Closed string disk amplitudes in the pure spinor formalism

Michael Haack (ASC Munich, LMU)

"*Strings, Geometry and the Swampland*"
Ringberg Castle, Nov. 10, 2021

2011.10392 (with Andreas Bischof)

ARNOLD SOMMERFELD
CENTER FOR THEORETICAL PHYSICS

LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Overview

- Motivation
- Review of pure spinor formalism
- Adaption to purely closed string disk amplitudes
- Application to 2- and 1-point functions

Why disk amplitudes?

- First quantum correction to sphere level

- Higher derivative corrections to DBI action?

  E.g. $\sim e^{-\Phi}\epsilon_{10}\epsilon_{10}R^4$ leads to correction to 4D EH-term

  [Antoniadis, Ferrara, Minasian, Narain (1997)]

- Heterotic / Type I duality seems to predict such a term

  [Green, Rudra (2016)]

- Type II:

$$\underbrace{(\zeta(3)e^{-2\Phi}+\frac{\pi^2}{6})t_8t_8R^4}_{\text{leads to correction to 4D kinetic terms of scalars}} - \underbrace{(\zeta(3)e^{-2\Phi} \overset{\text{IIB (inhereted by I)}}{\underset{\text{IIA}}{\pm}} \frac{\pi^2}{6})\epsilon_{10}\epsilon_{10}R^4}_{\text{leads to correction to 4D EH-term}}$$

[Antoniadis, Ferrara, Minasian, Narain (1997)]

- Heterotic:

$$(\zeta(3)e^{-2\Phi}+\frac{\pi^2}{6})t_8t_8R^4 - \zeta(3)e^{-2\Phi}\epsilon_{10}\epsilon_{10}R^4$$

- How is this compatible with heterotic / type I duality?

$$g^{(I)}_{\mu\nu} = e^{-\Phi_{het}} g^{(het)}_{\mu\nu} \quad , \quad \Phi_I = -\Phi_{het}$$

- Possible answer: $S^{(het)}$ in 10D contains

[Green, Rudra (2016)]

$$\underbrace{\sqrt{g^{(het)}} e^{-\Phi_{het}/2} J_0 E_{3/2}(e^{-\Phi_{het}})}_{\to \sqrt{g^{(I)}} e^{-\Phi_I/2} J_0 E_{3/2}(e^{-\Phi_I})} - \underbrace{\sqrt{g^{(het)}} \frac{\pi^2}{6} \mathcal{I}_2}_{\to \sqrt{g^{(I)}} \frac{\pi^2}{6} e^{-\Phi_I} \mathcal{I}_2}$$

Disk level!

★ $J_0 = t_8 t_8 R^4 - \frac{1}{8}\epsilon_{10}\epsilon_{10} R^4$

[de Roo, Suelmann, Wiedemann (1993)]

★ $\mathcal{I}_2 = -\frac{1}{8}\epsilon_{10}\epsilon_{10} R^4 + \ldots$

★ $E_{3/2} = \zeta(3) e^{-3/2\Phi_{het}} + \frac{\pi^2}{6} e^{\Phi_{het}/2} + non - pert.$

S-duality invariant

- Origin: Disk and/or projective plane

- Direct check requires 5-point graviton amplitude!

- Alternatively: Check for EH-term correction from disk / projective plane in type I on Calabi-Yau Y_3

$$\sim e^{-\Phi}\chi_{Y_3}R$$

- How can disk / projective plane amplitude of gravitons depend on Euler number χ_{Y_3} of Calabi-Yau Y_3 ?

Pure spinor formalism

[Berkovits (2000)]

- Manifestly super-Poincare covariant
- Loop amplitudes calculable via non-minimal formulation
- Equivalence to RNS formalism shown in many examples (mainly amplitudes of massless states; for examples with massive states cf. [Chakrabarti, Kashyap, Verma (2018)])

- Can lead to simplified calculations of amplitudes

  - ★ Complete quartic effective action of type II at tree level [Policastro, Tsimpis (2006)]

  - ★ Closed string 4-point *3-loop* amplitude in type II at low energy [Gomez, Mafra (2013)]

  - ★ Arbitrary n-point amplitude of massless open strings on the disk [Mafra, Schlotterer, Stieberger (2011)]

CFT

- Type IIB-action for 10D flat space-time:

$$S = \frac{1}{2\pi}\int \mathrm{d}^2 z \left(\tfrac{1}{2}\partial X^m \overline{\partial} X_m + p_\alpha \overline{\partial}\theta^\alpha + \overline{p}_\alpha \partial \overline{\theta}^\alpha + w_\alpha \overline{\partial}\lambda^\alpha + \overline{w}_\alpha \partial \overline{\lambda}^\alpha\right)$$

$$(\lambda\gamma^m\lambda) = 0$$

i.e. λ pure spinor

- λ^α, w_α commuting

- $h(\theta^\alpha) = h(\lambda^\alpha) = 0,\ h(p_\alpha) = h(w_\alpha) = 1$

- Supersymmetric fields (relevant for vertex operators):

$$\Pi^m = \partial X^m + \tfrac{1}{2}(\theta\gamma^m\partial\theta) \ ,$$

$$d_\alpha = p_\alpha - \tfrac{1}{2}\left(\partial X^m + \tfrac{1}{4}(\theta\gamma^m\partial\theta)\right)(\gamma_m\theta)_\alpha$$

(antiholomorphic analogs)

- OPEs (needed to perform contractions):

$$X^m(z,\overline{z})X^n(w,\overline{w}) = -\eta^{mn}\ln|z-w|^2$$

$$p_\alpha(z)\theta^\beta(w) = \frac{\delta_\alpha{}^\beta}{z-w} \qquad , \qquad w_\alpha(z)\lambda^\beta(w) = -\frac{\delta_\alpha{}^\beta}{z-w}$$

- Nilpotent BRST operator:

$$Q = \oint \frac{\mathrm{d}z}{2\pi i} \lambda^\alpha(z) d_\alpha(z)$$

$$(\lambda\gamma^m\lambda) = 0 \qquad \Longrightarrow \qquad Q^2 = 0$$

$$d_\alpha(z) d_\beta(w) = -\frac{\gamma^m_{\alpha\beta}\Pi_m(w)}{z-w}$$

- Cohomology of Q coincides with superstring spectrum

[Berkovits (2000)]

Massless vertex operators

- Open string
  - ★ Unintegrated

$$V^{(0)}(z) = \left[\lambda^\alpha A_\alpha(X,\theta)\right](z)$$

  - ★ Integrated

$$\tfrac{1}{2}(w\gamma^{mn}\lambda)$$

$$V^{(1)}(z) = \left[\partial\theta^\alpha A_\alpha(X,\theta) + \Pi^m A_m(X,\theta) + d_\alpha W^\alpha(X,\theta) + \tfrac{1}{2}N^{mn}\mathcal{F}_{mn}(X,\theta)\right](z)$$

  - ★ $QV^{(0)} = 0 \quad , \quad QV^{(1)} = \partial V^{(0)}$ (cf. RNS)

★ E.g.: Gauge field with polarisation ξ_m :

$$A_\alpha(X,\theta) = e^{ik\cdot X}\left\{\frac{\xi_m}{2}(\gamma^m\theta)_\alpha - \frac{1}{16}(\gamma_p\theta)_\alpha(\theta\gamma^{mnp}\theta)ik_{[m}\xi_{n]} + \mathcal{O}(\theta^5)\right\}$$

$$A_m(X,\theta) = e^{ik\cdot X}\left\{\xi_m - \frac{1}{4}ik_p(\theta\gamma_m{}^{pq}\theta)\xi_q + \mathcal{O}(\theta^4)\right\}$$

$$W^\alpha(X,\theta) = e^{ik\cdot X}\left\{-\frac{1}{2}ik_{[m}\xi_{n]}(\gamma^{mn}\theta)^\alpha + \mathcal{O}(\theta^3)\right\}$$

$$\mathcal{F}_{mn}(X,\theta) = e^{ik\cdot X}\left\{2ik_{[m}\xi_{n]} - \frac{1}{2}ik_{[p}\xi_{q]}ik_{[m}(\theta\gamma_{n]}{}^{pq}\theta) + \mathcal{O}(\theta^4)\right\}$$

- Closed string (G_{mn}, B_{mn}, Φ, i.e. NS-NS-fields)
  - ★ $\epsilon_{mn} = \xi_m \otimes \overline{\xi}_n$
  - ★ $V^{(a,b)}(z,\overline{z}) = V^{(a)}(z) \otimes \overline{V}^{(b)}(\overline{z})$, $\qquad a, b \in \{0, 1\}$
  - ★ $V^{(a,b)}$ contains factor

$$e^{ik\cdot X(z)} e^{ik\cdot \overline{X}(\bar{z})} = e^{ik\cdot [X(z) + \overline{X}(\bar{z})]} = e^{ik\cdot X(z,\bar{z})}$$

Tree level correlators

- After fixing conformal Killing group:

$$\mathcal{A}_{S^2}^{\text{closed}}(1,2,\ldots,n) = \left\langle V_1^{(0,0)}(z_1,\bar{z}_1)\prod_{i=2}^{n-2}\int \mathrm{d}^2 z_i V_i^{(1,1)}(z_i,\bar{z}_i) V_{n-1}^{(0,0)}(z_{n-1},\bar{z}_{n-1}) V_n^{(0,0)}(z_n,\bar{z}_n)\right\rangle$$

$$\mathcal{A}_{D_2}^{\text{open}}(1,2,\ldots,n) = \left\langle V_1^{(0)}(z_1)\prod_{i=2}^{n-2}\int_{z_{i-1}}^{z_{n-1}} \mathrm{d}z_i V_i^{(1)}(z_i) V_{n-1}^{(0)}(z_{n-1}) V_n^{(0)}(z_n)\right\rangle$$

take this
as example

- Integrate out non-zero modes via Wick's theorem, using

$$\langle X^m(z) X^n(w)\rangle = -\eta^{mn}\ln(z-w)$$

$$\langle p_\alpha(z)\theta^\beta(w)\rangle = \frac{\delta_\alpha{}^\beta}{z-w} \quad , \quad \langle w_\alpha(z)\lambda^\beta(w)\rangle = -\frac{\delta_\alpha{}^\beta}{z-w}$$

- Tree level: only $h=0$ fields $X^m, \theta^\alpha, \lambda^\alpha$ have zero modes

$\Longrightarrow$ All $h=1$ fields $\partial\theta^\alpha, \Pi^m, d_\alpha, N^{mn}$ have to be integrated out via Wick's theorem

$\Longrightarrow$ After Wick contractions and integrating out X^m zero modes, one ends up with:

$$\left\langle V_1^{(0)}(z_1)\prod_{i=2}^{n-2}V_i^{(1)}(z_i)V_{n-1}^{(0)}(z_{n-1})V_n^{(0)}(z_n)\right\rangle = \delta(\Sigma_{i=1}^n k_i)\,\langle\lambda^\alpha\lambda^\beta\lambda^\gamma f_{\alpha\beta\gamma}(\theta;z_i)\rangle_0$$

zero mode integration of X^m

zero mode integration of $\theta^\alpha, \lambda^\alpha$

- Zero mode prescription:

$$\underbrace{\langle(\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta)\rangle_0}_{\text{unique element of } Q\text{-cohomology with 3 factors of } \lambda} = 1$$

- Projects out coefficients of $\lambda^3\theta^5$-terms of

$$\lambda^\alpha\lambda^\beta\lambda^\gamma f_{\alpha\beta\gamma}(\theta; z_i)$$

- Need to integrate over z_i , $i = 2, \ldots, n-2$

Closed string disk amplitudes

[Bischof, M.H.]

- Earlier results on 3-pt fct of 1 closed and 2 open states in type I [Alencar (2009); Alencar, Tahim, Landim, Costa Filho (2011)]

- Type IIB with Dp-brane along $X^1, \ldots, X^p$

- $V^{(a,b)}(z,\bar{z}) = V^{(a)}(z)\overline{V}^{(b)}(\bar{z}) \quad , \quad z \in \mathbb{H}_+$

- Employ doubling trick: $\overline{X}^m(\bar{z}) = D^m{}_n X^n(\bar{z})$

$$D^{mn} = \begin{cases} \eta^{mn} & m,n \in \{0,1,\ldots,p\} \\ -\eta^{mn} & m,n \in \{p+1,\ldots,9\} \\ 0 & \text{otherwise} \end{cases}$$

- Doubling trick for spinors

[Garousi, Myers; Hashimoto, Klebanov; Gubser, Hashimoto, Klebanov, Maldacena (1996)]

$$\overline{\Psi}^{\alpha}(\overline{z}) = M^{\alpha}{}_{\beta}\Psi^{\beta}(\overline{z}) \quad , \quad \overline{\Psi}_{\alpha}(\overline{z}) = N_{\alpha}{}^{\beta}\Psi_{\beta}(\overline{z})$$

- Relations between D, M, N (e.g. $N = (M^T)^{-1}$) allow rewriting:

$$\overline{V}^{(0)}(\overline{z}) = \Big(\overline{\lambda}^{\alpha}\overline{A}_{\alpha}[\overline{\xi}, k](\overline{X}, \overline{\theta})\Big)(\overline{z}) = \Big(\lambda^{\alpha}A_{\alpha}[D\cdot\overline{\xi}, D\cdot k](X, \theta)\Big)(\overline{z})$$

$$\overline{V}^{(1)}(\overline{z}) = \Big(\overline{\partial}\theta^{\alpha}A_{\alpha}[D\cdot\overline{\xi}, D\cdot k](X,\theta) + \Pi^{m}A_{m}[D\cdot\overline{\xi}, D\cdot k](X,\theta)$$
$$+ d_{\alpha}W^{\alpha}[D\cdot\overline{\xi}, D\cdot k](X,\theta) + \frac{1}{2}N^{mn}\mathcal{F}_{mn}[D\cdot\overline{\xi}, D\cdot k](X,\theta)\Big)(\overline{z})$$

$\Longrightarrow$ Can use same contractions as above, but allow both z and $\bar{z}$

- Conformal Killing group $PSL(2,\mathbb{R})$ only allows to fix one and a half closed string vertex operators

$$\Longrightarrow \mathcal{A}^{\text{closed}}_{D_2}(1,\ldots,n) =$$

following [Hoogeveen, Skenderis (2007)]

$$= 2ig_c^n\tau_p\int_0^1 \mathrm{d}y\,\left\langle V_1^{(0)}(iy)\overline{V}_1^{(1)}(-iy)\prod_{j=2}^{n-1}\int_{\mathbb{H}_+}\mathrm{d}^2z_j V_j^{(1,1)}(z_j,\overline{z}_j)V_n^{(0)}(i)\overline{V}_n^{(0)}(-i)\right\rangle$$

D-brane tension

cf. also [Grassi, Tamassia (2004); Alencar, Tahim, Landim, Costa Filho (2011)]

2-pt fct

$$\mathcal{A}^{\text{closed}}_{D_2}(1,2) = 2ig_c^2\tau_p \int_0^1 \mathrm{d}y \left\langle V_1^{(0)}(iy)\overline{V}_1^{(1)}(-iy)V_2^{(0)}(i)\overline{V}_2^{(0)}(-i) \right\rangle$$

$$= 2ig_c^2\tau_p \int_0^1 \mathrm{d}y \Big\langle (\lambda A_1[\xi_1,k_1])(iy)\Big(\overline{\partial}\theta^\alpha A_{1\alpha}[D\cdot\overline{\xi}_1, D\cdot k_1] + \Pi^m A_{1m}[D\cdot\overline{\xi}_1, D\cdot k_1]$$

$$+d_\alpha W_1^\alpha[D\cdot\overline{\xi}_1, D\cdot k_1] + \frac{1}{2}N^{mn}\mathcal{F}_{1mn}[D\cdot\overline{\xi}_1, D\cdot k_1]\Big)(-iy)(\lambda A_2[\xi_2,k_2])(i)(\lambda A_2[D\cdot\overline{\xi}_2, D\cdot k_2])(-i)\Big\rangle$$

kinematic factors

$$\sim g_c^2\tau_p \int_0^1 \mathrm{d}y \; \underbrace{\left(\frac{4y}{(1+y)^2}\right)^{k_1\cdot D\cdot k_1} \left(\frac{(1-y)^2}{(1+y)^2}\right)^{k_1\cdot k_2}}_{\text{Koba-Nielsen factor}} \left(\frac{d_1}{2y} + \frac{d_2}{1+y} + \frac{d_3}{1-y}\right)$$

- E.g.

$$d_1 = \langle i(\lambda A_1[\xi_1, k_1])k_1\cdot A_1[D\cdot\overline{\xi}_1, D\cdot k_1](\lambda A_2[\xi_2, k_2])(\lambda A_2[D\cdot\overline{\xi}_2, D\cdot k_2])$$
$$+A_{1m}[\xi_1, k_1](\lambda\gamma^m W_1[D\cdot\overline{\xi}_1, D\cdot k_1])(\lambda A_2[\xi_2, k_2])(\lambda A_2[D\cdot\overline{\xi}_2, D\cdot k_2])\rangle_0$$

we applied zero mode prescription using *Cadabra* [Peeters]

- $$\mathcal{A}^{\text{closed}}_{D_2}(1,2) \sim g_c^2\tau_p \frac{\Gamma(-t/2)\Gamma(2q^2)}{\Gamma(1-t/2+2q^2)}\left(2\overset{\frac{1}{2}k_1\cdot D\cdot k_1}{\overbrace{q^2}}a_1 + \frac{\overset{-2k_1\cdot k_2}{\overbrace{t}}}{2}a_2\right)$$

$$a_1 = \mathrm{Tr}(\epsilon_1\cdot D)k_1\cdot\epsilon_2\cdot k_1 - k_1\cdot\epsilon_2\cdot D\epsilon_1\cdot k_2 - k_1\cdot\epsilon_2\cdot\epsilon_1^T\cdot D\cdot k_1$$
$$-k_1\cdot\epsilon_2^T\cdot\epsilon_1\cdot D\cdot k_1 - k_1\cdot\epsilon_2\cdot\epsilon_1^T\cdot k_2 + q^2\mathrm{Tr}(\epsilon_1\cdot\epsilon_2^T) + \{1\leftrightarrow 2\}$$

Same as RNS

[Hashimoto, Klebanov; Garousi, Myers (1996)]

$$a_2 = \mathrm{Tr}(\epsilon_1\cdot D)(k_2\cdot D\cdot\epsilon_2\cdot D\cdot k_2 + k_1\cdot\epsilon_2\cdot D\cdot k_2 + k_2\cdot D\cdot\epsilon_2\cdot k_1)$$
$$+k_1\cdot D\cdot\epsilon_1\cdot D\cdot\epsilon_2\cdot D\cdot k_2 - k_2\cdot D\cdot\epsilon_2\cdot\epsilon_1^T\cdot D\cdot k_1 + q^2\mathrm{Tr}(\epsilon_1\cdot D\cdot\epsilon_2\cdot D)$$
$$-q^2\mathrm{Tr}(\epsilon_1\cdot\epsilon_2^T) - (q^2 - \tfrac{t}{4})\mathrm{Tr}(\epsilon_1\cdot D)\mathrm{Tr}(\epsilon_2\cdot D) + \{1\leftrightarrow 2\}$$

1-pt fct

- Earlier work in bosonic theory and RNS [Douglas, Grinstein (1987); Liu, Polchinski (1988); Ohta (1987)]
- Pure spinor: At most 2 factors of λ^α (in $V^{(0,0)}$) !?
- Alternative zero mode prescription (equivalent for higher-point tree amplitudes): [Berkovits (2016)]

$$\langle 1 \rangle_0 = 1$$

corresponds to only alternative scalar element of Q-cohomology

- $$\mathcal{A}^{\text{closed}}_{D_2}(1) = g_c \tau_p \int_{\mathbb{H}_+} \frac{\mathrm{d}^2 z}{V_{\text{CKG}}} \Big\langle \underbrace{V^{(1)}(z,\bar{z})}_{V^{(1)}(z)\overline{V}^{(1)}(\bar{z})} \Big\rangle$$

infinite

$\Longrightarrow$ Fix position of $V^{(1)}$ to $z = i$ and divide by volume of $K \subset PSL(2,\mathbb{R})$ leaving $z = i$ invariant

$$K = \left\{ \left(\begin{array}{cc} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{array} \right) \middle| \; \theta \in [0, 2\pi] \right\} \quad , \quad \frac{\cos\theta \; i + \sin\theta}{-\sin\theta \; i + \cos\theta} = i$$

volume: 2π

$$\Longrightarrow \quad \mathcal{A}^{\text{closed}}_{D_2}(1) \sim g_c \tau_p \mathrm{Tr}(\epsilon \cdot D)$$

For graviton this corresponds to linearisation of $\tau_p \int d^p x \sqrt{-G}$

- Alternative approach [Kashyap (2020)]

Outlook

- RR-fields & fermions
- Projective plane cf. [Garousi (2006)] for RNS
- Higher n-points
- Higher derivative corrections to DBI

Thank you!