Non-extremal Black Holes and the Swampland

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Geometry, Strings and the Swampland

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Work in progress with **M. Dierigl, A. Gnecchi, D. Lüst, M. Scalisi**

Introduction

- Black Holes encode properties of quantum gravity
- Important role in the swampland program
- They are spacetime geometries
- They are thermodynamical objects

Black holes: geometry

- Solutions of GR

$$ds^2 = -f(r,\alpha)dt^2 + f(r,\alpha)^{-1}dr^2 + r^2 dS_2^2, \qquad \alpha = M, Q, \dots$$

with horizon r_H, such that $f(r_H) = 0$.

- Add scalars: warped geometry

$$f(r,\alpha) \to e^{2U(\phi)} f(r,\alpha)$$

- Attractor mechanism (extremal BH): asymptotic value of the scalars not detected at horizon, since it is at **infinite distance** in spacetime.

Black holes: thermodynamics

- Entropy given by the horizon Area

$$S = \frac{A}{4}$$

- Temperature given by the surface gravity

$$T = \frac{\kappa}{2\pi} = \frac{1}{4\pi} f'(r_H)$$

- Extremality parameter

$$c = 2ST$$

Extremal BH: $c = 0$

Motivation

- Can large/small **entropy** limits be at infinite distance in moduli space? What is the associated physical towers of states? (See also [Hamada, Montero, Vafa, Valenzuela '21] and I. Valenzuela's talk.)

- Can large/small **temperature** limits be at infinite distance in moduli space? What is the associated physical towers of states?
  Temperature in Euclidean compact time related to Matsubara modes, whose role in the swampland program is not clear.

Distance in the space of metrics

The geometric distance

Given a family of metrics

$$g_{\mu\nu} = g_{\mu\nu}(\alpha), \qquad \alpha = M, Q, \ldots$$

we can define a geometric distance [DeWitt '67]

$$\Delta(g) = \int_{\alpha_i}^{\alpha_f} d\alpha \left(\frac{1}{Vol_M} \int_M \sqrt{g}\, g^{\mu\nu} g^{\rho\sigma} \frac{\partial g_{\mu\rho}}{\partial\alpha} \frac{\partial g_{\nu\sigma}}{\partial\alpha} \right)^{\frac{1}{2}}$$

It is a distance in the parameter space of metrics.

- Used in [Lüst, Palti, Vafa '19] to derive ADC and in [Kehagias, Lüst, Lüst '19] in relation to Ricci-flows.
- Used systematically in [Bonnefoy, Ciambelli, Lüst, Lüst '19] for various BH solutions.
- Used in [Lüben, Lüst, Metidieri '20] for charged BHs in dS.

Infinite entropy and infinite distance

In [Bonnefoy, Ciambelli, Lüst, Lüst '19] it is found that generically

$$\Delta(g) \sim |\log S|,$$

thus it is postulated the existence of a tower of states with mass

$$m_S \sim S^{-\lambda}, \qquad \lambda \sim \mathcal{O}(1), \text{ positive.}$$

This has been named **Black Hole Entropy Distance Conjecture**.

- What is the nature of the tower of states?

Another example

[NC, Dierigl, Gnecchi, Lüst, Scalisi, in progress]

We can calculate the distance between extremal and non-extremal Reissner-Nordstrom BHs

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 dS_2^2, \quad f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2},$$

along the extremality parameter

$$c = \sqrt{M^2 - Q^2} = 2ST.$$

In the **near horizon** limit, it gives

$$\Delta \sim |\log c| \to \infty \qquad \text{for} \qquad c \to 0.$$

Are $AdS_2 \times S_2$ and $Mink_2 \times S_2$ are at infinite distance?

Possible caveats

We must be careful, since

- **Technically**
  The distance formula is not diffeomorphisms invariant. See [Bonnefoy, Ciambelli, Lüst, Lüst '19] for more on this and for a prescription to deal with it.

- **Physically**
  The physical meaning of the space of metrics is not clear.

Eventually we would like distances in moduli space.

Thus, we look at supergravity models, since they automatically contain scalars and can have string theory origin.

Distance in moduli space

Scalars in supergravity

(Extended) supergravity theories come naturally with scalars

$$e^{-1}\mathcal{L}_{bos} = R - \frac{1}{2}g_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j \\ + \mathrm{Im}\mathcal{N}_{\Lambda\Sigma}(\phi)F^{\Lambda}_{\mu\nu}F^{\Sigma\,\mu\nu} + e^{-1}\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}(\phi)\epsilon^{\mu\nu\rho\sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma}_{\rho\sigma}$$

parametrising a manifold with metric $g_{ij}(\phi)$ constrained by supersymmetry. This is the **moduli space**.

For $N > 1$:

- Isometry group embedded into $Sp(2n_V + 2, \mathbb{R})$ as a consequence of duality rotations [Gaillard, Zumino '81].
- The whole Lagrangian is covariant, but physical quantities such as entropy must be symplectic invariant.

N=2, D=4 supergravity

- Specified by a prepotential (or symplectic sections)

$$F = F(X^\Lambda) \qquad \Lambda = 0, 1, \ldots, n_V.$$

- Physical scalars from vectors (and hypers)

$$z^i = \frac{X^i}{X^0}, \qquad i = 1, \ldots, n_V$$

spanning a product manifold

$$\mathcal{M}_{scal} = \mathcal{M}_{SK} \otimes \mathcal{M}_{QK}$$

- Electric and magnetic charges (q_Λ, p^Λ)

$$p^\Lambda = \frac{1}{4\pi}\int_{S^2} F^\Lambda, \qquad q_\Lambda = \frac{1}{4\pi}\int_{S^2}(\mathrm{Re}\mathcal{N}_{\Lambda\Sigma}F^\Sigma - \mathrm{Im}_{\Lambda\Sigma} * F^\Sigma)$$

Black holes in supergravity

Terrific amount of work in past and recent years. Just recall:

- **Extremal**, $T = 0$: well understood, protected against corrections if SUSY, attractor mechanism. Reviews [Andrianopoli, D'Auria, Ferrara, Trigiange '06; Dall'Agata '11].
- **Non-extremal**, $T > 0$: less understood, break SUSY. Work by [Behrndt, Cardoso, Cvetic, Grass, Gruss, Horowitz, Kastor, Mohaupt, Sabra, Strominger, Vaughan, Youm, Win, ...]
  Proposal to construct them as deformation of extremal solutions [Galli, Ortín, Perz, Shahbazi '11]

Challenging but interesting to study (non-extremal) non-supersymmetric cases, to avoid bias from supersymmetry.

From extremal to non-extremal

[Galli, Ortín, Perz, Shahbazi '11]

Consider the metric [Ferrara, Gibbons, Kallosh '97]

$$ds^2 = -e^{2U}dt^2 + e^{-2U}\left[\frac{c^4 d\rho^2}{sinh^4(c\rho)} + \frac{c^2}{sinh^2(c\rho)}d\Omega_2^2\right]$$

wtih horizon at $\rho \to -\infty$, interpolating between $c = 0$ and $c > 0$.

- If the extremal solution is specified by harmonic function $\mathcal{I}$

$$z^i = z^i_{ext}(\mathcal{I}(\rho)), \qquad U(\rho) = U_{ext}(\mathcal{I}(\rho)),$$

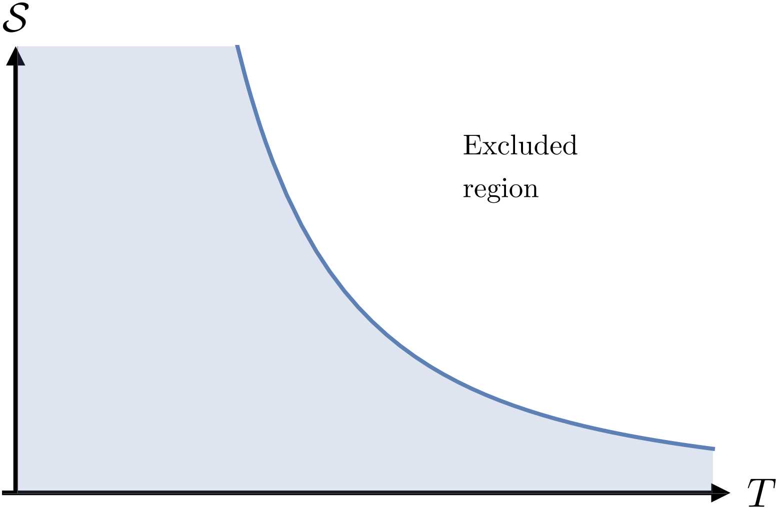
- then the non-extremal solution is

$$z^i = z^i_{ext}(\hat{\mathcal{I}}(\rho)), \qquad U(\rho) = U_{ext}(\hat{\mathcal{I}}(\rho)) + c\rho,$$

where $\hat{\mathcal{I}} = a + be^{2c\tau}$, with a, b integration constants.

Our procedure

- Choose S and T as parameters
- In supergravity, these will be functions of the moduli
- Investigate all limits $S, T \to 0, \infty$
- Map these limits to moduli space


$\mathcal{S}$
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region
T

An example, in some detail: STU model

[NC, Dierigl, Gnecchi, Lüst, Scalisi, in progress]

The setup

Consider BH in 4d N=2 SUGRA with prepotential

$$F = X^1 X^2 X^3 / X^0,$$

stemming from type IIA on CY_3 with D0-D4
[Bellucci, Ferrara, Marrani, Yeranyan '08]

- Solution specified by harmonic functions

$$\mathcal{I}_0,\, \mathcal{I}^{1,2,3}, \qquad \text{with} \qquad \mathcal{I}_4 = \mathcal{I}_0 \mathcal{I}^1 \mathcal{I}^2 \mathcal{I}^3$$

- 3 vector multiplets scalars

$$s = \frac{\mathcal{I}_0 \mathcal{I}^1}{\sqrt{\mathcal{I}_4}}, \quad t = \frac{\mathcal{I}_0 \mathcal{I}^2}{\sqrt{\mathcal{I}_4}}, \quad u = \frac{\mathcal{I}_0 \mathcal{I}^3}{\sqrt{\mathcal{I}_4}}$$

- Warp factor

$$e^{-2U} = 4\sqrt{\mathcal{I}_4}$$

Non-extremal deformation

- Replace harmonic functions and warp factor

$$\mathcal{I}^\Lambda \to \hat{\mathcal{I}}^\Lambda = a^\Lambda + b^\Lambda e^{2c\rho},$$
$$e^{-2U} \to e^{-2U_{ext}(\hat{\mathcal{I}})-2c\rho},$$

where $e^{-2U_{ext}(\hat{\mathcal{I}})} = 4\sqrt{\hat{\mathcal{I}}_0\hat{\mathcal{I}}^1\hat{\mathcal{I}}^2\hat{\mathcal{I}}^3}$.

- a^Λ, b^Λ fixed by EOMs to be (schematically)

$$\begin{pmatrix} a^\Lambda \\ b^\Lambda \end{pmatrix} \sim \left(1 \pm \frac{1}{c}\sqrt{c^2 + (q^\Lambda)^2}\right), \qquad c = 2ST$$

- Entropy and temperature at horizon ($\rho \to -\infty$)

$$S = 16\pi\sqrt{a_0 a^1 a^2 a^3}\,c^2, \qquad T = \frac{1}{32\pi}\frac{1}{c\sqrt{a_0 a^1 a^2 a^3}}$$

From parameters to moduli

The volume of the internal manifold

$$Vol = s\,t\,u$$

can be expressed in terms of S and T (schematically)

$$Vol \sim \frac{\left(ST + \sqrt{q_0^2 + S^2T^2}\right)^2}{S}$$

It connects **space of metrics** (S,T) to **moduli space** (Vol).
In particular, it gives information on the KK masses

$$M_{KK} \sim \frac{1}{Vol^{\frac{1}{6}}}$$

We study what happens if $S, T \to 0, \infty$

Some cases

One must be careful with the order of the limits.

Systematic analysis in [NC, Dierigl, Gnecchi, Lüst, Scalisi, in progress].

Two possibilities are:

1. $T \to \infty$ and $S \to 0$
$$Vol \to \infty$$
KK masses becomes light and EFT breaks down. Internal manifold becomes non-compact.

2. $T \to 0$ then $S \to \infty$
$$Vol \to 0$$
KK masses becomes heavy. Internal manifold shrinks and SUGRA breaks down.

Consistency check

- In the 2nd limit, we should get back [Bonnefoy, Ciambelli, Lüst, Lüst '19].
- However, for $S \to \infty$ they have

$$Vol \sim S^3/(p^1p^2p^3)^2 \to \infty,$$

while we have

$$Vol \sim q_0^2/S^2 \to 0.$$

- Mismatch due to different symplectic frame, electric vs. magnetic. Taking this into account, the results agree.
- This is T-duality, since

$$Vol \to 1/Vol.$$

Temperature duality

We can interpret this as **temperature duality** in parameter space.

- $T \to \infty$ (and $S \to 0$)

$$Vol \to \infty, \qquad m_{KK} \to 0$$

- $T \to 0$ (then $S \to \infty$)

$$Vol \to 0, \qquad m_{KK} \to \infty$$

The two limits are related by

$$T \to 1/T$$

In each direction we have a light tower of states. In the second case the tower becoming light is the winding tower.
Related works [Agrawal, Gukov, Obied, Vafa, '20; Blumenhagen, Kneißl, Makridou '21].

Some final speculation

- It is tempting to draw an analogy

$$T \longleftrightarrow m_{3/2}, \qquad S \longleftrightarrow \Lambda$$

- Both $m_{3/2}$ and Λ (AdS) should be arbitrary small

- What are the ADC and the GMC [NC, Lüst, Scalisi '21; Castellano, Herraez, Ibanez, Font, '21] telling us about BH thermodynamics?

Conclusion

- Black holes teach lessons about quantum gravity
- Going from extremal to non-extremal is challenging but interesting
- Thanks to scalars we can map the distance in the space of metrics to distance in moduli space
- We are studying various limits and finding connections with swampland conjectures

Thank you!