The anomaly that was not meant IIB

Based on:

2107.14227 with Arun Debray, Jonathan J. Heckman, and Miguel Montero

(+ to appear, see also 2012.00013 with J.J. Heckman)

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The anomaly that was not meant IIB

Is the **duality of type IIB anomalous**?

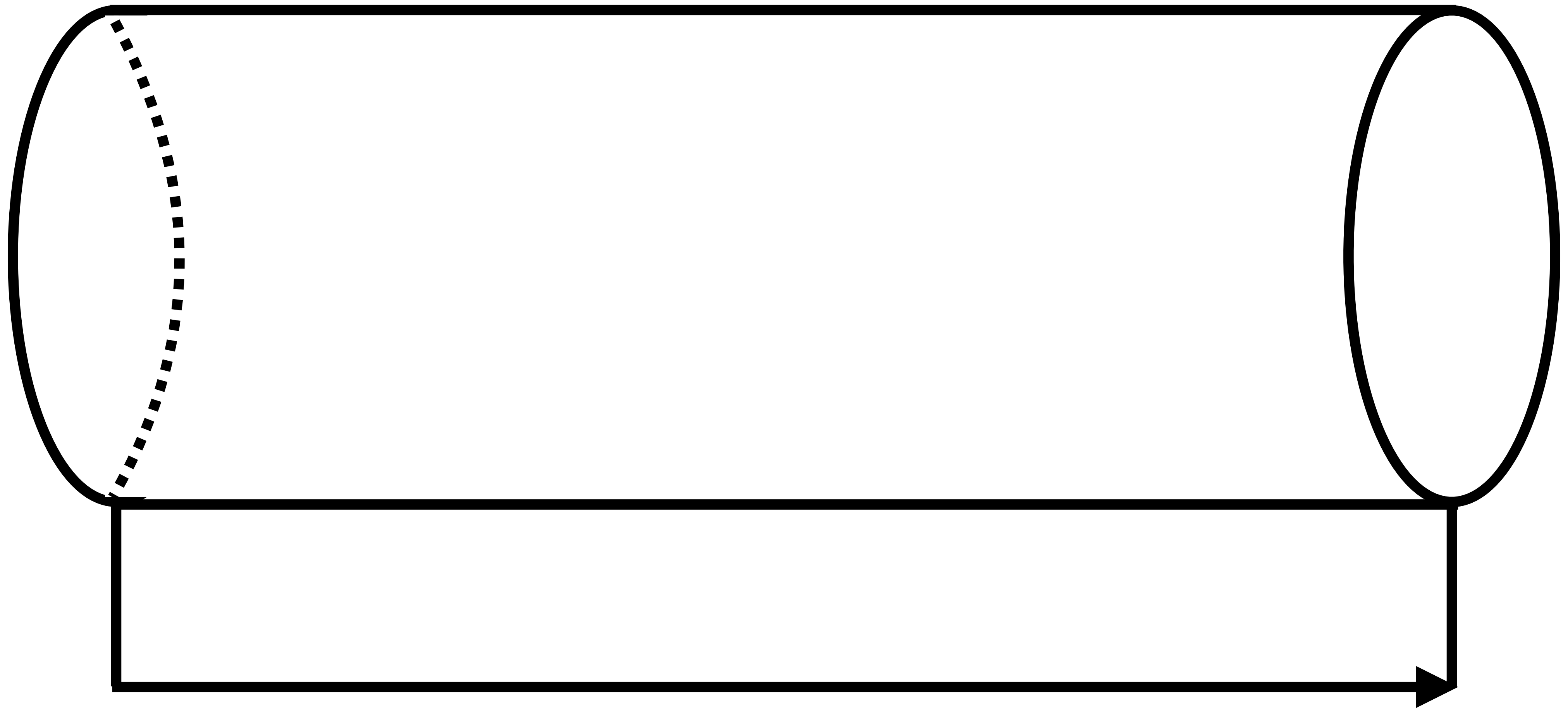
(Broken by quantum corrections)

Anomaly:

- **Couple symmetry to background connection** A
- **Move in configuration space** $A \to A^g$
- **Calculate partition function**

$$Z[A] \neq Z[A^g]$$

Geometrize



$$A \to A^g$$

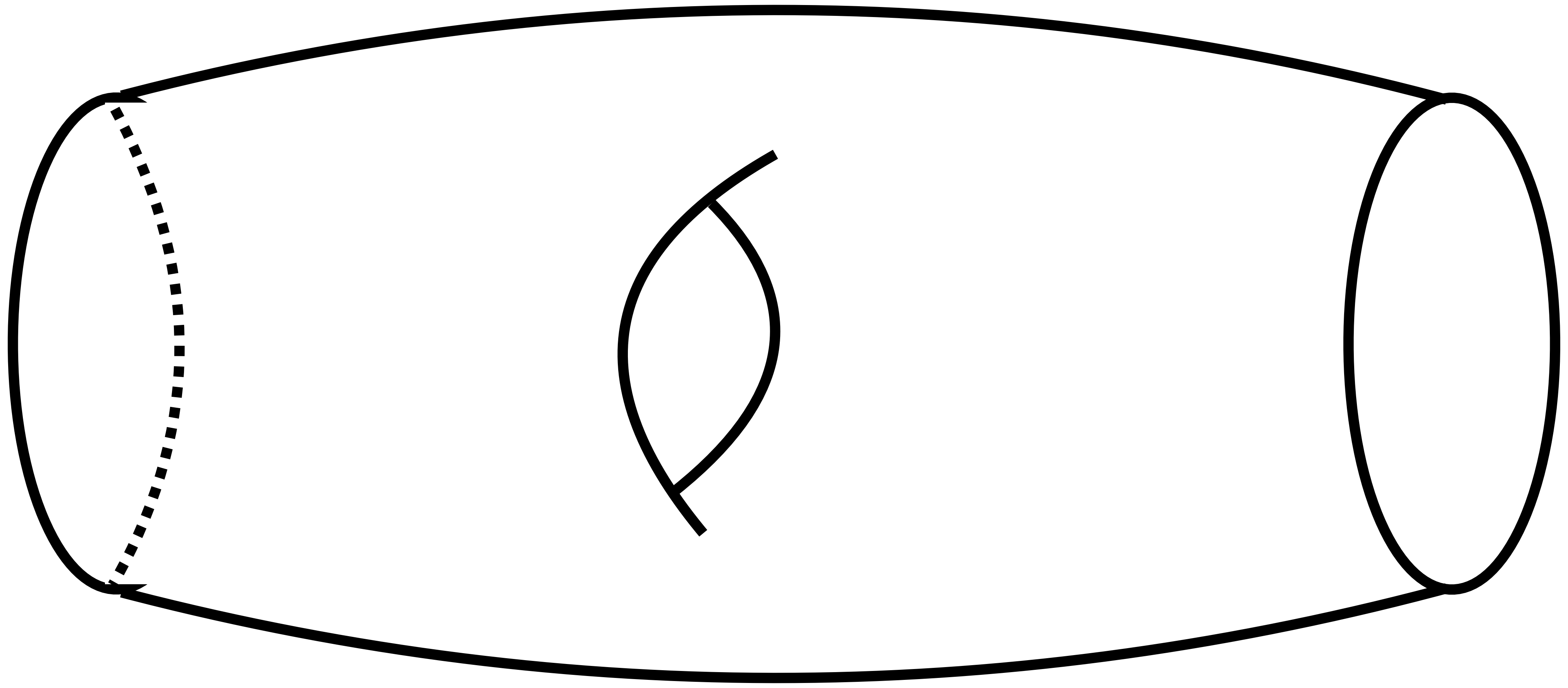
- **Small variations** (contractible paths): **perturbative anomalies**
- **Large variations** (non-contractible paths): **global anomalies**

➔ (d+1)-dimensional manifold (mapping tori)

[Witten '82]

Dai-Freed anomaly

[Dai, Freed ’94], [Witten ’15], [Yonekura ’16], see also [Montero, Garcia-Etxebarria ’18] for a great review



We demand absence of these!

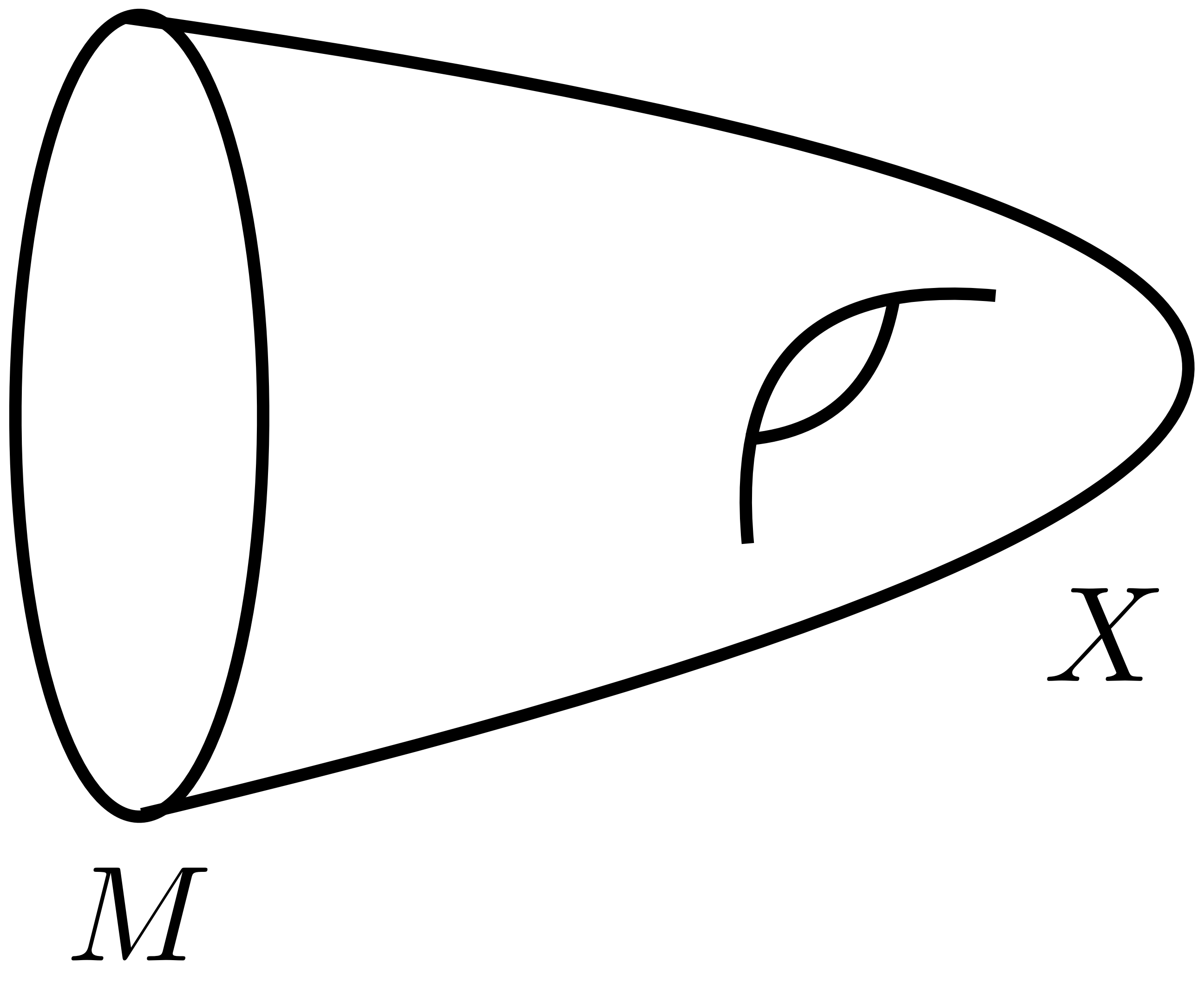
- **Topology changes along path ➔ ‘quantum gravity’ flavor**
- Forms **(d+1)-dimensional manifold** with given structure
- **Detected** by evaluation of **(d+1)-dimensional anomaly theory**

Anomaly field theory

e.g. [Freed, Teleman '14]

$$Z[M] = e^{2\pi i \mathcal{A}(X)}\,, \quad \partial X = M$$

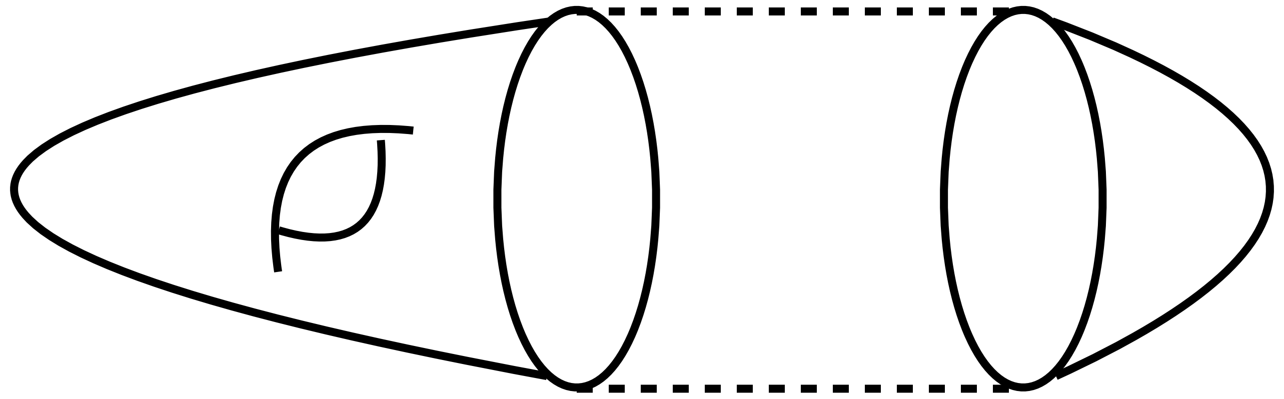
- **Physical data extends** from M to X
- **No Dai-Freed anomalies if it does not depend on extension**



$$\rightarrow \; e^{2\pi i \mathcal{A}[X]} = 1$$

for **closed manifolds** with wanted **structure**
up to deformations

$$\rightarrow \quad \Omega_{d+1}^{\text{structure}}$$



classified by **bordism groups**

Duality anomaly for type IIB

- **Find correct bordism group**

$$\Omega_{11}^{\text{IIB-duality}}$$

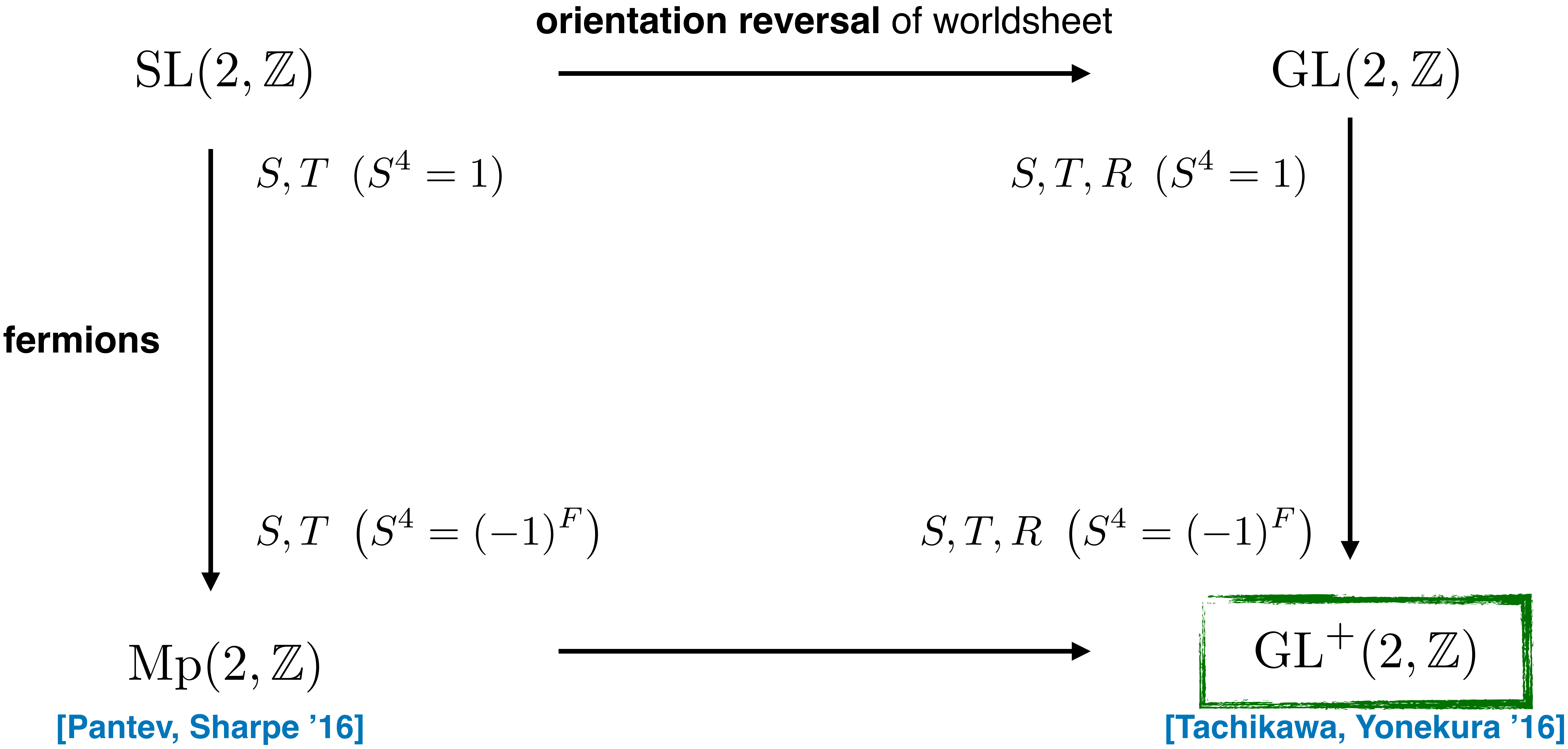
- **Find generator of bordism group**
- **Determine the anomaly theory**

$$\mathcal{A}^{\text{IIB}}$$

- **Make sure that**

$$e^{2\pi i \mathcal{A}^{\text{IIB}}} = 1$$

Duality group for type IIB



The bordism group

- **Decomposition into easier parts** (amalgam structure, p-equivalences, …)
- **Atiyah-Hirzebruch** and **Adams spectral sequences** (highly non-trivial)

$$\Omega_{11}^{\text{Spin-GL}^+(2,\mathbb{Z})} = (\mathbb{Z}_2)^{\oplus 9} \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_{27}$$

Manifolds with twisted GL(2,Z) structure

➛ **Many potential anomalies!**

The generators

- Hints from **eta-invariants** and **cohomology classes**
- A lot of **(educated) guess work**

  - **Lens spaces** L_k^n
  - **Lens space bundles** Q_4^{11}
  - **Real projective spaces**
  - **Arcanum XI** $X_{11}, \widetilde{X_{11}}$

  with appropriate bundles

Factor	Generator
$\mathbb{Z}_{27}$	L_3^{11}
$\mathbb{Z}_3$	$\mathbb{HP}^2 \times L_3^3$
$\mathbb{Z}_8$	Q_4^{11}
$\mathbb{Z}_2$	$\mathbb{HP}^2 \times L_4^3$
$\mathbb{Z}_2$	$\mathbb{RP}^{11}$
$\mathbb{Z}_2$	$\widetilde{\mathbb{RP}^{11}}$
$\mathbb{Z}_2$	$\mathbb{HP}^2 \times \mathbb{RP}^3$
$\mathbb{Z}_2$	$\mathbb{HP}^2 \times \widetilde{\mathbb{RP}^3}$
$\mathbb{Z}_2$	$X_{10} \times S^1$
$\mathbb{Z}_2$	$X_{10} \times \widetilde{S^1}$
$\mathbb{Z}_2$	X_{11}
$\mathbb{Z}_2$	$\widetilde{X_{11}}$

The anomaly theory

[Hsieh, Tachikawa, Yonekura '20]

$$\mathcal{A}(X) = \underbrace{\eta_1^{\mathrm{RS}}(X) - 2\eta_1^{\mathrm{D}}(X) - \eta_{-3}^{\mathrm{D}}}_{\text{fermions}}(X) - \frac{1}{8}\underbrace{\eta_-^{\mathrm{sig}}(X) + \mathrm{Arf}(X) - \tilde{\mathcal{Q}}}_{\text{4-form}}(\breve{c})$$

- Contribution from **signature operator to index theorem** $\eta_-^{\mathrm{sig}}(X)$
- **Requires the introduction of quadratic refinement** $\tilde{\mathcal{Q}}$

of bilinear pairing in differential cohomology

$$\mathrm{Arf}(\tilde{\mathcal{Q}}) = \frac{1}{2\pi}\arg\Big(\sum_{a\in A} e^{2\pi i\tilde{\mathcal{Q}}(a)}\Big)$$

topologically non-trivial part

$\tilde{\mathcal{Q}}(\breve{c})$ coupling to background, e.g.

$C_4 \wedge F_3 \wedge H_3$

not considered here

Physical assumption: there is a **canonical choice for** $\tilde{\mathcal{Q}}$

The duality anomaly

$$\mathcal{A}(X) = 0 \bmod \mathbb{Z}$$

$\mathbb{Z}_2$	$\mathbb{RP}^{11}$
$\mathbb{Z}_2$	$\widetilde{\mathbb{RP}^{11}}$
$\mathbb{Z}_2$	$\mathbb{HP}^2 \times \mathbb{RP}^3$
$\mathbb{Z}_2$	$\mathbb{HP}^2 \times \widetilde{\mathbb{RP}^3}$
$\mathbb{Z}_2$	$X_{10} \times S^1$
$\mathbb{Z}_2$	$X_{10} \times \widetilde{S^1}$

→ $\mathcal{A}(X) = 0$

$\mathbb{Z}_2$	X_{11}
$\mathbb{Z}_2$	$\widetilde{X_{11}}$

→ $\mathcal{A}(X) = ? \quad (0 \text{ or } \frac{1}{2})$

we don't know how to evaluate, let's assume it is OK

The duality anomaly

$\mathbb{Z}_{27}$	L_3^{11}	➡	$\mathcal{A}(X) = \frac{1}{3}$
$\mathbb{Z}_3$	$\mathbb{HP}^2 \times L_3^3$	➡	$\mathcal{A}(X) = \frac{1}{3}$
$\mathbb{Z}_8$	Q_4^{11}	➡	$\mathcal{A}(X) = \frac{k}{4}$
$\mathbb{Z}_2$	$\mathbb{HP}^2 \times L_4^3$	➡	$\mathcal{A}(X) = \frac{1}{2}$

➡ **Duality is anomalous**

Meant IIB: The anomaly that was not

Anomaly cancellation

Remember: $\widetilde{\mathcal{Q}}(\breve{c})$ Choose $\breve{c}$ in terms of bordism invariants such that **anomaly is cancelled**

- **Characteristic classes of bundle**

$$\mathbb{Z}_3 : \quad a\,, \beta(a)\,, \qquad \mathbb{Z}_4 : \quad b\,, \beta(b)$$

$a, b :$ discrete gauge fields $\quad \beta(a), \beta(b) :$ discrete fieldstrengths

- **Reductions of characteristic classes of manifold**

$$(p_k)_n\,, w_k\,, \mathcal{P}(w_k)\ \ldots$$

Anomaly cancellation

Let's take: $S^{11}/\mathbb{Z}_3$ which generates the $\mathbb{Z}_{27}$ factor

Quadratic refinement is given by:

$$\widetilde{\mathcal{Q}}(n) = \frac{1}{3}n^2$$

Anomaly is given by:

Chance: 1 in 26

$$\mathcal{A}[S^{11}/\mathbb{Z}_3] = \frac{1}{3} - \widetilde{\mathcal{Q}}(\breve{c})$$

$\breve{c} = \beta(a)^2 \cup a$ evaluates to 1 on $S^{11}/\mathbb{Z}_3$ → Anomaly can be cancelled!

Anomaly cancellation

	Class	$\mathcal{A}$	Arf	$\tilde{\mathcal{Q}}$	$\beta(a)^2 \cup a$	$\frac{(p_1)_3}{2} \cup a$	$\beta(b)^2 \cup b$	$\frac{1}{2}\left[(p_1)_4 - \mathcal{P}(w)\right] \cup b$
	L_3^{11}	$1/3$	$1/4$	$n^2/3$	1	0	0	0
	$L_3^3 \times \mathbb{HP}^2$	$1/3$	$1/4$	$n^2/3$	0	1	0	0
$2Q_4^{11} \longrightarrow$	L_4^{11}	$1/2$	$3/8$	$3n^2/8$	0	0	0	2
	$L_4^3 \times \mathbb{HP}^2$	$1/2$	$3/8$	$3n^2/8$	0	0	0	2

All the (calculable) **anomalies canceled** if we include: $(C_4, \check{c}_0) \approx F_5 \wedge \ldots$

$$\tilde{\mathcal{Q}}(\check{c}_0) \text{ with } \check{c}_0 = \left(\lambda_1 \beta(a)^2 + \lambda_2 \frac{(p_1)_3}{2}\right) \cup a + \frac{\lambda_3}{2}\left[(p_1)_4 - \mathcal{P}(w_2)\right] \cup b + \kappa \beta(b)^2 \cup b$$

$\lambda_i \in \{-1, +1\}\,, \;\; \kappa \in \mathbb{Z} \bmod 4$ (other physical systems (S-folds) suggest $\lambda_{1,3} = 1$)

The anomaly meant: That was not (type) IIB?!?

Alternatives

[Garcia-Etxebarria, Hayashi, Ohmori, Tachikawa, Yonekura '17]

Add sector **without local degrees of freedom** Ξ :

$$Z_\Xi(M) = e^{-2\pi i \mathcal{A}(X)}$$

$$\rightarrow \; Z_\Xi(M) Z_{\mathrm{IIB}}(M) \text{ invariant}$$

E.g. higher-dimensional '**BF theory**':

$$S_{\mathrm{BF}} \sim (B, A) \approx \int_M B \wedge F$$

B (d-p)-form

$F = dA$

A (p-1)-form

where B and A know about duality and tangent bundle

Modifications to theory

A and B **couple** naturally to **extended objects** (**completeness** hypothesis)

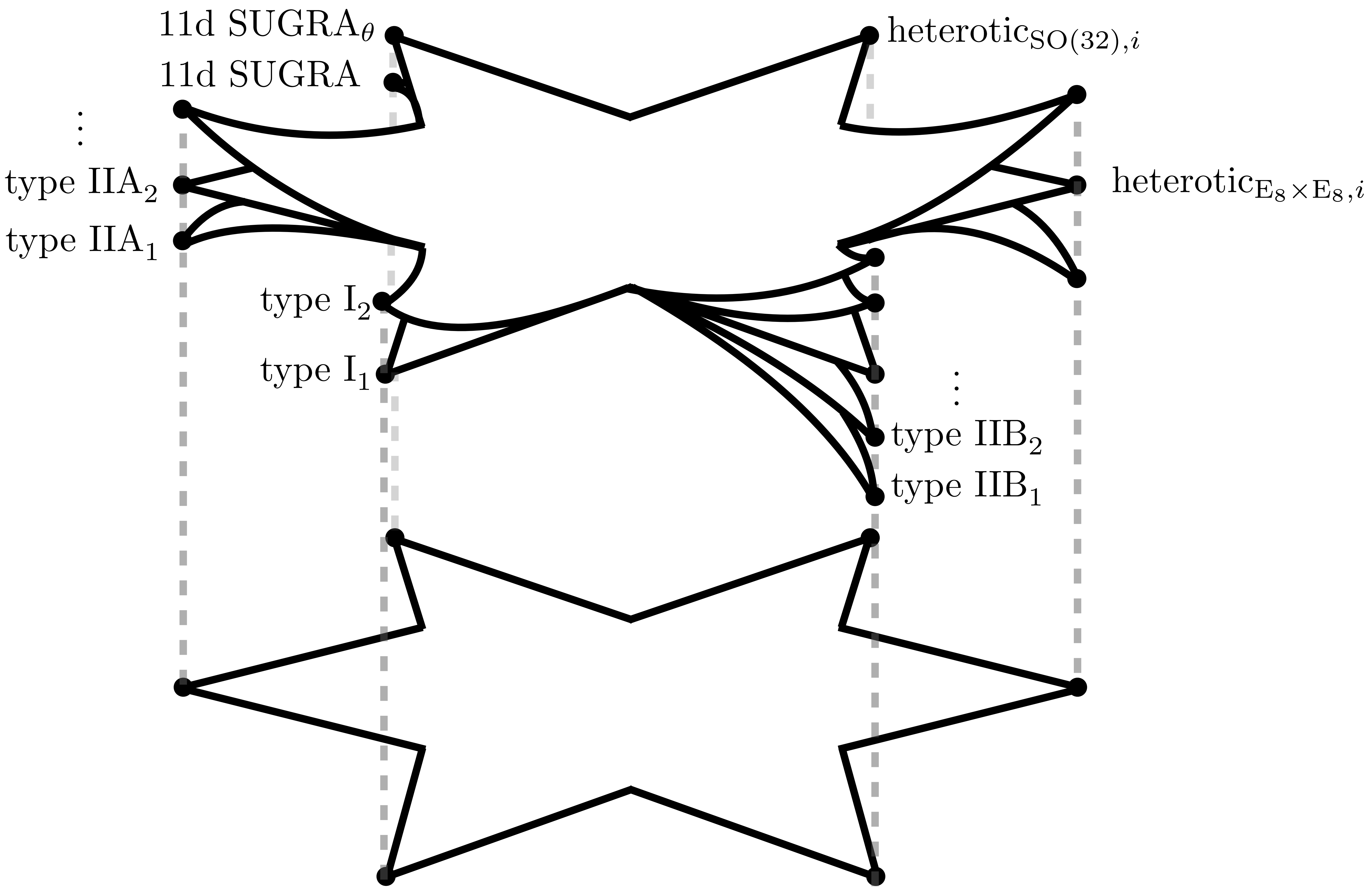
$$\exp\left(2\pi i \int_{\Sigma_{p-1}} A\right)$$

$$\exp\left(2\pi i \int_{\Sigma_{d-p}} B\right)$$

- **New objects**
- **Known IIB backgrounds forbidden by ‘tadpole cancellation’**

Two possibilities:

- **Topological** GS in the **Swampland** ➔ Why?
- Alternative consistent UV completions ➔ **Discrete Landscape**



Domain walls connecting the different possibilities (cobordism conjecture)

[McNamara, Vafa ’19], [Montero, Vafa ’20]

Outlook

[Debray, Dierigl, Heckman, Montero soon(ish)]

What about

$$\Omega_d^{\text{Spin}}\big((BSL(2,\mathbb{Z})\big)\,, \quad \Omega_d^{\text{Spin-Mp}(2,\mathbb{Z})}\,, \quad \Omega_d^{\text{Spin-GL}^+(2,\mathbb{Z})}$$

for d < 11

Cobordism Conjecture tells us that Ω_{QG} **should vanish**

[McNamara, Vafa ’19], [Montero, Vafa ’20]

- **Non-trivial classes forbidden** (‘selection rules’, tadpole cancellation)
- **New defects killing ‘breaking’ the non-trivial classes**

When you open Polchinski you should be aware,
There likely are terms that are not yet in there,
If you want a fully invariant action,
Under duality and worldsheet reflection.

The quadratic refinement can help us to see,
The duality action is anomaly-free.
But it seems there are also alternative ways,
To get rid of the theory’s anomalous phase.

A variety,
Of anomaly-free,
Type IIB
String theory?

But wait! There might be a solution at hand
If we discre(e)tely put them in the swampland.

What would physics be,

if all was about results,

and not also fun

Conclusions

- We have the necessary **tools to calculate anomalies** (bordism classes, eta invariants, self-dual fields, a lot of new results there)
- **The 'textbook version' of IIB is anomalous**
- Can be **cancelled by quadratic refinement term** (very subtle new topological term in the action)
- **Alternative cancellations** via topological GS (modifications of spectrum of extended objects)
- **Discrete Landscape** or **Topological Swampland**

P.S.

Maybe the term

$$\tilde{\mathcal{Q}}(\breve{c}_0) \text{ with } \breve{c}_0 = \left(\lambda_1 \beta(a)^2 + \lambda_2 \frac{(p_1)_3}{2}\right) \cup a + \frac{\lambda_3}{2}[(p_1)_4 - \mathcal{P}(w_2)] \cup b + \kappa\beta(b)^2 \cup b$$

appears naturally in the way one chooses the **quadratic refinement** on $\mathrm{Spin\text{-}GL}^+$ manifolds

➔ Part of **defining data of type IIB** rather than anomaly cancellation