The Tadpole Problem

Iosif Bena

IPhT, CEA-Saclay

Johan Blåbäck, Callum Brodie, Alex Buchel, Oscar Dias, Mariana Graña, Nicholas Kovensky, Severin Lüst, Gabriele Lo Monaco, Stefano Massai, Ruben Minasian, Andrea Puhm, Raffele Savelli, Dimitrios Toulikas, David Turton, Bert Vercnocke, Thomas Van Riet

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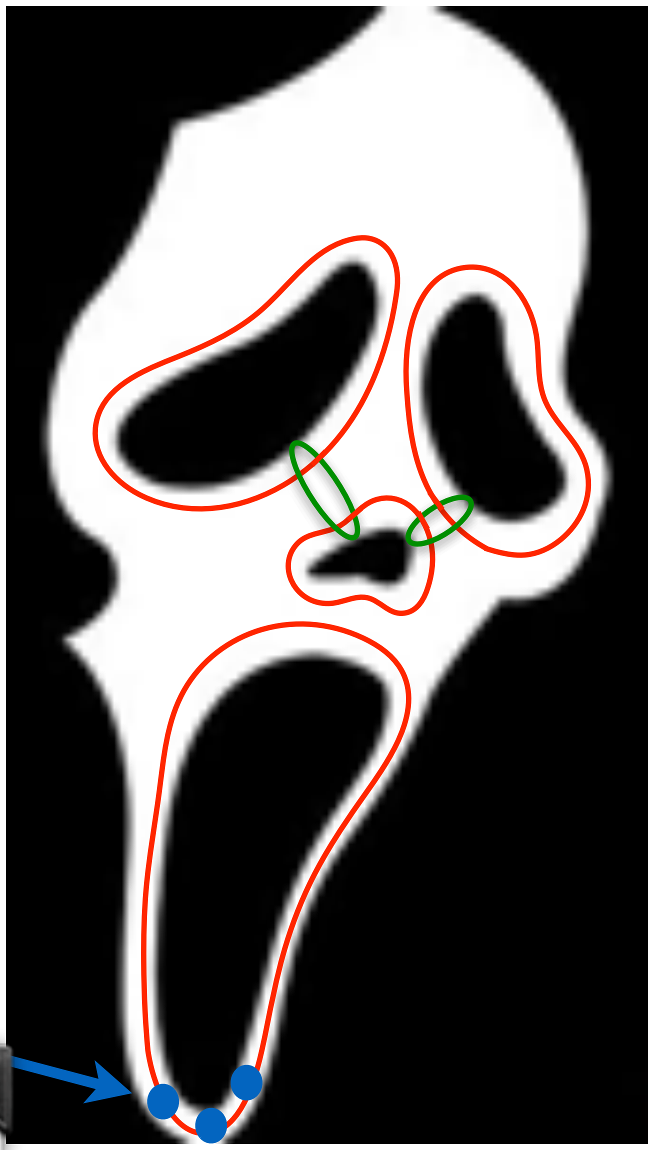
String Cosmology - Standard Lore

Compactify to 4D on 6D manifold (Calabi-Yau)
Lots of unphysical massless scalars (moduli)

add fluxes + gaugino cond. → fix moduli → 10^{500} stable AdS vacua (negative cosmological constant)

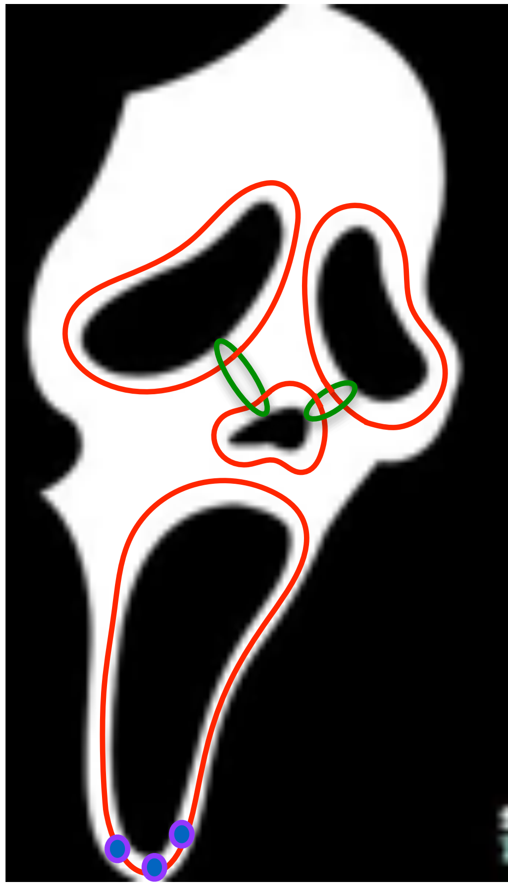
anti-D3 down long throats → redshift → very-small energy → lift AdS to dS KKLT, ~2500 others



THE LANDSCAPE

3 stages of de Sitter construction

KKLT



CY moduli:
- Complex-structure
- Kähler

1. Fix Complex-structure moduli: fluxes on 3-cycles
2. Fix Kähler moduli: D3 instantons or gaugino condensation on D7 branes
3. Add antibranes: uplift cosmological constant

The problem

Steps 1,2,3: low-energy effective field theory
using String-Theory-derived ingredients

Nontrivial interactions in String Theory

1 → ~~3~~

Bena, Graña, Kuperstein, Massai

3 → ~~1~~

Bena, Dudaş, Graña, S. Lüst
Bena, Blåbäck, Graña, S. Lüst

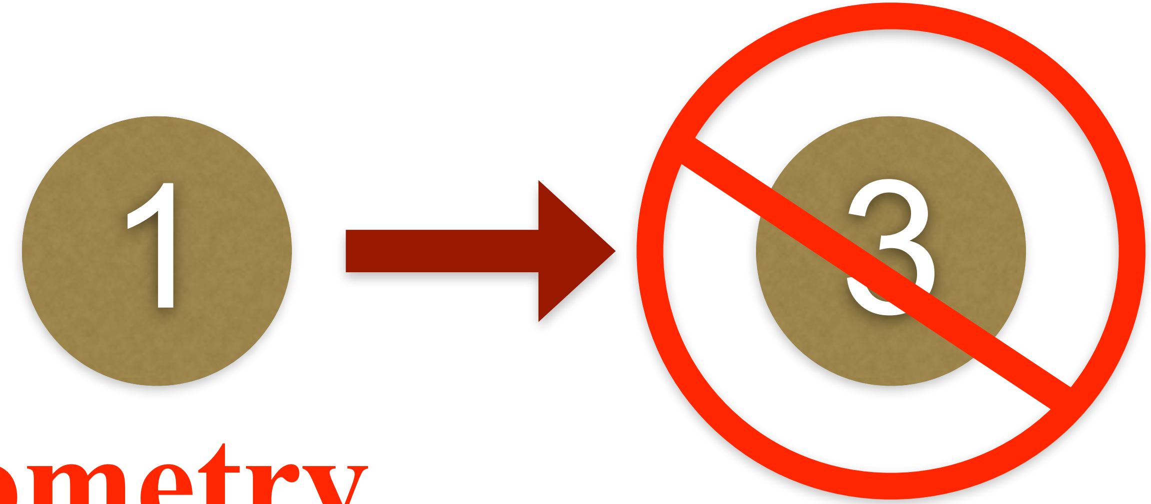
1 → ~~1~~

Bena, Blåbäck, Graña, S. Lüst
Bena, Brodie, Graña (to appear)

3 → ~~2~~

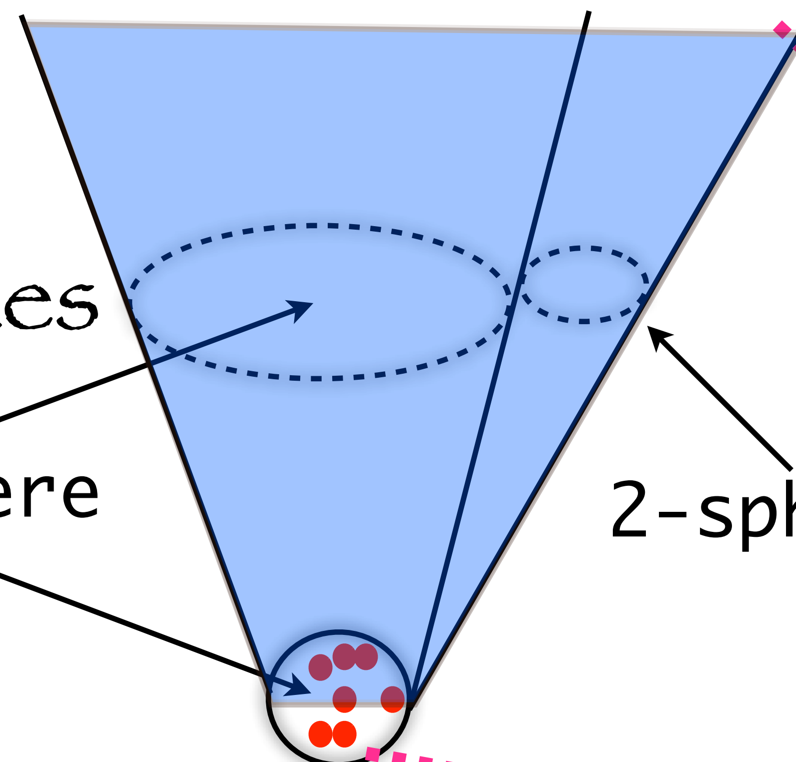
Moritz, Retolaza, Westphal
Bena, Graña, Kovensky, Retolaza,
Bena, Dudaş, Graña, Lo Monaco, Toulikas (in progress)

The First Result



Klebanov-Strassler geometry

Bena, Graña, Kuperstein, Massai



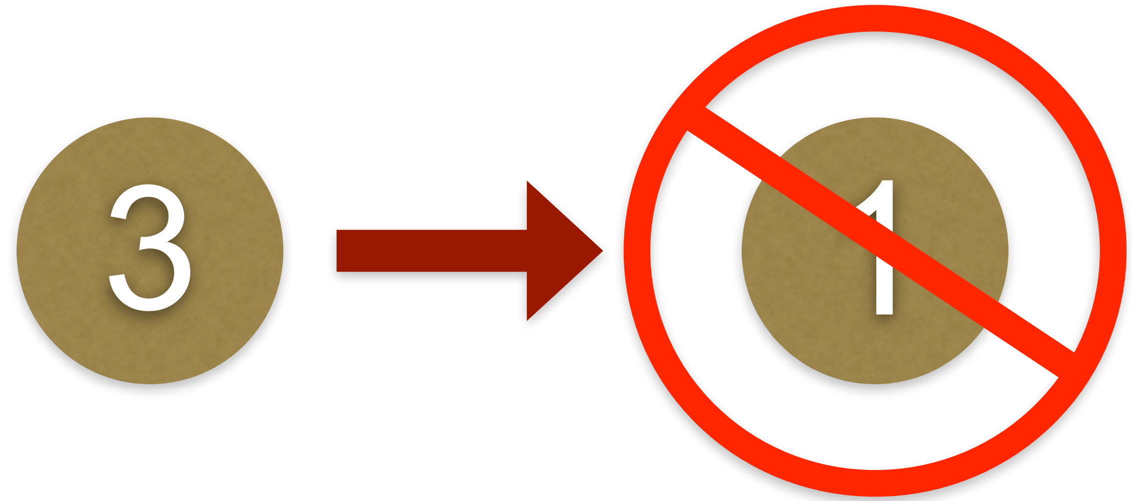
“probe” anti-D3 → NS5 = metastable

Kachru, Pearson, Verlinde; Armas, Nguyen, Niarchos, Obers, Van Riet

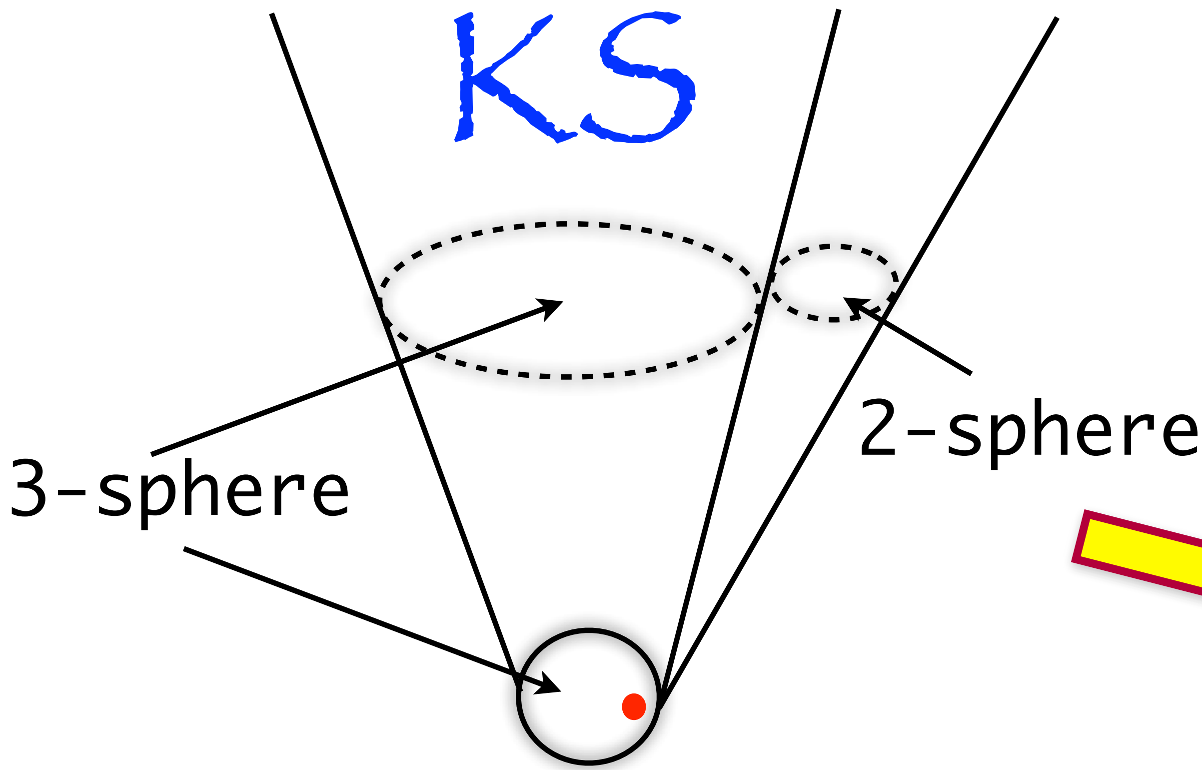
anti-D3 backreacted geometry: Coulomb-branch tachyon

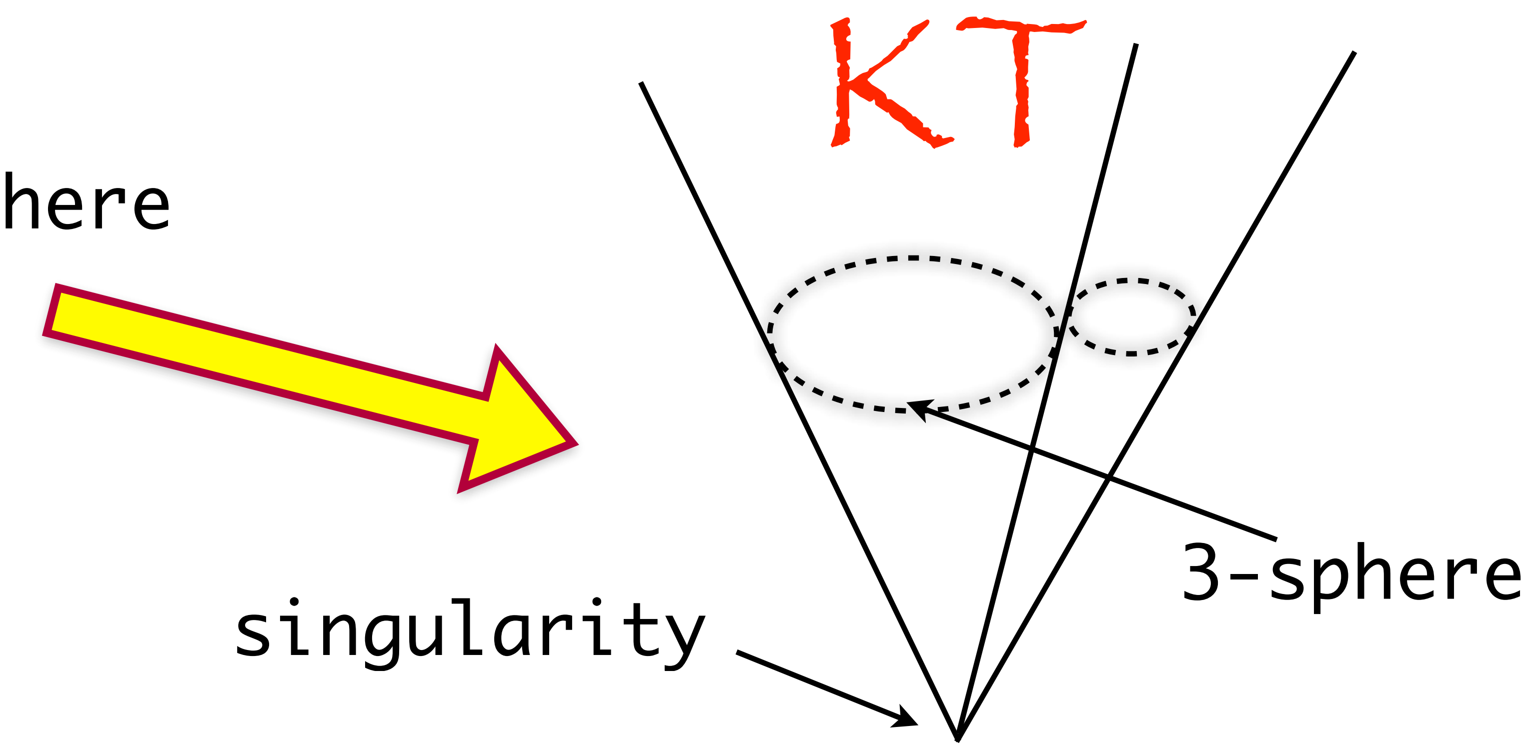
Bena, Graña, Halmagyi, Kuperstein, Massai, 2009-2014

Second Result



Bena, Dudaş, Graña, S. Lüst
Bena, Blåbäck, Graña, S. Lüst





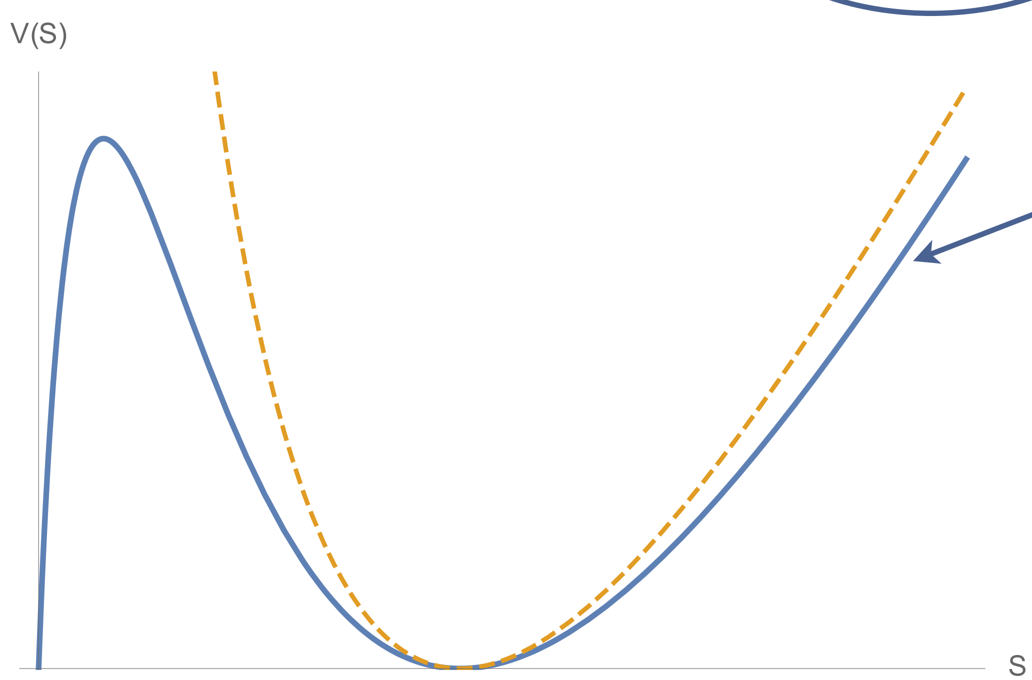
- 3-sphere size = conifold deformation modulus
- Fixed in flux compactification, flat in ∞ KS
- anti-D3 energy + flat direction $\Rightarrow$ runaway to ∞ KT

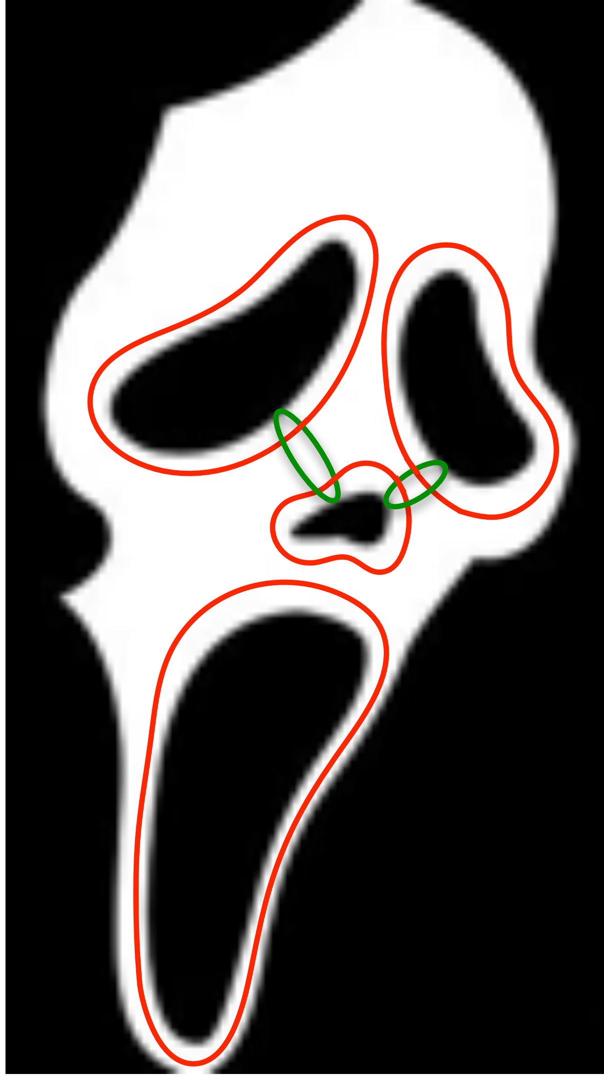
Stabilization in compactification

Douglas, Torroba

$$V_{KS} = \frac{\pi^{3/2}}{\kappa_{10}} \frac{g_s}{(\mathrm{Im}\,\rho)^3} \left[c \log \frac{\Lambda_0^3}{|S|} + c' \frac{g_s(\alpha' M)^2}{|S|^{4/3}} \right]^{-1} \left| \frac{M}{2\pi i} \log \frac{\Lambda_0^3}{S} + i \frac{K}{g_s} \right|^2$$

Quantum Correction

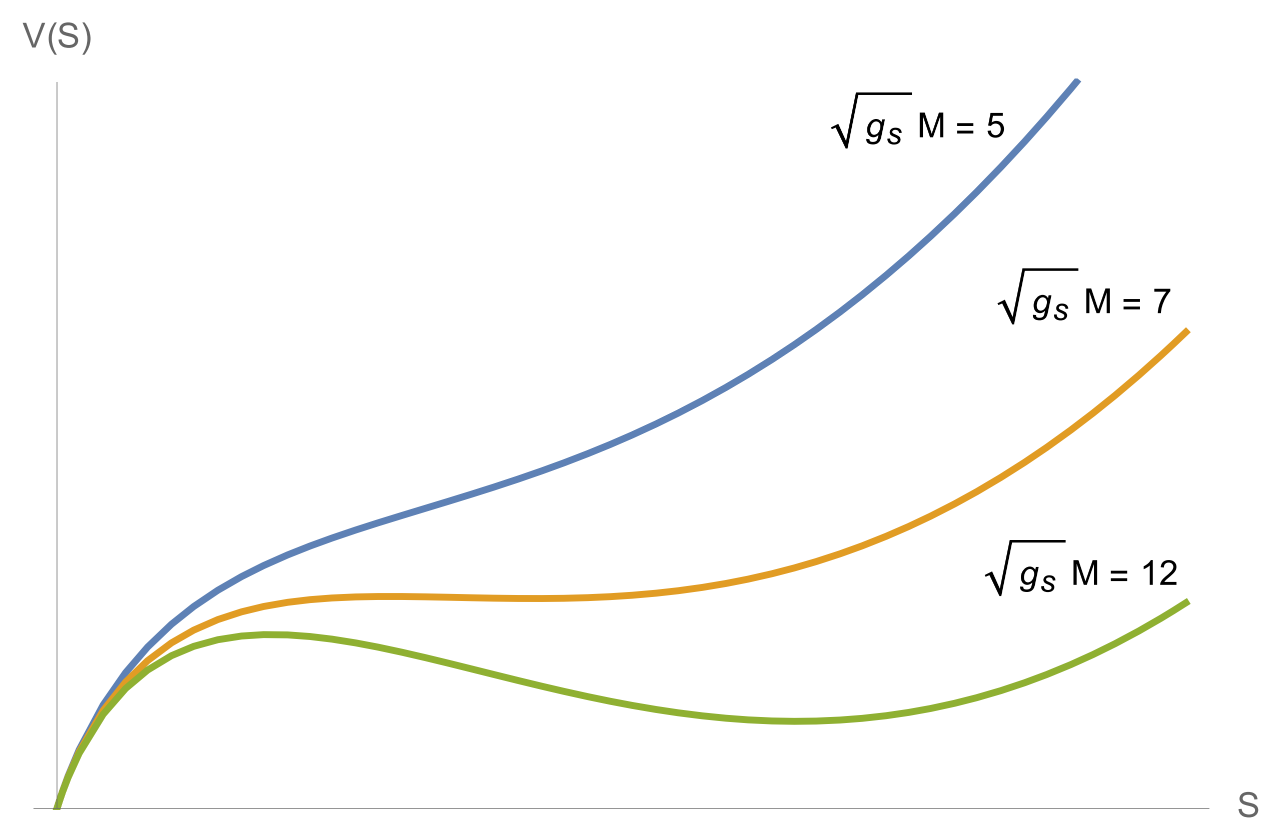




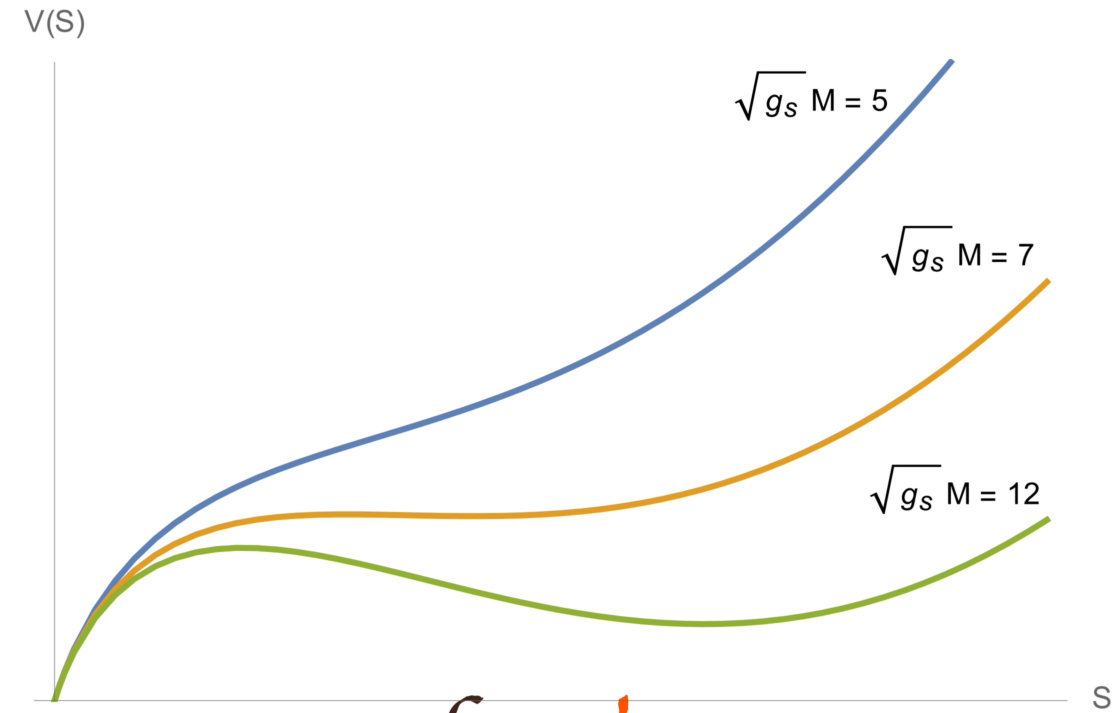
Add single anti-D3 brane:

$$V_{\overline{D3}} = \frac{\pi^{1/2}}{\kappa_{10}} \frac{1}{(\mathrm{Im}\,\rho)^3} \frac{2^{1/3}}{I(\tau)} \frac{|S|^{4/3}}{g_s(\alpha' M)^2}$$

Bena, Dudaș, Graña, S. Lüst



∞-throat KS destabilization



- anti-D3 does not destabilize ∞ KS for large M
- neither should a small BH horizon $\Rightarrow$
- conjecture: $\exists$ KS black hole   Bena, Dudaș, Graña, S. Lüst
- Deconfined, χSB. Confirmed by numerics   Buchel
- Existence range similar to metastable anti-D3:

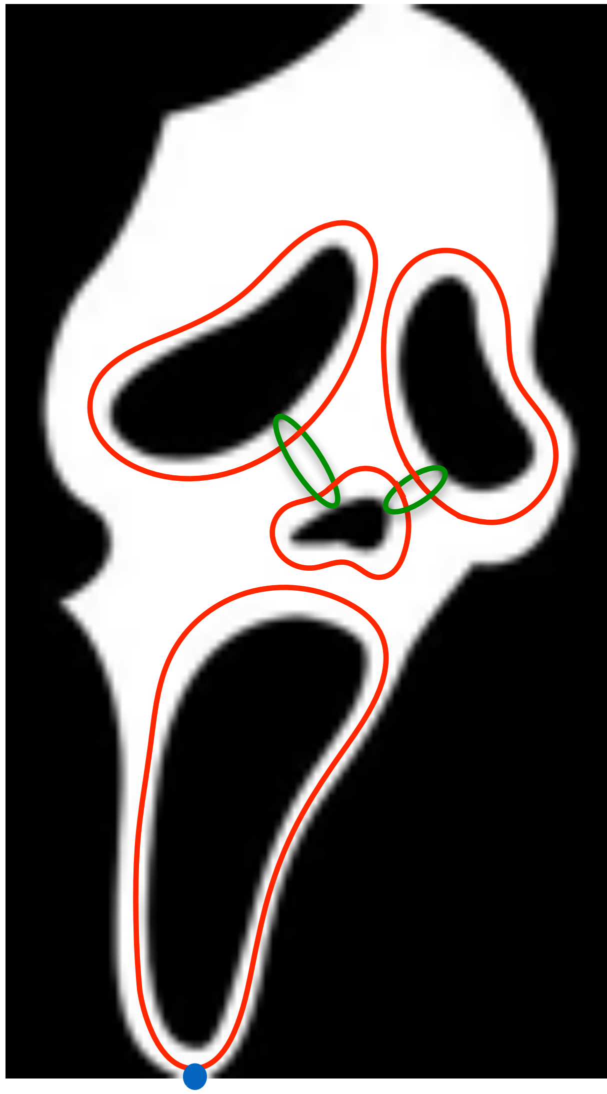
Bena, Buchel, S. Lüst

$$\sqrt{g_s}M \;\geq\; \gamma\sqrt{N_{\overline{D3}}} \qquad \gamma_{\overline{D3}} \approx 6.8 \qquad \gamma_{BH} \approx 4.16$$

UV-IR Hierarchy

Runaway mode ↔ jaw becoming longer and longer

Bena, Dudaş, Graña, S. Lüst



Goes away if $\boxed{\sqrt{g_s}M \;\geq\; 6.8\sqrt{N_{\overline{D3}}}}$

$$e^A|_{\text{bottom}} = \frac{\Lambda_{IR}}{\Lambda_{UV}} = \exp\left(-\frac{2\pi}{3}\frac{KM}{g_s M^2}\right) > e^{-\frac{2\pi}{3}\frac{Q^{\text{throat}}_{\text{flux}}}{6.8^2}} \sim \mathcal{O}(10^{-2})$$

Hierarchy requires $Q^{\text{throat}}_{\text{flux}} \gtrsim \mathcal{O}(100)$

But total charge on compact space has to be zero !

Lost 100 units of charge Master Calabi-Yau has ?
How embarrassing ? How embarrassing !

How to get -100 units of charge ?

- O3 planes - at most -32
- D7 planes on 4-cycle S with huge Euler number: $\chi(S)/24$
- F-theory : $\chi(CY_4)/24$

$\exists\, \chi\,(CY_4)=1\,820\,448$
$N_3 = 75\,852$

Taylor, Wang 15

~ 100

$$N_{\rm D3}^{\rm throat} + \frac{1}{2}\int_{CY_4} G\wedge G = \frac{\chi(CY_4)}{24}$$

$$\chi(CY_4) = 6(8+h^{1,1}+h^{3,1}-h^{2,1})$$

how much D3 charge ?
$> 100\,000$?

Add fluxes to stabilize
these 303 148 moduli

Tadpole question: How much tadpole is sourced by fluxes which stabilize n complex-structure moduli ?

What has been believed

e.g. Ashok, Denef, Douglas

- Just throw any fluxes on cycles and *everything will be fine.* $Q_{D3} = \frac{1}{2} N^A \eta_{AB} N^B$
- 10^{500} possibilities
- Fluxes that stabilize all complex-structure moduli can have any tadpole one wants
- One can turn on lots of fluxes and still have tadpole of order 1. For example:

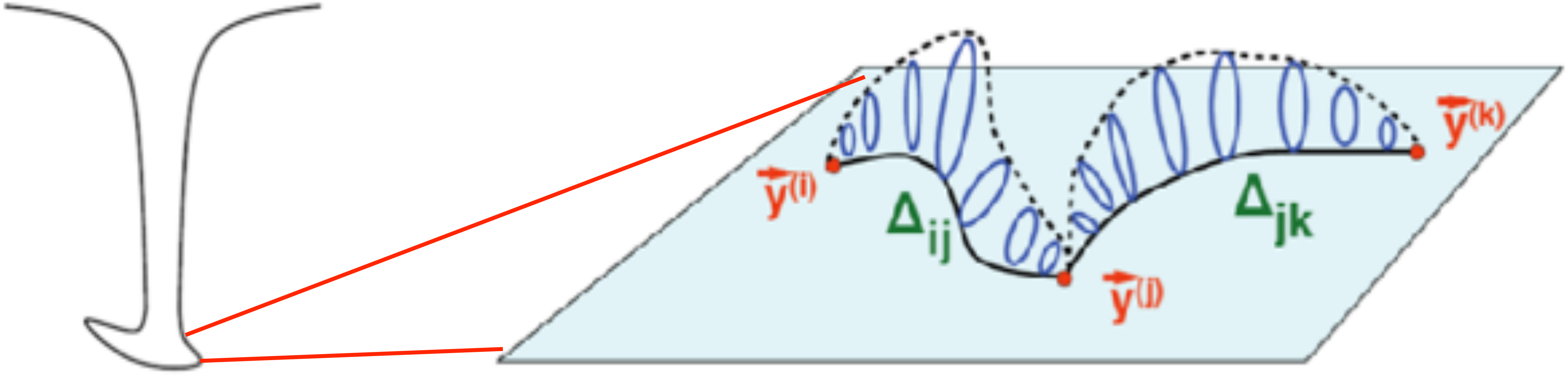
$$\begin{pmatrix} 1 & 0 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

I’m a Romanian ! mind tricks do not work on me !
Only supergravity solutions !

What has been constructed

- Lots of "bubbling geometries" with self-dual fluxes wrapping and stabilizing n cycles
- Fluxes source M2 charges ⇒ microstate geometries *(fuzzballs)* of M2-M2-M2 black hole

(Some can be dualized to Denef-type multicenter BHs)



$$Q_{M2} \sim n^2$$

Bena, Wang, Warner,

What happens when you try arbitrarily throwing fluxes:

- Warp factor: $d * d\, Z = G \wedge G$
- $G \wedge G$ will have positive and negative regions. Z wants to be negative
- Most flux choices $\Rightarrow$ Closed Timelike Curves
- $Q_{M2} \sim n^2$. Impossible to get a smaller charge from fluxes on n cycles

If $Q_{M2} \sim n^0$ one could blow an arbitrary nr. of bubbles@fixed charge $\Rightarrow$ overcount BH entropy

Tadpole Question: 3 incarnations

- IIB: $h^{2,1}$ complex-structure moduli stabilized by 3-form fluxes:

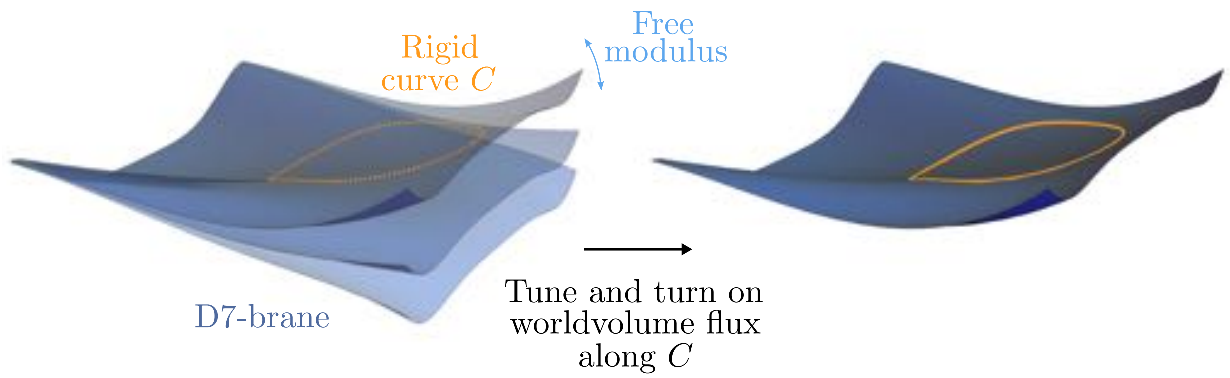
$$\int_{\alpha_I} F_3 = \underbrace{M^I}_{\in \mathbb{Z}} \qquad \int_{\alpha_I} H_3 = K^I \qquad Q_{\text{flux}} = \int F_3 \wedge H_3 = M^I K_I > 0$$

basis of 3-cycles

at minimum $H_3 = \star F_3$

How does Q_{flux} grow with $h^{2,1}$?

- IIB: D7 moduli stabilized by worldvolume F_2



How much Q_{flux} sourced by F_2 ?

Tadpole Question: 3 incarnations

- F-theory on CY_4

$$\left.\begin{array}{l} h^{2,1} \text{ complex structure moduli} \\ \text{D7-brane moduli} \end{array}\right\} \quad h^{3,1} \text{ complex structure moduli of } CY_4$$

$$\left.\begin{array}{l} \text{3-form fluxes } H_3, F_3 \\ \text{2-form fluxes } F_2 \text{ on D7} \end{array}\right\} \quad \text{4-form flux } G_4$$

Tadpole of fluxes

moduli stabilized by fluxes

$$Q_{\text{flux}} = \frac{1}{2}\int G_4 \wedge G_4 \leq \frac{\chi(CY_4)}{24} = \frac{1}{4}(h^{3,1} + h^{1,1} - h^{2,1} + 8) \sim \frac{1}{4} n_{\text{moduli}}$$

all the negative 3-charge from D7/O7

for large $h^{3,1}$

Tadpole Question - answers

Description	n_{moduli}	$Q_{\text{flux}}^{\text{min}}$	$\alpha = \frac{Q_{\text{flux}}^{\text{min}}}{n_{\text{moduli}}}$	Ref
IIB at highly symm pt in mod space	$h^{2,1} = 128$	48	0,38	Giryavets, Kachru, Tripathy, Trivedi 03
	$h^{2,1} = 272$	124	0,46	Demirtas, Kim, Mc Allister, Morritz 19
F-theory on sextic CY	$h^{3,1} = 426$	775/4	0,45	Braun, Valandro 20
F-theory on $\mathbb{CP}^3$ base	$n_7 = 3728$	1638	0,44	Collinucci, Denef Esole 08
F-theory on any weak-Fano base	$n_7 = 58c_1^3(B) + 16$	$\frac{7}{16}(58c_1^3(B) + 15)$	0,44	Bena, Brodie, Graña to appear
M-theory on K3xK3	57	25	0,44	Bena, Blåbäck, Graña, S. Lüst 20

Tadpole Conjecture(s)

Bena, Blåbäck, Graña, S. Lüst '20

1. Smallest charge sourced by the fluxes that stabilize n moduli grows *linearly* with n

2. $$\alpha \equiv \frac{Q_{\text{flux}}^{\text{min}}}{n_{\text{moduli}}} > \frac{1}{3}$$

Remarks:

- $Q_{\text{flux}}^{\text{min}}$ = minimum value. $Q_{\text{flux}}^{\text{generic}} \sim n_{\text{moduli}}^2$
- No corresponding sugra solution with no ctc's.

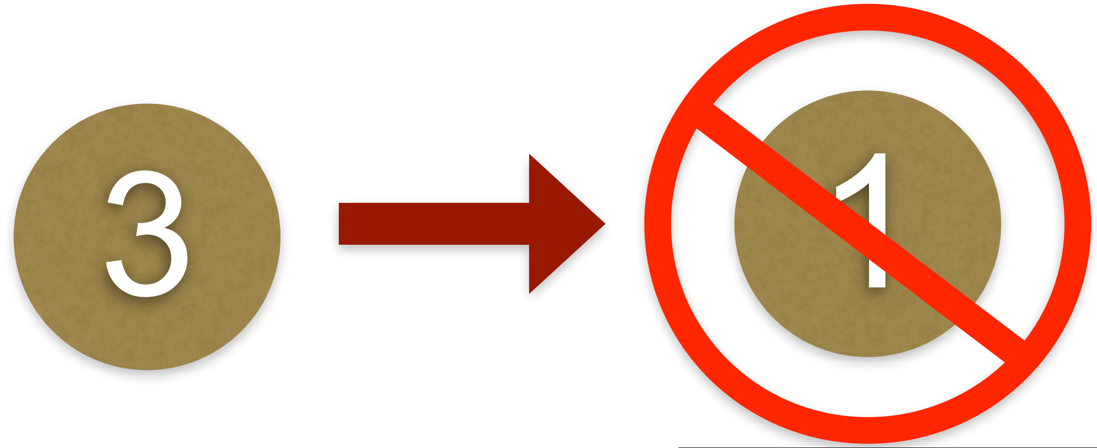
Implications 1 → ~~1~~

$$Q_{\text{flux}} = \frac{1}{2}\int G_4 \wedge G_4 \begin{cases} \leq \dfrac{\chi(CY_4)}{24} = \dfrac{1}{4}(h^{3,1} + h^{1,1} - h^{2,1} + 8) \sim \dfrac{1}{4} n_{\text{moduli}} \\ > \dfrac{1}{3} n_{\text{moduli}} \end{cases}$$

(tadpole conjecture)

- No F-theory stable vacua with large $h^{3,1}$.
  All $\mathbf{10^{272\,000}}$ vacua have flat directions !
- D7 brane wrapped on 4-cycle with large χ
  - negative tadpole contribution $-\dfrac{\chi}{24}$
  - D7 moduli stabilized by fluxes with tadpole $> \dfrac{\chi}{18}$
  - Such D7 branes increase the tadpole

Back to de Sitter



Uplifting with 1 antibrane ⇒ throat of charge >100

Bena, Dudaș, Graña, S. Lüst

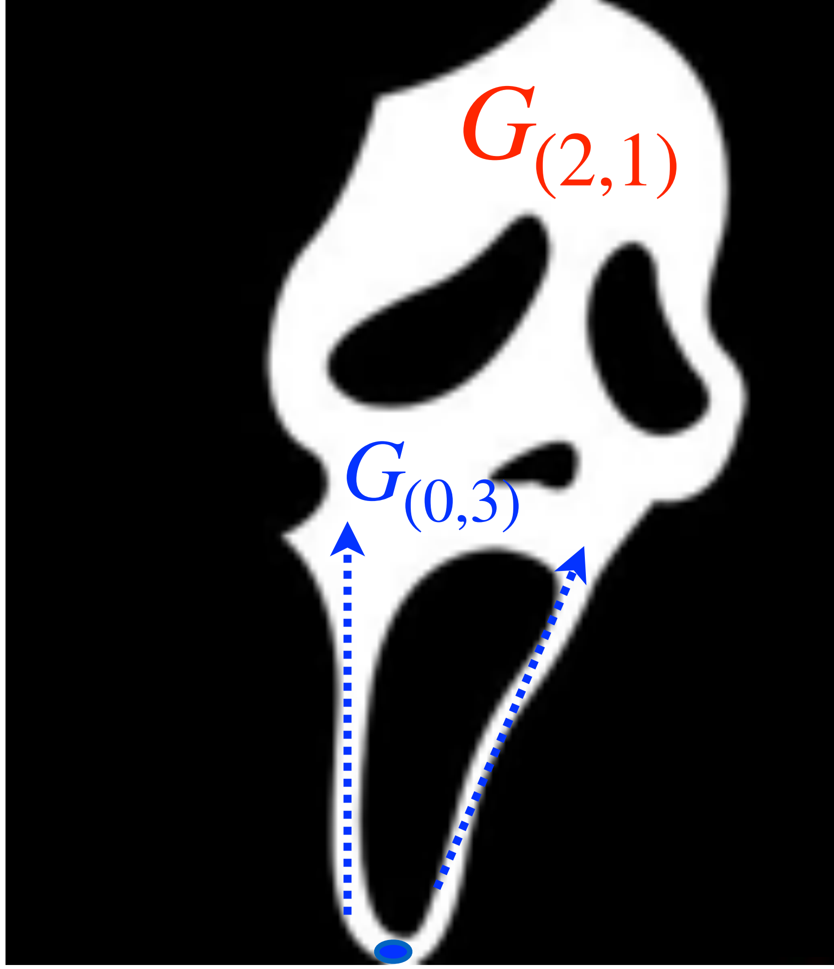
- Very hard/impossible to absorb so much tadpole with stabilized moduli:
- Cannot be done with O3, O7 or D7

- Hard to go around:
  - hierarchy crucial in KKLT for getting small positive Λ
  - maybe smaller hierarchy for LVS — Crinò, Quevedo, Valandro
  - de Sitter with no hierarchy — Bento, Chakraborty, Parameswaran, Zavala

Bare Bones de Sitter

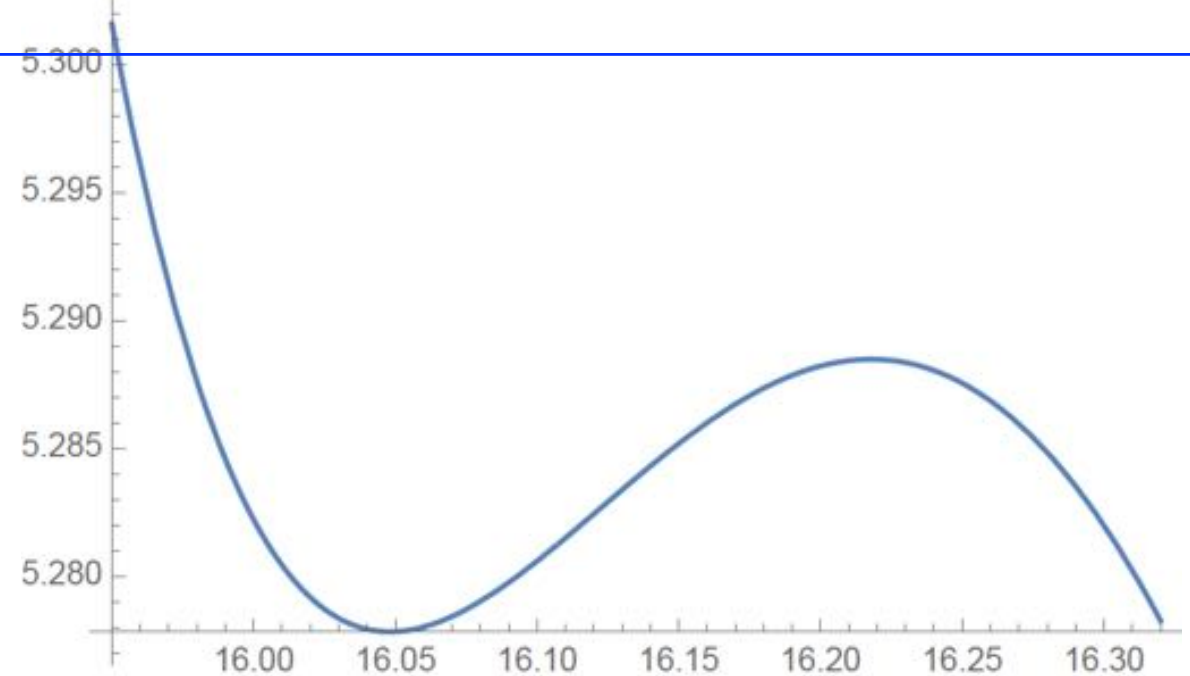
Bena, Dudaș, Graña, Lo Monaco, Toulikas (to appear)

- Top-down calculations also make some KKLT steps easier
- Step 1 : $G_{(2,1)}$ to fix moduli
- Add by hand $G_{(0,3)}$ **not necessary !**
- Break susy, need small $W_0 \neq 0$



anti-D3

antibrane fluxes source $G_{(0,3)}$ $\rightarrow$ nonzero W_0 !!!



Tadpole Caveats

- Only based on calculations on spaces with special properties
  - Maybe on less symmetric spaces things will be different
  - Not likely. Example of F-theory CY_4 fibered over B_3:
    $\mathbb{CP}^3$, toric $\sim \mathcal{O}(10^{15})$, Weak Fano Bena, Brodie, Graña, to appear

- Moduli stabilization scenarios Marchesano, Prieto, Wiesner '21

  Standard scenario $Q_{\rm flux} \sim n^2_{\rm moduli}$

  Non-standard scenario $Q_{\rm flux} = \mathcal{O}(1)$ - seems to violate tadpole conjecture

  Flux that enters the tadpole enters all equations for moduli

  Only worked out in detail for the small number of moduli.
  Genericity problems. S. Lüst; Grimm, Plauschinn, van de Heisteeg '21

  We believe it will be impossible to stabilize a large number of moduli with fluxes sourcing $\mathcal{O}(1)$ charge

  Bena, Brodie, Graña, S. Lüst, in progress

How can we see this ?

- $G_4 = N^A \omega_A$ where $\omega_A \in H^4(CY_4, \mathbb{Z})$
- Tadpole : $\frac{1}{2}\int_Z G_4 \wedge G_4 = \frac{1}{2} N^A \eta_{AB} N^B$
- Throw arbitrary $N^A \Rightarrow$ naively any tadpole
- However, at minimum $\Rightarrow G_4 = * G_4$
- Tadpole contribution = everywhere positive
- If many fluxes give zero or negative contributions $\Rightarrow$ trouble

Many fluxes + small tadpole

Example:

similar to Marchesano, Prieto, Wiesner

$$\boxed{N^A \eta_{AB} N^B}$$

$$\begin{pmatrix} 1 & \boxed{f_B} & 1 & 0 & 1 & 0 & \ldots \end{pmatrix} \times \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & \ldots \\ 1 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 1 & 0 & 0 & \ldots \\ 0 & 0 & 1 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & 0 & 1 & \ldots \\ 0 & 0 & 0 & 0 & 1 & 0 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \times \begin{pmatrix} 1 \\ \boxed{f_B} \\ 1 \\ 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix} = 2\boxed{f_B}$$

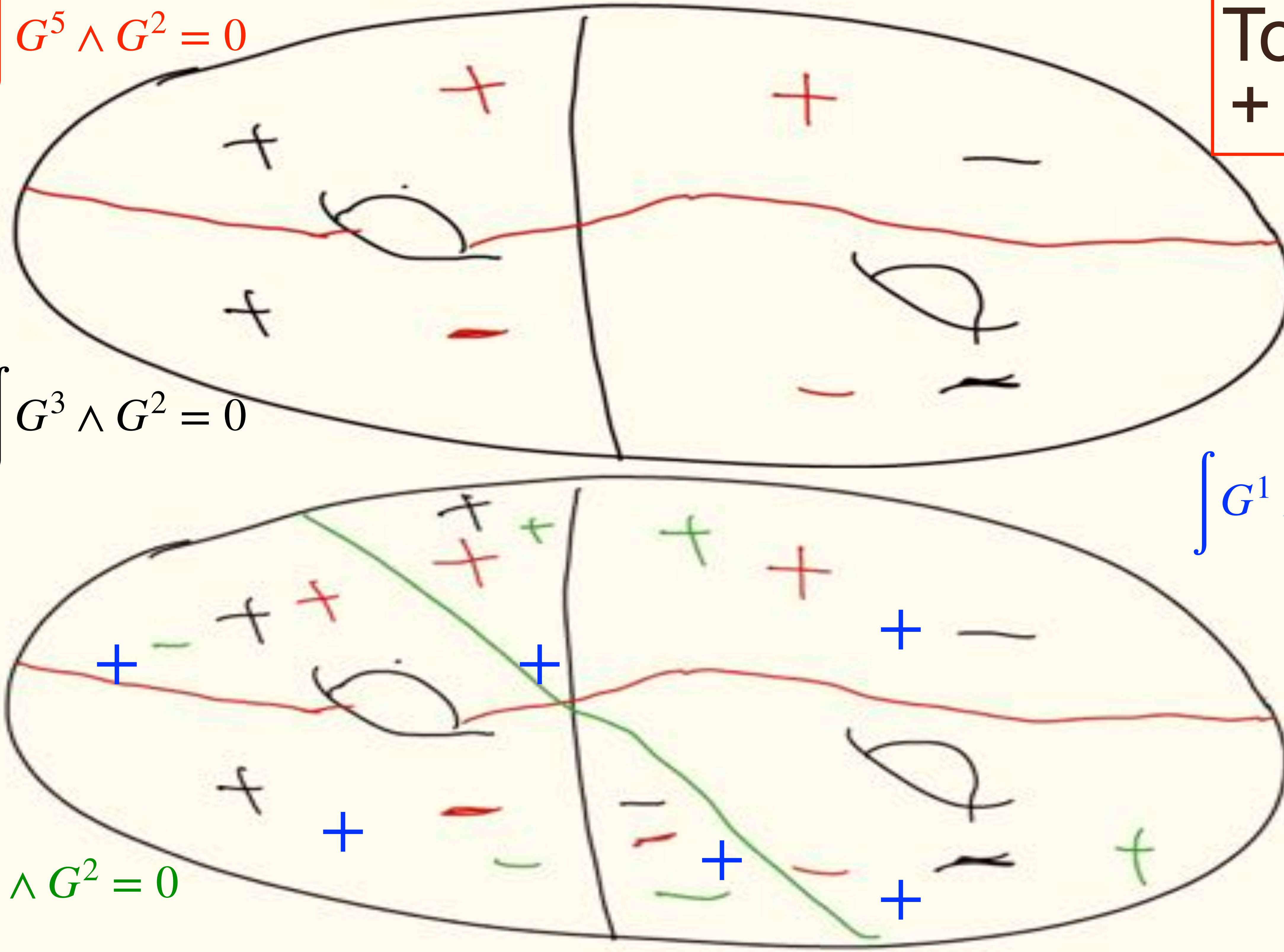
$$G = N^A \omega_A = G^1 + G^2 + G^3 + G^4 + G^5 + \ldots$$

$$\int G^3 \wedge G^2 = \int G^5 \wedge G^2 = \int G^7 \wedge G^2 = \ldots = 0$$

$$\int G^1 \wedge G^2 = \boxed{f_B}$$

$\mathcal{O}(n)$ vanishing integrals (at least)

Topology + Fluxes



2^n regions, some with n negative and 1 positive contributions

$G_4 = {}^*G_4$ forces metric to make $Q_{\text{flux}} > 0$ in each region

Metric screams $\Rightarrow$ negative warp factors, ctc's

Scapezilla's trail of destruction

- “all ant  es are OK” 2010
- “$g_s N_{antiD3} \ll 1$  ” 2012
- “a single anti-D3 is ( 2015
- tadpole > $\mathcal{O}(100)$ cancelec  D7s 2018
- “F-theory  the day” 2020

No de Sitter via antibrane uplift in warped throats

Impossible to stabilize large numbers of complex-structure moduli

Conclusions

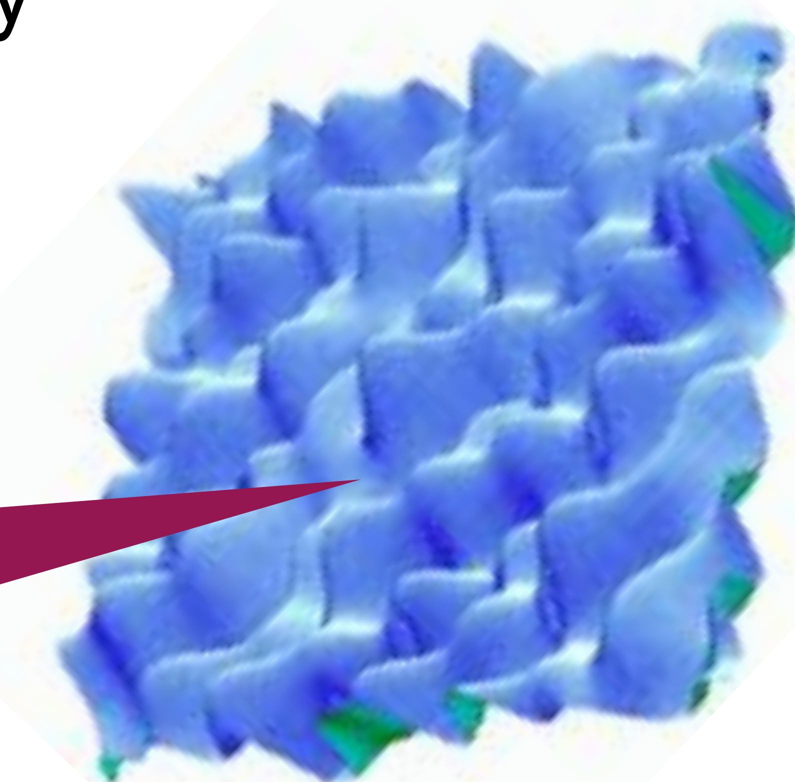
Calculations could have given pro-landscape results. They did not !

Romanian Proverb: if 3 people tell you that you are drunk, go and take a nap !

Physics Version: if 3 calculations tell you that something does not work, maybe it is time to give it up

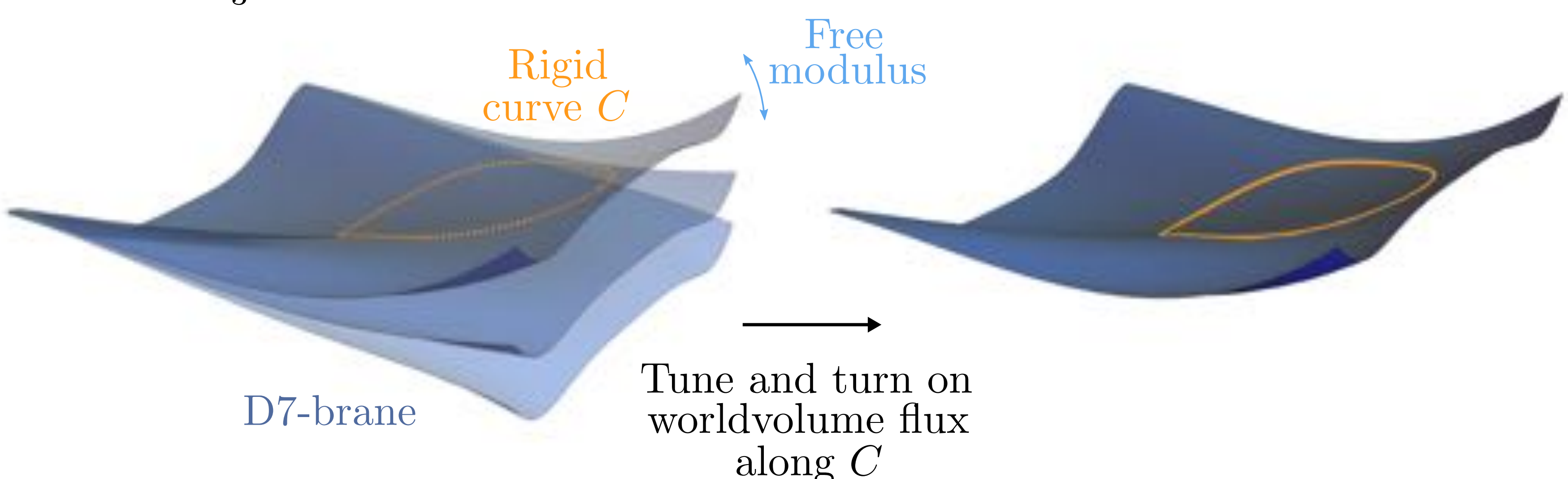
- No lazy anthropic solution to fine-tuning problems
- Back to drawing board in String Cosmology
- No controlled construction of de Sitter ☹️
- No string inflation model one can trust ☹️
- In agreement with Swampland programme
- Quintessence ? ☹️
  Non-perturbative ?

Where are we ?



D7 moduli stabilization

- F-theory on CY 4-fold fibered over a base B_3 in Sen limit
  - D7-brane moduli $$n_7 = 58 \int_{B_3} c_1(B_3)^3 + 16$$
- Stabilized by F_2



Tadpole cancelation condition

$$\boxed{Q_{\text{flux}} = \frac{1}{2}\int_S F_2 \wedge F_2} \leq \boxed{15 \int_{B_3} c_1(B_3)^3 + 12} \sim \frac{15}{58} n_7 \simeq \boxed{0.26\, n_7} \quad \text{for large } n_7$$

negative
3-charge from D7/O7

Tadpole conjecture

$$\boxed{\frac{1}{2}\int_S F \wedge F \,\Big|_{\text{all D7-moduli are stabilized}}} > \boxed{\frac{1}{3} n_7}$$

If true, cannot stabilize
a large number of moduli

D7 moduli stabilization

Bena, Brodie, Graña, to appear

- We verified tadpole conjecture for any Weak Fano base !

- Moduli stabilized by flux $F \leftrightarrow C$ complex curve

$$C \cdot (-K) = 4d \text{ for } \mathbb{CP}^3$$

$$n_{\text{stab.moduli}} \leq 8\tilde{d} + 1 \qquad Q_{\text{flux}} \geq \frac{7}{2}\tilde{d} + 1 - g$$

$$\left(n_7 = 58 \int c_1(B_3)^3 + 16 \right)$$

- For large n_7 and fixed genus, we recover $\alpha \geq \frac{7}{16} \simeq 0.44 > 0.26$ allowed by tadpole cancelation condition

$> \frac{1}{3}$ Tadpole conjecture

- Moduli cannot be stabilized within tadpole, Tadpole conjecture satisfied

- This reduces to the result for B₃= $\mathbb{CP}^3$, genus 0

Collinucci, Denef, Esole 08

$$n_{\text{stab.moduli}} = 32d + 1 \qquad Q_{\text{flux}} \geq 14d + 1 \qquad |Q_{\text{neg}}| = 972 \qquad \alpha \geq \frac{14}{32} \simeq 0.44$$
$$= 3728 \qquad \geq 1640$$