Holomorphic boundary conditions for topological field theories via branes in twisted supergravity

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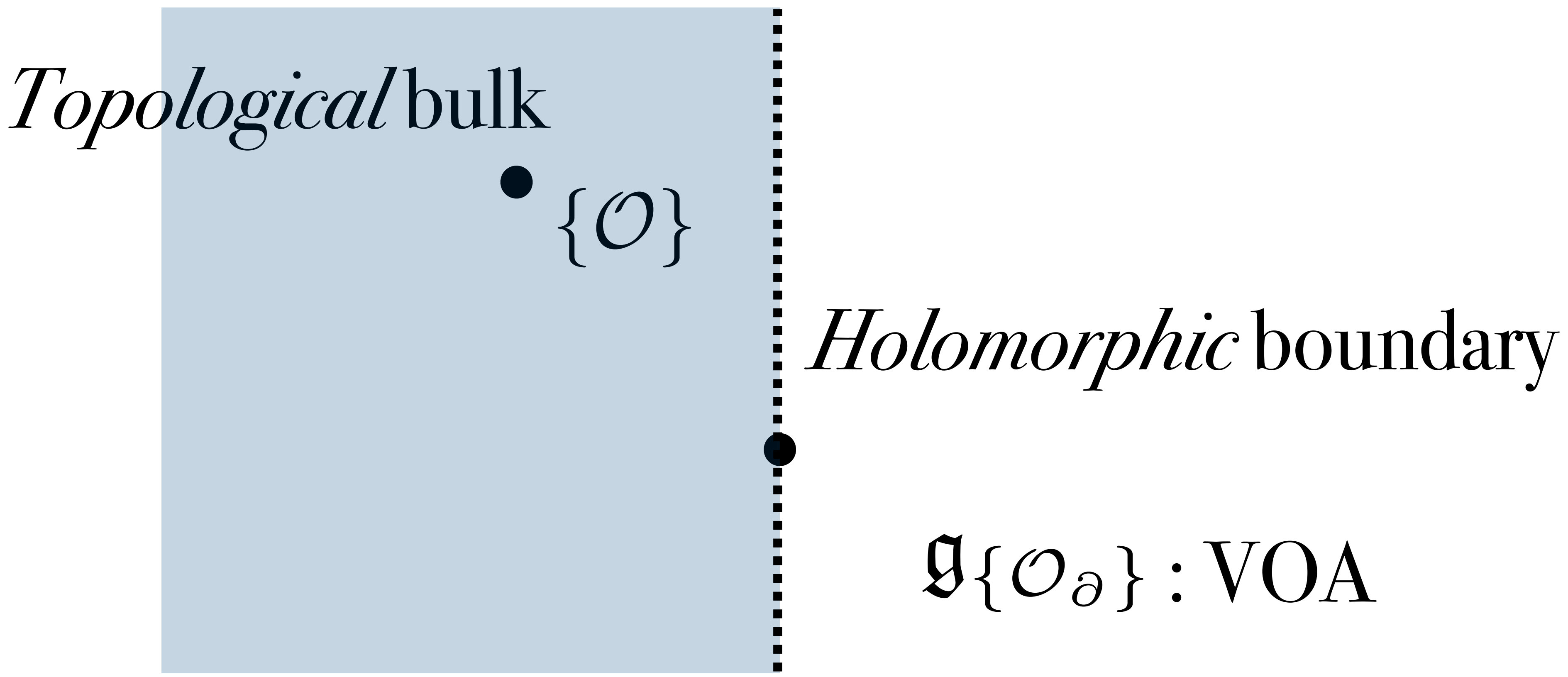
Motivation and Objective:

Twisting 3d $\mathcal{N}$=4 theories **with** boundaries:



- Very interesting implications:

  Susy compactifications on $\Sigma_{\mathfrak{g}}$

  Study bulk TFT using bdy VOA data

Gaiotto '18
Gaiotto - Costello - Creutzig '18

- Central issue:

  Compatibility of boundary with bulk topological twist via appropriate *deformation*

Gaiotto-Costello '18

In this talk:

Show how topological twist is implemented as a deformation of a holomorphically twisted theory, by engineering the theories in brane world volumes in a particular background

Outline:

- 3d N=4 theories with boundaries:
  - Topological and Holomorphic twists
  - Boundaries, deformations and compatibility
- Brane engineering:
  - Bulk and boundary theory
  - Twisting and background geometry
- Twisted IIB supergravity:
  - Holomorphic brane engineering
  - Background deformations and compatibility
- Summary, future directions and ongoing work

3d N=4 with boundaries:

Superconformal algebra:

$$\mathfrak{g}(3, \mathcal{N}=4) = \mathfrak{osp}(4|4) \supset \mathfrak{so}(3,2) \times \mathfrak{so}(4)_R$$

$$Q \in [\mathbf{2}]^{(\mathbf{2},\mathbf{2})} \qquad SO(4)_R \simeq SU(2)_C \times SU(2)_H$$

- 3d N=4 Hypermultiplet

Scalars: $(q, \tilde{q}) \in [\mathbf{1}]^{(\mathbf{1},\mathbf{2})}$

Fermions: $(\psi, \tilde{\psi}) \in [\mathbf{2}]^{(\mathbf{2},\mathbf{1})}$

Twisting:

- $\mathfrak{h}_{C,H} : SU(2)_L \to SU(2)_L \times SU(2)_{C,H}$
- $\mathcal{Q} : \mathcal{Q}^2 = 0, \quad Q_{\text{BRST}} \to \mathcal{Q} + Q_{\text{BRST}}$

$\mathcal{Q}_C, \mathcal{Q}_H \to$ Topological twist

$\mathcal{Q}_{\text{hol}} \to$ Holomorphic twist

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?

3d N=4 with boundaries:

Consider topological supercharges as *deformations* of the holomorphic one:

$$\mathcal{Q}_H = \mathcal{Q}_{\text{hol.}} + \zeta Q_H \qquad \mathcal{Q}_C = \mathcal{Q}_{\text{hol.}} + \zeta Q_C$$

Gaiotto-Costello '18

⟶ Apply to construction of appropriate boundary conditions!

- $\mathcal{N} = (0,4)$: Compatible with bulk *holomorphic* but *not topological* twist

  Deformable to be *compatible* with bulk topological twist

- $\mathcal{N} = (2,2)$: Compatible with both *holomorphic and topological* twist

Holomorphic twist

↓ Deform

Topologically twisted theory

3d N=4 with boundaries:

- (0,4) b.c for free hypermultiplet: Parametrized deformation,

$$\mathcal{Q}_{\text{hol.}} + \zeta Q_{H,C} \longrightarrow \mathcal{J}_{\perp}^{\text{hol}} + \zeta \mathcal{J}_{\perp}^{H,C} = 0$$

  - C : $\tilde{\mathcal{B}}_D \phi$: (0,4) Dirichlet b.c can be deformed to be compatible with C-twist
  - H : $\tilde{\mathcal{B}}_N \phi$: (0,4) Neumann b.c can be deformed to be compatible with H-twist

- $S_\partial \mapsto S_\partial + \mathcal{S}$, $\delta_{Q_{\text{hol}}} \mathcal{S} = \mathcal{J}_\partial^{Q_{\text{hol.}}}$

  Defo preserving H,C-deformed susy if:

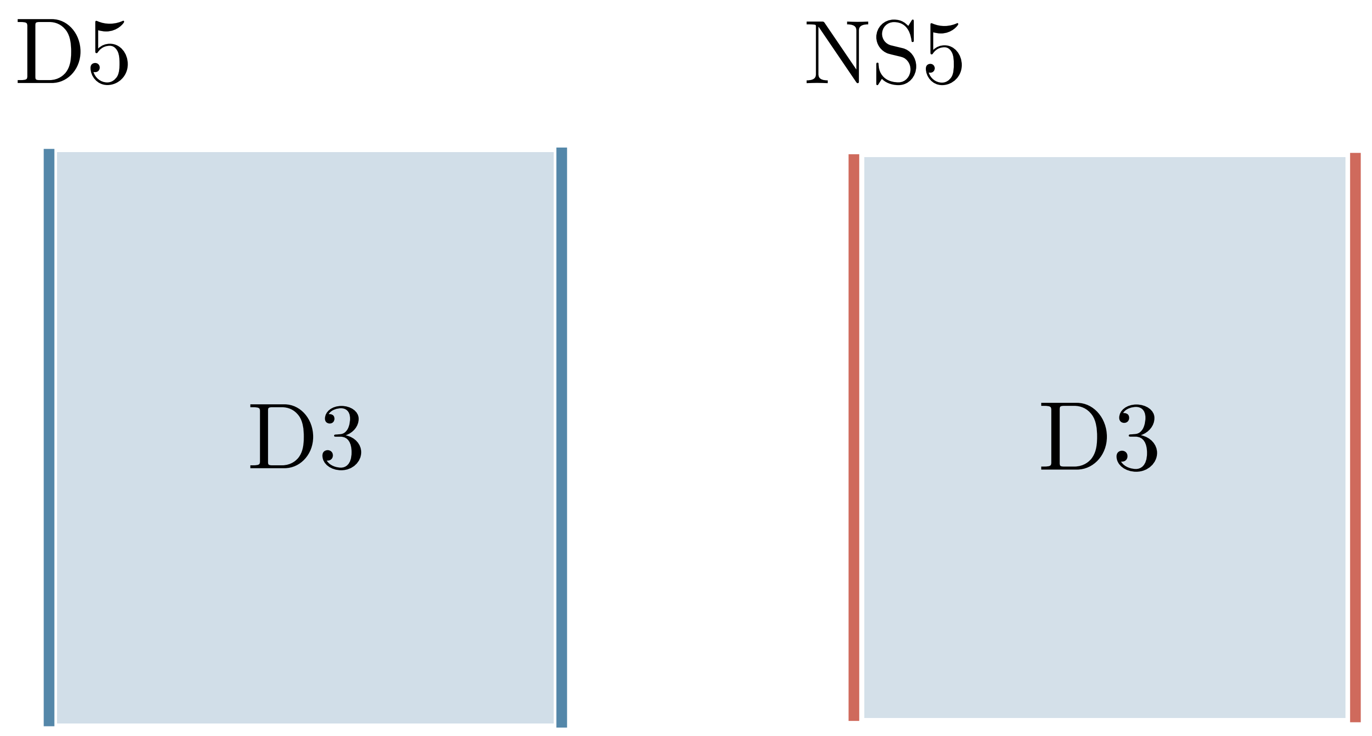
$$\delta_{Q_{\text{hol}}} \tilde{\mathbb{S}} = \mathcal{J}_\partial^{Q_{H,C}}$$

$\mathcal{N}$:	H-twist	$\tilde{\mathbb{S}} = \mathbb{S}_{Sb}$	Symplectic Boson VOA
$\mathcal{D}$:	C-twist	$\tilde{\mathbb{S}} = \mathbb{S}_{Fc}$	Symplectic Fermion VOA

Gaiotto-Rapčak '18

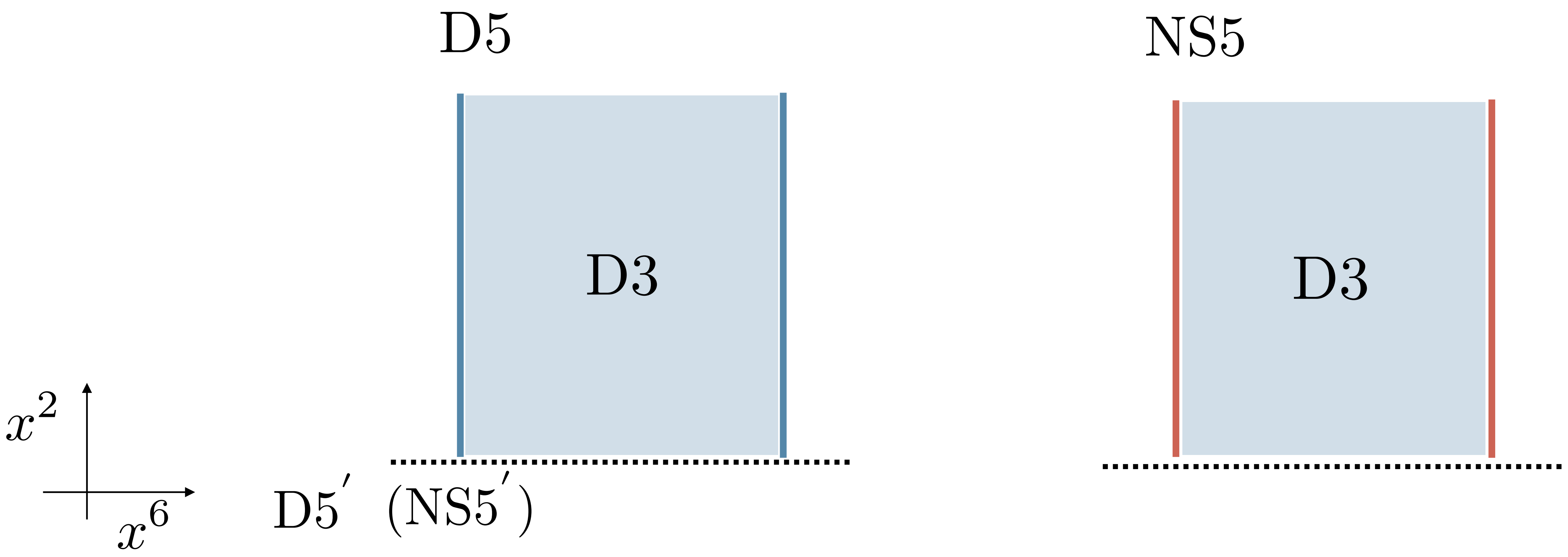
Brane engineering:

- 3d N=4 theories can be realized as brane world volume theories:



	0	1	2	3	4	5	6	7	8	9
D3	•	•	•				•			
D5	•	•	•					•	•	•
NS5	•	•	•	•	•	•				

- Boundaries realized from extra IIB five-branes:



$\mathcal{N} = (0, 4)$

	0	1	2	3	4	5	6	7	8	9
D5′	•	•		•	•	•	•			
NS5′	•	•					•	•	•	•

Hanany-Okazaki '18

Brane engineering:

Twisting homomorphisms:

- Appropriate choice of target space geometry

Bershadsky, Vafa, Sadov '96

	0	1	2	3	4	5	6	7	8	9
D3	•	•	•				•			
D5	•	•	•					•	•	•
NS5	•	•	•	•	•	•				

$$M_3 \hookrightarrow T^*_{H,C} M_3 \subset \mathrm{CY}_3$$

$$\Gamma_{\mu\nu} \to \Gamma_{\mu\nu} + \Gamma_{(\mu+3)(\nu+3)},$$

$$\mu, \nu = 0, 1, 2$$

$$\Gamma_{\mu\nu} \to \Gamma_{\mu\nu} + \Gamma_{(\mu+7)(\nu+7)},$$

Several points intracable:

- Q-cohomology?
- Holomorphic twist?
- Deformations?

Twisted IIB supergravity:

Defined as :

Supergravity background where, the bosonic ghost (q) of local supersymmetry acquires non-vanishing v.e.v

- EoM satisfied for q being a cov. const. spinor: $q = \mathcal{Q}$
- Susy theory on this background : $Q_{BRST} + \mathcal{Q}$

Different choice of bosonic ghosts $\longrightarrow$ different twists

Twisted IIB supergravity:

Interested in a particular vev, giving the *holomorphic* twist of the IIB background:

Holo twist of IIB supergravity: BCOV theory on $\mathbb{C}^5$

Costello, Li '16

- BCOV theory is the closed string field theory of the topological B-model

$\rightarrow$ deformations of CY-complex structure

Bershadsky, Cecotti, Ooguri, Vafa '94

- Fields of the theory in terms of *polyvector* fields:

$$PV^{i,j}(\mathcal{X}) = \Omega^{0,j}(\mathcal{X}, \wedge^i T\mathcal{X}) \qquad PV^{i,j}(\mathcal{X}) \xrightarrow{\Omega \text{v}} \Omega^{d-i,j}(\mathcal{X})$$

- Residula supersymmetries: Focus on $PV^{2,0}, PV^{0,0}$

Twisted IIB supergravity:

Holomorphic Hanany-Witten: the case of the hypermultiplet

$$\begin{array}{cccccccccc} \text{bulk:} & \mathbb{C}_{z_1} & \times & \mathbb{C}_{w_2} & \times & \mathbb{C}_{z_2} & \times & \mathbb{C}_{w_1} & \times & \mathbb{C}_{z_3} \\ & \cong & & \cong & & \cong & & \cong & & \cong \\ & \mathbb{R}^2_{01} & \times & \mathbb{R}^2_{26} & \times & \mathbb{R}^2_{34} & \times & \mathbb{R}^2_{59} & \times & \mathbb{R}^2_{78} \\ \hline \text{D3:} & \mathbb{C}_{z_1} & \times & \mathbb{C}_{w_2} & \times & 0 & \times & 0 & \times & 0. \end{array}$$

✓ Holomorphic configuration on bulk and brane

Next: need to add the D5s

Twisted IIB supergravity:

The D5 support is not along a complex submanifold of $\mathbb{C}^5$

Deform, using the bivector $\partial_{w_1} \wedge \partial_{w_2}$

bulk:	$\mathbb{C}_{z_1}$	$\times$	$\mathbb{C}_{w_2}$	$\times$	$\mathbb{C}_{z_2}$	$\times$	$\mathbb{C}_{w_1}$	$\times$	$\mathbb{C}_{z_3}$
	$\cong$		$\cong$		$\cong$		$\cong$		$\cong$
	$\mathbb{R}^2_{01}$	$\times$	$\mathbb{R}^2_{26}$	$\times$	$\mathbb{R}^2_{34}$	$\times$	$\mathbb{R}^2_{59}$	$\times$	$\mathbb{R}^2_{78}$
D3:	$\mathbb{C}_{z_1}$	$\times$	$\mathbb{C}_{w_2}$	$\times$	0	$\times$	0	$\times$	$0.$
D5:	$\mathbb{C}_{z_1}$	$\times$	$\mathbb{R}_2 \times \partial I_6$	$\times$	0	$\times$	$\mathbb{R}_9$	$\times$	$\mathbb{C}_{z_3}$

Theory topological along $w_{1,2}$

Next: $\mathcal{N} = (0,4)$ boundary

Twisted IIB supergravity:

Introduce *Dirichlet* boundary conditions: D5'

bulk:	$\mathbb{C}_{z_1}$	$\times$	$\mathbb{C}_{w_2}$	$\times$	$\mathbb{C}_{z_2}$	$\times$	$\mathbb{C}_{w_1}$	$\times$	$\mathbb{C}_{z_3}$
	$\cong$		$\cong$		$\cong$		$\cong$		$\cong$
	$\mathbb{R}^2_{01}$	$\times$	$\mathbb{R}^2_{26}$	$\times$	$\mathbb{R}^2_{34}$	$\times$	$\mathbb{R}^2_{59}$	$\times$	$\mathbb{R}^2_{78}$
D3:	$\mathbb{C}_{z_1}$	$\times$	$(\mathbb{R}_+)_2 \times I_6$	$\times$	0	$\times$	0	$\times$	$0.$
D5:	$\mathbb{C}_{z_1}$	$\times$	$\mathbb{R}_2 \times \partial I_6$	$\times$	0	$\times$	$\mathbb{R}_9$	$\times$	$\mathbb{C}_{z_3}$
D5$'$:	$\mathbb{C}_{z_1}$	$\times$	$0 \times \mathbb{R}_6$	$\times$	$\mathbb{C}_{z_2}$	$\times$	$\mathbb{R}_5$	$\times$	0

Which are the admissible deformations?

- Look among residual deformations and control which is compatible with the above:

Twisted IIB supergravity:

Pin down the following two :

$$\mathcal{Q}_H = z_2 \quad \& \quad \mathcal{Q}_C = \partial_{z_1} \wedge \partial_{z_2}$$

✔ • Deforming by $\mathcal{Q}_C$ retains moduli along z_3 while making the rest z-directions topological.

• Holomorphic solutions to the eom

✘ • Deforming by $\mathcal{Q}_H$ retains the z_2 moduli

• No solutions to the eom!

Results in agreement with the original compatibility criterion

(and of course with our field theory analysis c.f. closing remarks ;))

Concluding remarks

- Geometrization of holomorphic twist and on its deformation to topological
- Applicable to more general theories
- Field theory analysis BV

To be studied:

- Defect bulk operators and boundary VOAs
- Mirror-Symmetry checks (branches, holomorphic twist and dual pairs)?
- Theories with higher supersymmetry? (e.g ABJM)

Thank you for your attention…!