(asymptotically) deSitter space in String Theory

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אוניברסיטת בן-גוריון

- Idea: stringy dual of (asymptotically) dS/Inflation
- Closed strings in the Hagedorn phase
  - Winding modes and the thermal scalar EFT
  - Thermal scalar condensate
- Correspondence to an asymptotically dS space

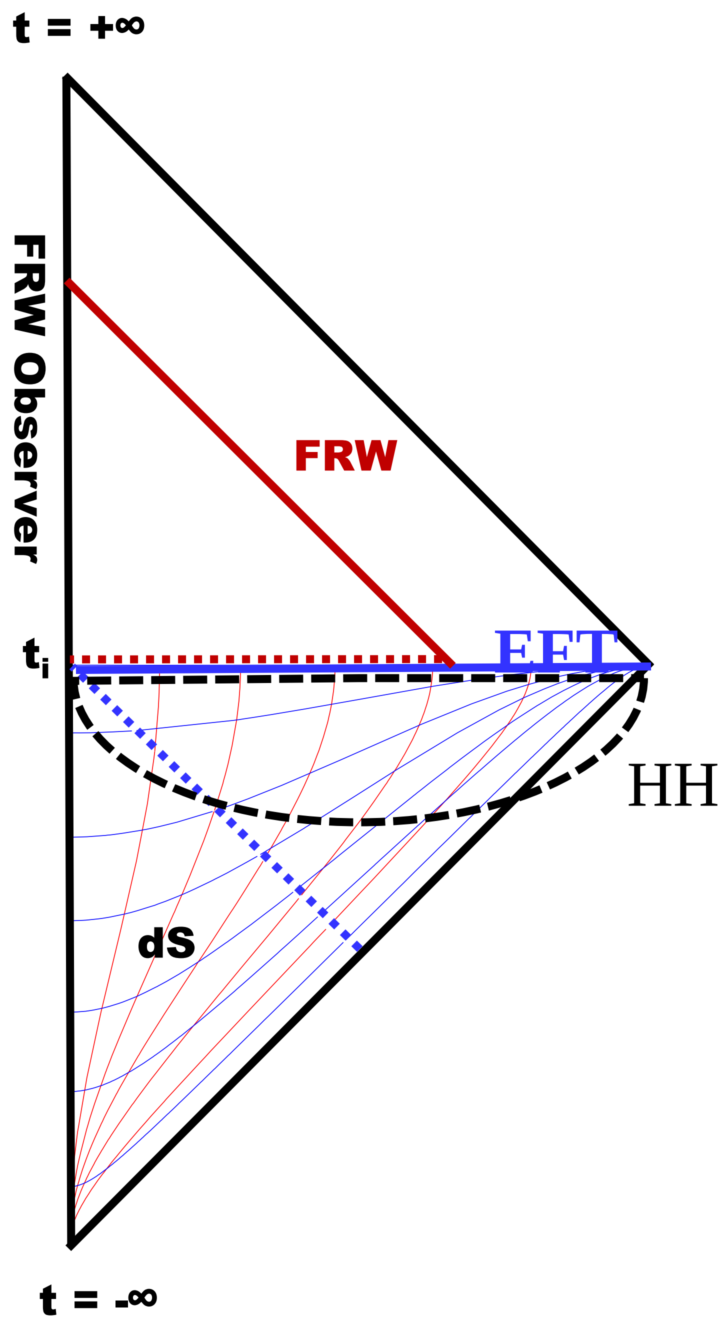
RB, Zigdon
2101.07836
To appear
RB, Medved
2005.09321
1906.00989
=========
RB, Medved
Black holes as collapsed polymers
1602.07706 + …
RB, Zigdon
2107.09001

Idea: “dual of inflation”

Strings ←→ Wavefunction ←→Infation

Initial state of the Universe defined by

- Classical inflation: a long **classical** evolution
- Wavefunction of the Universe (HH) : a long **semiclassical** evolution
- Strings: a **quantum** state @/near Hagedorn temperature



Consistency with swampland constraints ?

- Eembedding a dS (or a dS-like) solution in string theory
- UV completion of a theory that has a stable dS space solution by a weakly coupled (semiclassical) quantum gravity
- "Species scale" -- energy scale @ which gravity becomes strongly coupled

  (my perspective: entropy bounds become saturated)

Idea: dS space emerges in string theory as a state whose energy density is **exactly** @ the species scale

Why this idea could work:
prepare a state with the properties of dS

Asymptotic dS/Inflation	Hagedorn strings EFT
Energy & entropy	**Energy & entropy ←→ hot strings**
dS isometries/conformal invariance	**Generalized conformal structure**
Translation invariance Puzzles solved by inflation (Agrawal et al '20)	**Translation invariant state**
Scale invariant perturbations	**Scale invariance ← 2D nature + generalized conformal structure**

Highly excited (Hagedorn) phase of strings

$$Z = \mathrm{Tr}\, e^{-\beta H} \sim \int_0^\infty dm\, \exp(4\pi m \alpha'^{1/2}) \exp(-m/T)$$

$$n(m) \approx \exp(4\pi m \alpha'^{1/2})$$

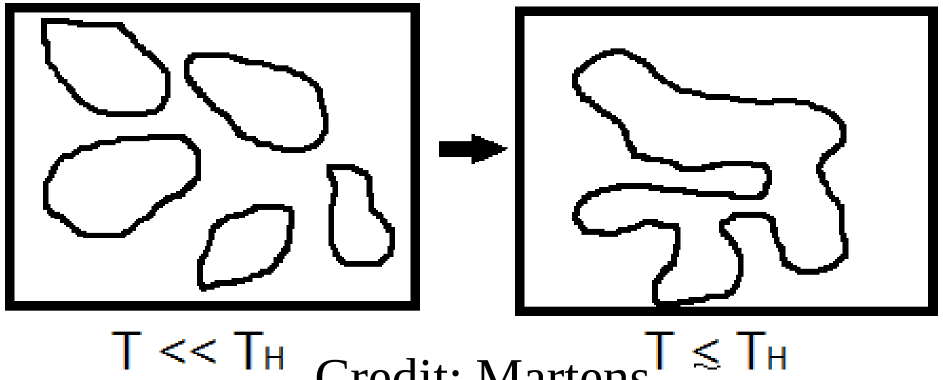
Hagedorn divergence $T_{Hag} \sim 1/l_s$

$$\omega(\varepsilon) \approx \frac{V \exp(\beta_H \varepsilon)}{\varepsilon^{D/2+1}},$$

Long string: Energy, Entropy ~ Length

T< T_{Hag} , Energy dominates

T ~ T_{Hag} , Entropy dominates (strong coupling)

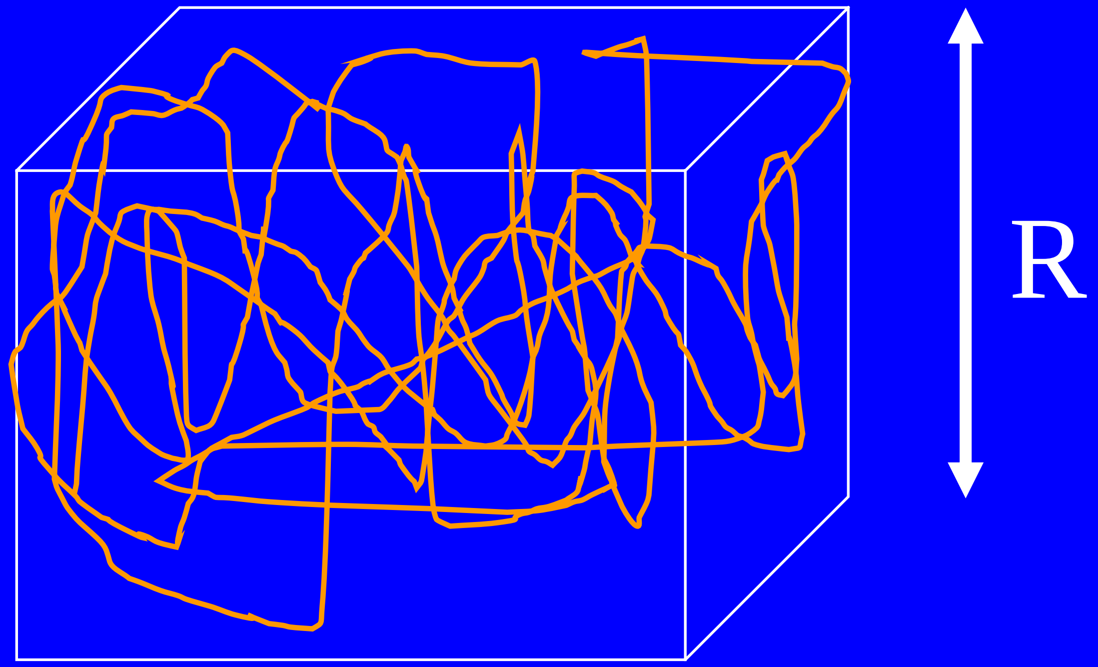


Credit: Martens

Dominated by long string(s) : entropically favourable

Free long string ←→ Random walk

$$R = \sqrt{L}$$



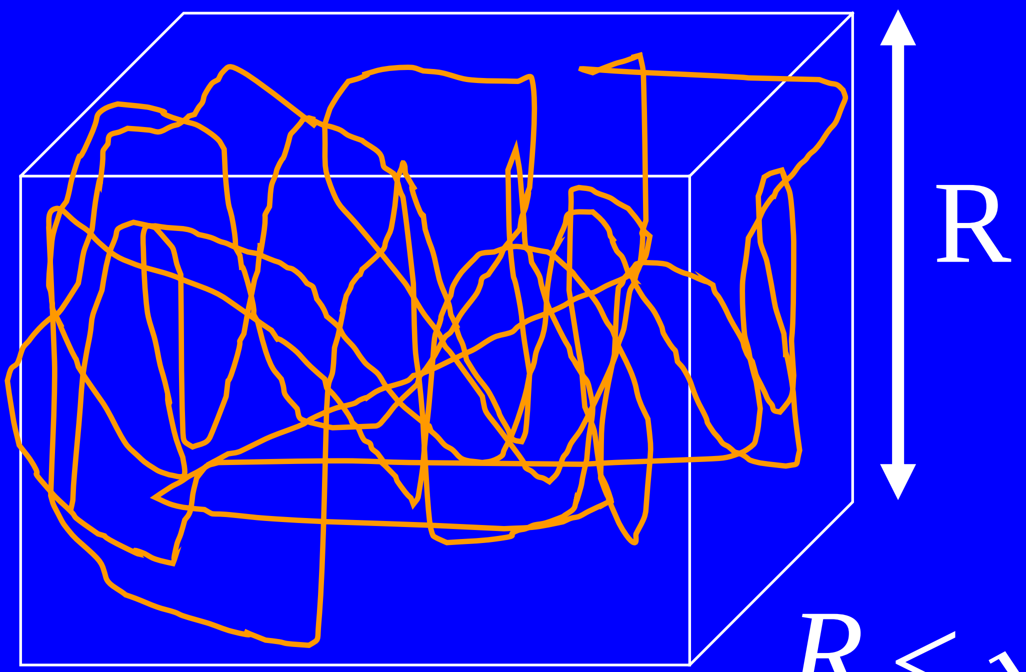
Highly excited strings in a bounded region

Salomonson & Skagerstan ‘86
Low+Thorlacius ’94
===================
Horowitz+Polchinski ’97
Damour + Veneziano ‘99

String-BH correspondence

Closed strings

- Total length L
- Entropy L/l_s



$R < \sqrt{L}$

Dominated by long strings

$\ell \sim L$

Plan

- Thermal scalar of closed strings @ Hagedorn temperature
  - Euclidean field theory of the thermal scalar condensate
  - Free energy & effective 2D CFT

- Correspondence to an asymptotically dS space
  - Parameters & Fields
  - Free energy
  - Correlations (preliminary)
- Conclusion

Atick, Witten '88
Brandenberger, Vafa '89
Horowitz, Polchinski '97
Barbon, Rabinovici '01-'04
Many more …
Recently: Chen, Maldacena
Chen, Maldacena, Witten

Thermal scalar

- Closed strings @ $T \sim T_{Hag}$ $\quad \frac{1}{T} = \oint_0^\beta d\tau \sqrt{g_{\tau\tau}}$
- Compactify on a Euclidean time circle ➔ "timeless" theory on a d-dimensional spatial hypersurface
- +1 winding mode ϕ, -1 winding mode ϕ^* become massless @ T_{Hag}

$$Z_{strings} = Tr\, e^{-\beta H} = \int [d\phi] e^{-S_E(\phi,\phi^*)}$$

Euclidean path integral of the thermal scalar -- an effective (but complete) description of the multi-string partition function when $T \sim T_{Hag}$

Self-gravitating long strings

Horowitz, Polchinski '97
…
Chen, Maldacen ‘21
Chen, Maldacena, Witten ‘21

$$S = \beta \int d^d x \sqrt{G} e^{-2\psi} \left\{ -\frac{1}{2\kappa^2} R + \frac{1}{\kappa^2}\Lambda - \frac{2}{\kappa^2} G^{\mu\nu}\partial_\mu\psi\partial_\nu\psi \right.$$
$$\left. + \; G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi^* - 2c_2\epsilon T^2\phi\phi^* + c_2\kappa^2 T^2(\phi\phi^*)^2 \right\}.$$

$\varepsilon = T - T_H$

$\epsilon = \frac{T-T_H}{T_H}$

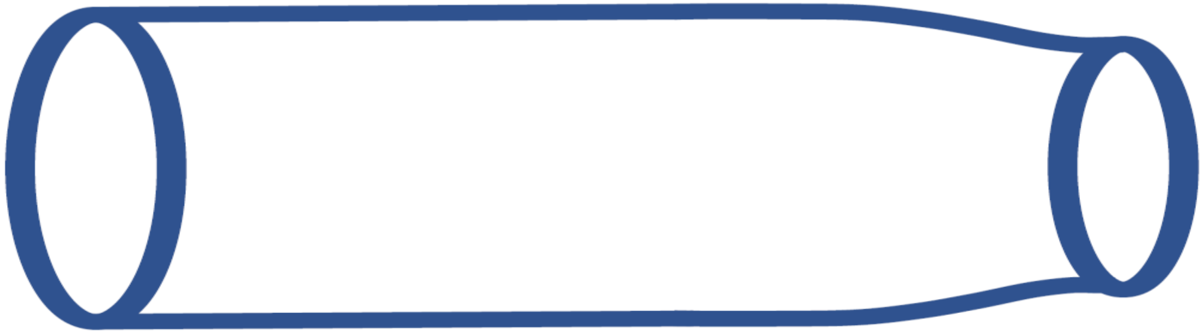
$\kappa^2 = 8\pi G_N$

$T < T_H$, $\varepsilon < 0$ $\text{mass}^2 > 0$

$T > T_H$, $\varepsilon > 0$ $\text{mass}^2 < 0$

Winding modes/Thermal scalar condense $\phi\phi^* \sim \epsilon^2/\kappa^2$

← $L \sim 1/\sqrt{\epsilon}$ →



“String-BH correspondence” ?

Thermal scalar condensate

Thermal equilibrium

$$G_{\tau\tau} = e^{2\sigma}$$

$$S_{\phi,\phi^*} = \beta \int d^d x \sqrt{G} e^{-2\psi} \left(G^{\mu\nu}\partial_\mu \phi \partial_\nu \phi^* + \frac{\beta^2 e^{2\sigma} - \beta_H^2}{4\pi^2 (\alpha')^2} \phi\phi^* + \frac{2\kappa^2}{\alpha'} (\phi\phi^*)^2 \right)$$

$$S_{NS-NS} = -\frac{\beta}{2\kappa^2} \int d^d x \sqrt{G} e^{-2\psi} \left(R - 2\Lambda + 4G^{\mu\nu}\partial_\mu \psi \partial_\nu \psi + \right.$$
$$\left. -G^{\mu\nu}\partial_\mu \sigma \partial_\nu \sigma - \frac{1}{4} e^{-2\sigma} H_{\tau\mu\nu} H_\tau{}^{\mu\nu} \right).$$

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"Flux compactification"
flux stabilizes the thermal circle

required hierarchy

$$\epsilon \ll \tilde{g}_s^2 < 1$$

Thermal scalar condensate

Thermal equilibrium

$\varepsilon = T - T_H$

$\epsilon = \frac{T - T_H}{T_H}$

$\kappa^2 = 8\pi G_N$

$$G_{\tau\tau} = e^{2\sigma}$$

EOM

$$\frac{e^{2\psi}}{\sqrt{G}}\partial_\mu\left(e^{-2\psi}\sqrt{G}G^{\mu\nu}\partial_\nu\phi\right) = \frac{\beta^2 e^{2\sigma} - \beta_H^2}{4\pi^2(\alpha')^2}\phi + \frac{4\kappa^2}{\alpha'}(\phi)^2\phi^*.$$

$$\frac{e^{2\psi}}{\sqrt{G}}\partial_\mu\left(e^{-2\psi}\sqrt{G}\partial^\mu\sigma\right) = -\frac{1}{4}e^{-2\sigma}H_{\tau\mu\nu}H_\tau{}^{\mu\nu} + \frac{\beta^2\kappa^2}{2\pi^2(\alpha')^2}\phi\phi^* e^{2\sigma}$$

Solutions

$$\phi\phi^* = \frac{\epsilon}{\kappa^2} \qquad \sigma = -\epsilon$$

$$H_{\tau ij} = h\epsilon^{S^2}_{\tau ij} \qquad h^2 = \frac{8\epsilon}{\alpha'}$$

Thermal scalar condensate

Thermal equilibrium

$\varepsilon = T - T_H$

$\epsilon = \frac{T-T_H}{T_H}$

$\kappa^2=8\pi G_N$

EOM

$$R_{\mu\nu} - \partial_\mu\sigma\partial_\nu\sigma + 2\nabla_\mu\nabla_\nu\psi - \tfrac{1}{2}H_{\mu\lambda\tau}H_\nu{}^{\lambda\tau} = \quad 2\kappa^2\partial_\mu\phi\partial_\nu\phi^*$$

$$R - 2\Lambda + 4\nabla^2\psi - 4\partial^\mu\psi\partial_\mu\psi - \partial_\mu\sigma\partial^\mu\sigma - \tfrac{1}{4}H_{\tau\mu\nu}H^{\tau\mu\nu} = \\ = 2\kappa^2\left[\partial^\mu\phi\partial_\mu\phi^* + \tfrac{\beta^2e^{2\sigma}-\beta_H^2}{4\pi^2(\alpha')^2}\phi\phi^* + \tfrac{2\kappa^2}{\alpha'}(\phi\phi^*)^2\right].$$

$$\phi\phi^* = \frac{\epsilon}{\kappa^2} \qquad \sigma = -\epsilon$$

$$H_{\tau ij} = h\epsilon^{S^2}_{\tau ij} \qquad h^2 = \frac{8\epsilon}{\alpha'}$$

Solutions

$$\frac{1}{r_0^2}G_{ij} = \frac{h^2}{2}G_{ij},\ i,\ j = 1,2$$
$$G_{\mu\nu} = \delta_{\mu\nu}\ \mu,\ \nu \neq 1,2.$$

$$G_{ij} = G_{ij}^{S2}\ , \Lambda = 2\epsilon/\alpha'$$
$$\psi = const.$$

$$r_0 = \frac{\sqrt{\alpha'}}{2\sqrt{\epsilon}}$$

Also, a linear dilaton solution

Thermal scalar condensate

Thermal equilibrium 👍☺

$\varepsilon = T - T_H$

$\epsilon = \frac{T - T_H}{T_H}$

$\kappa^2 = 8\pi G_N$

$$G_{\tau\tau} = e^{2\sigma}$$

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$$\phi\phi^* = \frac{\epsilon}{\kappa^2}$$

$$H_{\tau ij} = h\epsilon^{S^2}_{\tau ij} \qquad h^2 = \frac{8\epsilon}{\alpha'}$$

$$\begin{aligned} \frac{1}{r_0^2} G_{ij} &= \frac{h^2}{2} G_{ij}, \; i, \; j = 1, 2 \\ G_{\mu\nu} &= \delta_{\mu\nu} \; \mu, \; \nu \neq 1, 2. \end{aligned}$$

$$r_0 = \frac{\sqrt{\alpha'}}{2\sqrt{\epsilon}}$$

$$G_{ij} = G_{ij}^{S2}, \Lambda = 2\epsilon/\alpha'$$

$$\psi = const.$$

$$S^1_\beta \times S^2 \times R^{d-2} \qquad S^1_\beta \times S^2 \times R_\psi \times R^{d-3}$$

Translation invariance 👍☺

Alternatives to dilaton stabilization:

- A linear dilaton solution
- Possibly, a one-loop thermal mass

Thermodynamics

$$S_{\phi,\phi^*} = \beta \int d^d x \sqrt{G} e^{-2\psi} \left(G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi^* + \frac{\beta^2 e^{2\sigma} - \beta_H^2}{4\pi^2 (\alpha')^2} \phi\phi^* + \frac{2\kappa^2}{\alpha'} (\phi\phi^*)^2 \right)$$

$$\phi\phi^* = \frac{\epsilon}{\kappa^2}$$

$$s = (\beta\partial_\beta - 1)\, \mathcal{L}_{tot} = \frac{2\beta^3 e^{-2\psi+2\sigma}}{(2\pi\alpha')^2} \phi\phi^*$$

$$p = -\mathcal{L}_{tot}$$

$$s(T) = 2\, c_2 \frac{\epsilon T}{\kappa^2}$$

$$p(T) = c_2 \frac{\epsilon^2 T^2}{\kappa^2}$$

$$\rho(T) = c_2 \frac{\epsilon^2 T^2}{\kappa^2}$$

$$\rho = f + \varepsilon s$$

$$\rho + p = \varepsilon s$$

$$T_H = (8\pi^2 \alpha')^{-\frac{1}{2}}$$

$$c_2 = 16\pi^2$$

$$\varepsilon = T - T_H$$

$$\epsilon = \frac{T - T_H}{T_H}$$

ε – effective temperature,
d/dε = d/dT for fixed ϵ

$$p = \rho$$

$$s = 2\sqrt{\frac{c_2 \rho}{\kappa^2}}$$

"maximal entropy"
"species scale"

Saturation of the causal entropy bound, RB, Veneziano '99

Energy + entropy 👍☺

Generalized conformal structure

$$S_{\phi,\phi^*} = \beta \int d^dx \sqrt{G} e^{-2\psi} \left(G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi^* + \frac{\beta^2 e^{2\sigma} - \beta_H^2}{4\pi^2(\alpha')^2}\phi\phi^* + \frac{2\kappa^2}{\alpha'}(\phi\phi^*)^2 \right)$$

$$= \beta \int d^dx \sqrt{G} e^{-2\psi} \{ G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi^* - 2c_2\epsilon T^2\phi\phi^* + c_2\kappa^2 T^2(\phi\phi^*)^2 \}$$

Generalized conformal structure:

A dimensionful parameter transforms – temperature (coupling)

$$\sqrt{G} \to \Omega^d \sqrt{G}$$
$$\phi\phi^* \to \Omega^{-(d-1)}\phi\phi^*$$
$$T^2 \to \Omega^{-2}T^2$$
$$\epsilon \to \epsilon$$

$$G_{\mu\nu} \to \Omega^2 G_{\mu\nu},$$
$$\phi \to \Omega^{-\frac{d-1}{2}}\phi,$$
$$T \to \Omega^{-1}T$$
$$\beta \to \Omega\beta$$

The generalized conformal structure guarantees the structure of 2- and 3-point functions

Free energy, thermodynamics & effective thermal 2D CFT

El-Showk, Papadodimas '11
Iliesiu + … '18

$$
\begin{aligned}
F_2/V &= f_2 \beta^{-2}, \\
\rho_2 &= -\tfrac{1}{2}(b_\beta)_2\, \beta^{-2}, \\
s_2 &= -(b_\beta)_2\, \beta^{-1}
\end{aligned}
$$

$$\varepsilon = 1/\beta$$

$$\frac{F}{V} = \rho - \varepsilon s = -\, c_2 \frac{\epsilon^2 T^2}{\kappa^2}$$

$$s(\mathrm{T}) = 2\, c_2 \frac{\epsilon T}{\kappa^2}$$

$$p(T) = c_2 \frac{\epsilon^2 T^2}{\kappa^2}$$

$$\rho(T) = c_2 \frac{\epsilon^2 T^2}{\kappa^2}$$

ε – effective temperature

$$f_2 = \tfrac{1}{2}(b_\beta)_2 \qquad \rho = \tfrac{\pi}{6} c \beta^{-2} \qquad s = \frac{\pi}{3} c \beta^{-1}$$

2D+ generalized conformal structure allows scale-invariance of 2-point functions

Why this idea could work:
prepare a state with the properties of dS

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Energy & entropy	**Energy & entropy from strings**
dS isometries/conformal invariance	**Generalized conformal structure**
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Scale invariant perturbations	**Scale invariance ← 2D nature + generalized conformal structure**

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Strings ←→ Wavefunction ←→Infation

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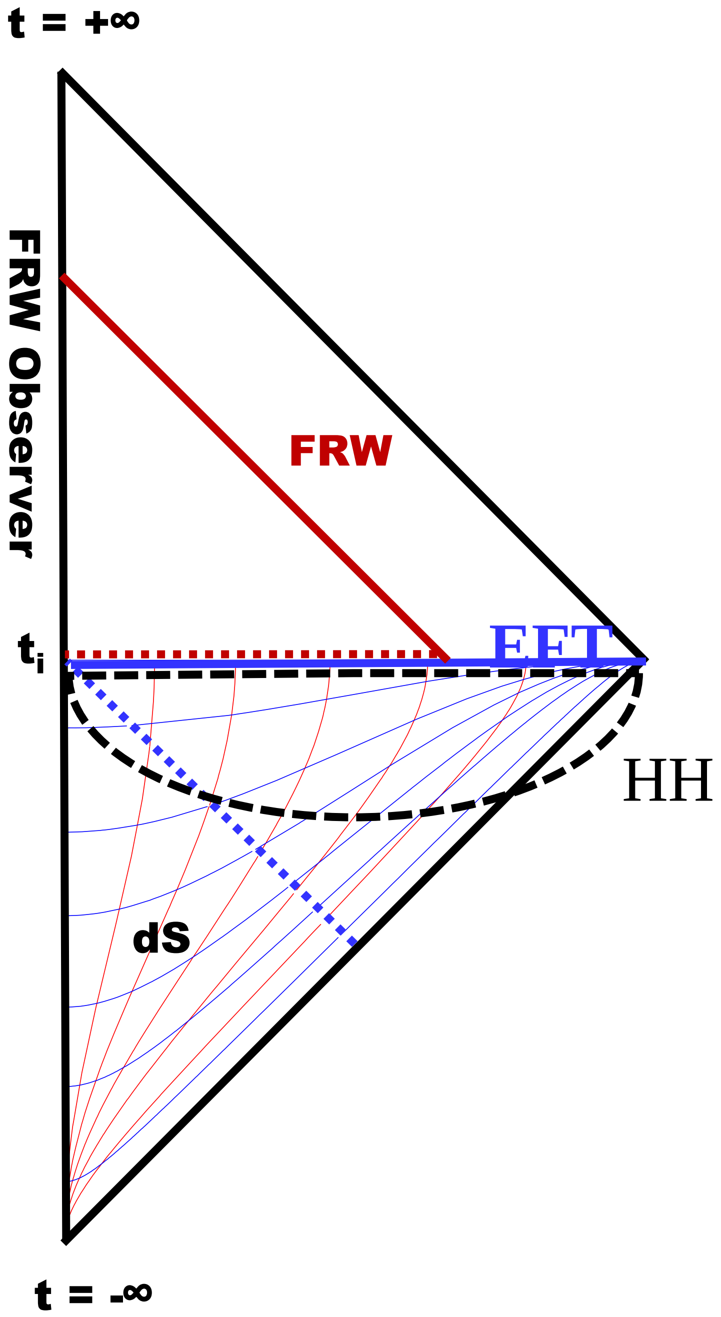

Correspondence to asymptotically dS space

Parameters & Fields

Dimensional parameters

EFT	dS
ε	H
κ^2	G_N

EFT	asdS	Cosmological observables
$\left(\frac{\nabla_i \nabla_j}{\nabla^2} - \frac{1}{d}\delta_{ij}\right)\delta s$	h_{ij}	P_T
$\int \delta s$	ζ	P_S

Correspondence to asymptotically dS space

Cosmological perturbations

RB, Medved 1906.00989
2005.09321
RB, Zigdon, to appear

$$s = 2c_2 T|\phi|^2$$

$$\delta s \ll T\frac{\epsilon}{\kappa^2}$$

$$\sigma \equiv \frac{\kappa}{c_2 T}\sqrt{\frac{1}{8\epsilon}}\delta s$$

$$\sigma \ll \frac{\sqrt{\epsilon}}{\kappa}$$

$$S_\phi = \beta \int d^d x \left(\frac{2c_2 T + c_3 \kappa^2 s}{16 c_2^2 T^2 s} \partial_\mu s \partial^\mu s - \epsilon T s + \frac{\kappa^2}{4c_2} s^2 \right)$$

$$S[\sigma] = \beta \int d^d x \left(\tfrac{1}{2} \left(1 - \tfrac{\sqrt{2}\kappa}{\sqrt{\epsilon}} \sigma \right) \partial_\mu \sigma \partial^\mu \sigma \right.$$
$$+ 2\epsilon T^2 \left[c_2 + \epsilon \left(3c_3 + c_{4\epsilon} \right) + c_{1loop} g_s^2 \right] \sigma^2 +$$
$$\left. + \sqrt{8} c_3 \epsilon^{\frac{3}{2}} \kappa T^2 \sigma^3 + 4 c_2 \kappa^2 T^2 \epsilon^2 \sigma^4 \right) .$$

Can be used to calculate σ-correlation functions

Correspondence to asymptotically dS space

Cosmological observables
(preliminary – scaling results)

RB, Medved 1906.00989
2005.09321
RB, Zigdon, to appear

$$\left(\frac{\delta\rho}{\rho}\right)^2_{|H} = \frac{1}{S_H} \sim H^2 M_P^2$$

$$\zeta_{st} \equiv \delta N_{e-folds}$$

$$\langle\zeta_{st}^2\rangle_H \sim \int dN_{e-fold}\delta_\rho^2(H) \sim \int d\ln n_H\, \delta_\rho^2(H)$$

d=3

$$\langle\zeta_{st}^2\rangle_H \sim N_{e-folds}\delta_\rho^2(H)$$

$$r = \frac{\delta_\rho^2(H)}{\langle\zeta_{st}^2\rangle_H} \simeq \frac{1}{N_{e-folds}}$$

Correspondence to asymptotically dS space

Duration & Scale of "inflation"

"Minimal inflation"

$$e^{3N_{e-folds}} = \frac{V_{tot}}{H^{-3}} = n_H = \frac{S_{tot}}{S_H}$$

$$\tau_{inflation} = H^{-1}N_{e-folds} \sim l_s(g^2 S_H)^{1/2} \ln\left(\frac{S_{tot}}{S_H}\right)$$

$$N_{e-folds} \sim \frac{1}{3}\ln\left(\frac{S_{tot}}{S_H}\right)$$

$$\tau_{inflation} \sim \tau_{scrambling}$$

$$\tau_{scrambling} = \frac{1}{H}\ln\frac{S_{tot}}{S_H}$$

Correspondence to asymptotically dS space

Scale and duration of "inflation"

"String scale inflation" $3H^2 \sim \frac{1}{m_P^2} T_{Hag}^4$

d=3

In our universe $\in \sim 10^{-4}$, $g_s^2 > \in$

$$T_{Hag}^4 \sim N_{species} T_{rad}^4$$

Summary & Conclusion

- State of an asymptotically de Sitter space/inflation is “dual” to interacting closed strings in the Hagedorn phase
- Dimensional parammeters : κ^2 , $T - T_H$
- A “microscopic” description of inflation
- High scale, “minimal” inflation
- Can be repeated for black holes

Comparison to "similar" models

String gas cosmology

- Below Hagedorn temperature
- Requires specific number of compact and large dimensions
- Mechanism of exit from inflation (needs compact dimensions to unwind)
- Cosmological perturbations have different properties
- Predictions for cosmological observables are different.

Holographic cosmology

- Similar maximal entropy, $p=\rho$ EOS
- Framed in a semiclassical setup, we argue that , $p=\rho$ could not be described in terms of semiclassical physics.
- A difference in predictions for cosmological observables, in particular, the prediction for r, which in holographic cosmology has a parametrically small value

dS/CFT

- Involves a semiclassical saddle point.
- Similar in that larger scales are associated with earlier
- Resolves the smoothness and flatness issues in a conceptually similar way