U(1) \rtimes \mathbb{Z}_2^{\mathrm{C}},$$

gauge theory + charge 3 matter, an incomplete theory. We have

$$Z(O(2)) = \{\pm 1\}, \quad Z_{O(2)}(U(1)) = U(1).$$

The only topological G-W operator is $T_{[\omega]}$ for $\omega^3 = 1$, a non-invertible operator which protects flux strings of charge $\neq 0$ mod 3.

Non-Invertible Symmetry and Completeness

Suppose we add matter to fully break the non-invertible symmetry. Then

$$\rho = \mathrm{Adj} \oplus \{\text{Matter reps}\},$$

is faithful, since any $g \in \ker(\rho)$ would define a topological G-W operator.

But ρ faithful $\implies$ tensor products of ρ and $\bar{\rho}$ generate a complete spectrum.

No Non-Invertible Global Symmetries $\implies$ **Completeness Hypothesis**

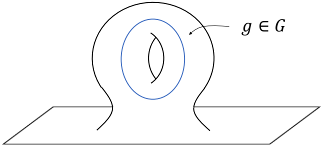
Gravitational Solitons

Charged Gravitational Solitons

Naively, making the topology dynamical adds no new charges, so how could anything change?

Claim: This intuition is wrong, and gravitational solitons may carry charges beyond those generated by EFT particles.

Basic Mechanism: In quantum gravity, *global* properties of field space are probed *locally*:



We may support holonomy on a 1-cycle of a gravitational soliton.

Allowed Charges

The wavefunction $\psi(g)$ lives in $L^2(G)$. Under a gauge transformation h,

$$\psi(g) \to \psi(hgh^{-1}),$$

so the charges live in the conjugation action on $L^2(G)$.

Example: Under rotation by an angle θ, the disconnected component of $O(2)$ rotates by 2θ. Thus, we have

$$L^2\,(\text{Disconnected component of } O(2)) = \mathrm{triv} \oplus 2 \oplus 4 \oplus \cdots$$

and gravitational solitons realize states of every even charge. In contrast, the EFT includes only photons, transforming in the $\det$ representation.

Gravitational Solitons and Completeness

We have seen that gravitational solitons provide more charges than the EFT. Correspondingly, they break the non-invertible symmetry to

$$\ker(L^2(G)) = Z(G),$$

the center symmetry.

All that remains for completeness is to break the center symmetry, and so

$$\textbf{No Global Symmetries} \implies \textbf{Completeness Hypothesis}$$

if we count gravitational solitons as charged particles.

Breaking and F-Moves

How does gravity break the non-invertible symmetry? Like this:

Non-invertible topological operators need only be invariant under isotopy, not homology.

In QG, the possibility of virtual gravitational solitons means the non-invertible symmetry is broken even in “flat” space.

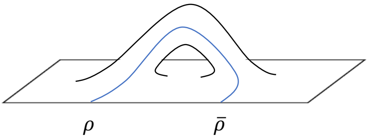
Generalizations and Conclusions

Wormholes and the Adjoint Subcategory

By the Peter-Weyl theorem, we have that

$$L^2(G) = \bigoplus_{\rho} \rho \otimes \bar{\rho},$$

where ρ ranges over all irreps of G. This description is evocative: our solitons are wormholes threaded by electric flux ρ.



Easily generalizes to any tensor category $\mathcal{C}$ of line operators: gravitational solitons populate the *adjoint subcategory*, generated by fusions $\rho \otimes \bar{\rho}$.

Solitons and Cobordism Groups

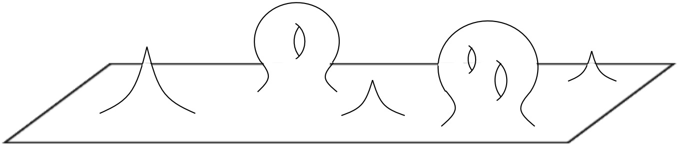
Cobordism conjecture [M., Vafa '19]: every wormhole can pinch off. In this case, each end is an independent particle, of charges ρ and $\bar{\rho}$. But ρ was arbitrary, so

$$\textbf{Cobordism Conjecture} \implies \textbf{Completeness Hypothesis}$$

The cobordism groups of quantum gravity exactly measure the symmetries that remain once gravitational solitons are included.

Comments on Mass Scales

Generically, we might expect gravitational solitons to have Planckian mass.



This expectation is semi-classical, and may break down once gravity is strongly interacting. In fact, the distinction between gravitational soliton and point particle is likely meaningless in a full theory of quantum gravity.

Question: Can gravitational solitons become light enough to enter a dual EFT description? Can they satisfy the Weak Gravity Conjecture?

Thank You!