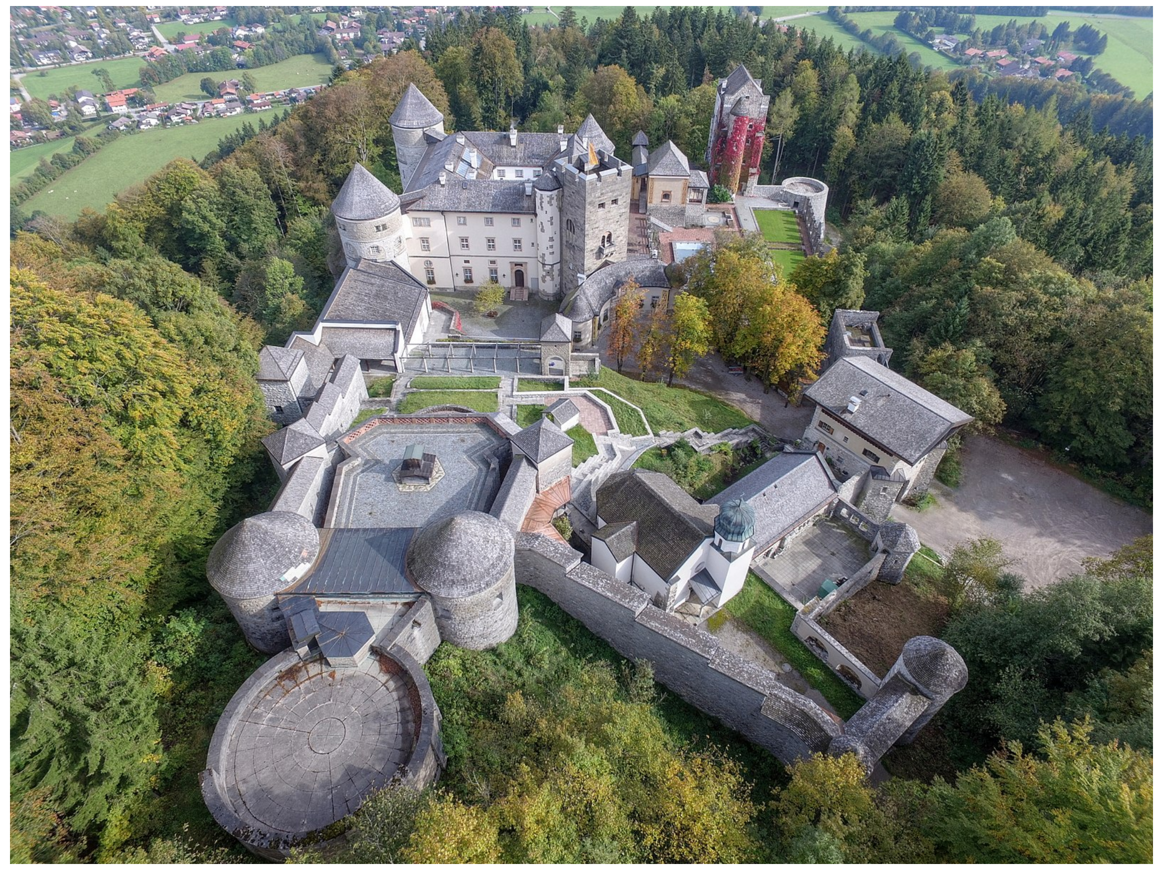
RINGBERG CASTLE, 12/11/2021



SWAMPLAND, STRING DEFECTS AND (BRANE) SUPERSYMMETRY BREAKING

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Based on arXiv:2007.12722
in collaboration with Bonnefoy, Condeescu, Dudas + …

WE LEARN IN BOOKS THAT A THEORY TO BE CONSISTENT MUST BE FREE FROM ANOMALIES

HOWEVER, THIS IS NOT ENOUGH TO BE IN THE LANDSCAPE

HOW TO FURTHER CONSTRAIN IT?

KIM, SHIU AND VAFA SUGGESTED TO LOOK AT THE CONSISTENCY OF EFFECTIVE THEORY ON DEFECTS

[Kim, Shiu, Vafa 2019]

THIS APPROACH IS PARTICULARLY EFFECTIVE IN D=10 AND D=6 DIMENSIONS WHERE ANOMALIES ARE VERY RICH

(IN D=10 KSV MANAGED TO RULE OUT $U(1)^{496}$ AND $E_8 \mathrm{x} U(1)^{248}$)

OUTLINE

The $\mathcal{N}$=(0,1) (SUSY) vacuum

String defects and anomaly inflow

A conjecture

(Brane) Supersymmetry Breaking

Reducible anomaly and couplings in the LEEA

THE $\mathcal{N}$=(0,1) VACUUM IN D=6

A generic 6d vacuum with minimal supersymmetry
has four different types of matter

Gravity multiplet: $\{g_{\mu\nu}, C^{+}_{\mu\nu}; \psi_{\mu\,\alpha}\}$

Tensor multiplet: $\{\phi, C^{-}_{\mu\nu}; \tau_{\dot{\alpha}}\}$

Vector multiplet: $\{A_{\mu}; \lambda_{\alpha}\}$ $\longrightarrow \; G = \prod_i G_i$

Hyper multiplet: $\{4\phi; \zeta_{\dot{\alpha}}\}$ $\longrightarrow \; R_G$

THE $\mathcal{N}$=(0,1) VACUUM IN D=6

Absence of irreducible (gravitational) anomaly (trR^4):

$$273 - 29\, n_{\rm T} + n_{\rm V} - n_{\rm H} = 0$$

The reducible anomaly polynomial

$$I_8 = \tfrac{1}{2}\, X_4^{\alpha}\, \Omega_{\alpha\beta}\, X_4^{\beta}$$

$SO(1, n_T)$

$$X_4^{\alpha} = \tfrac{1}{2}\, a^{\alpha}\, {\rm tr} R^2 + \tfrac{1}{2} \sum_i \frac{b_i^{\alpha}}{\lambda_i}\, {\rm tr} F_i^2$$

THE $\mathcal{N}$=(0,1) VACUUM IN D=6

The reducible anomaly is then cancelled by the *Sagnotti-Green-Schwarz* counter-term

$$S_{\mathrm{GS}} = \int \Omega_{\alpha\beta} C_2^\alpha \wedge X_4^\beta$$

which implies a redefinition of the two-form curvature

$$H_3 = dC_2 + \omega_3$$

Supersymmetry relates this term to other couplings in the action

$$S \supset \int \sum_i J(\phi) \cdot b_i \, \mathrm{tr}(F_i^2)$$

$A \cdot B = \Omega_{\alpha\beta} A^\alpha B^\beta$

$$J \cdot J > 0 \,, \qquad J \cdot b_i > 0 \,, \qquad J \cdot a < 0$$

STRING DEFECTS AND ANOMALY INFLOW

One can (should) consider one-dimensional defects in these theories.

If BPS they preserve four supercharges and have $\mathcal{N}$=(0,4) supersymmetry on the two-dimensional world sheet

KSV derived consistency conditions for the consistency of the 2D/6D system

[Kim, Shiu, Vafa 2019]

STRING DEFECTS AND ANOMALY INFLOW

$$S_{\mathrm{D1}} \supset -Q^{\alpha}\Omega_{\alpha\beta}\int C_2^{\beta}$$

and from the anomalous transformation of C_2 so that the d.o.f. on the defect must cancel the anomaly inflow

$$\begin{aligned} I_4 &= \Omega_{\alpha\beta}Q^{\alpha}\left(X_4^{\beta} + \tfrac{1}{2}Q^{\beta}\chi(N)\right) \\ &= \tfrac{1}{2}Q\cdot a\,\mathrm{tr}R^2 + \tfrac{1}{2}\frac{Q\cdot b_i}{\lambda_i}\,\mathrm{tr}F_i^2 + \tfrac{1}{2}Q\cdot Q\,\chi(N) \end{aligned}$$

STRING DEFECTS AND ANOMALY INFLOW

From the anomaly inflow we can extract the CFT data

$$c_{\mathrm{L}} - c_{\mathrm{R}} = 6Q \cdot a + 2 \,, \qquad k_i = Q \cdot b_i$$

If one can (unambiguously) identify a SU(2)$_R$ symmetry

$$\begin{aligned} c_{\mathrm{L}} &= 3Q \cdot Q - 9Q \cdot a + 2 \,, \\ c_{\mathrm{R}} &= 3Q \cdot Q - 3Q \cdot a \,. \end{aligned}$$

KSV CONSISTENCY CONDITIONS

[Kim, Shiu, Vafa 2019]

$J \cdot J > 0$

tensor metric positivity

$J \cdot a < 0$

Gauss-Bonnet positivity

$J \cdot b_i > 0$

vector metric positivity

conditions on 6D SUGRA

$Q \cdot J \geq 0$

positivity D1 tension

$Q \cdot Q \geq -1$

$Q \cdot Q + Q \cdot a \geq -2$

$k_i \geq 0$

positivity of central charges (c_R and k_l)

non-degenerate strings

unitarity $$\sum_i \frac{k_i \dim G_i}{k_i + h_i^\vee} \leq c_\mathrm{L}$$

A CONJECTURE

[C.A., Bonnefoy, Condeescu, Dudas 2019]

In all consistent (F-theory or orientifold) constructions we have analysed we have always found defects with ***null-charge***

$$Q \cdot Q = 0$$

In string compactification this clearly follows from the D1 string existing in 10D

What about really non-geometric constructions?

Null-charge defects should exist for any model in the landscape

(clearly, for vacua with at least one tensor multiplet)

New conjecture

A CONJECTURE

[C.A., Bonnefoy, Condeescu, Dudas 2019]

Example 1

$$n_{\mathrm{T}} = 1\,, \qquad G = \mathrm{SU}(N)\,, \qquad \tfrac{1}{2}N(N+1) + (N-8) \times N$$

$a = (-3,1)\,, \quad b = (0,-1)\,, \quad Q = (\pm q, q)$ does not satisfy KSV constraints

Previous restrictions
$N < 31$ (anomaly)
N < 118 (KSV)

Example 2

$$n_{\mathrm{T}} = 1\,, \qquad G = \mathrm{SU}(24) \times \mathrm{SO}(8)\,, \qquad 3 \times 276$$

$a = (-3,1)\,, \quad b_1 = (1,0)\,, \quad b_2 = (0,-2)\,, \quad Q = (\pm q, q)$ does not satisfy KSV constraints

NO previous
restrictions

Examples from:
[Kumar, Morrison, Taylor 2010]

These models are ruled out by the null-charge conjecture

BREAKING SUPERSYMMETRY IN THE VACUUM

DO KSV CONSISTENCY CONDITIONS HOLD
WHEN SUPERSYMMETRY IS BROKEN?

spontaneous breaking

continuous deformation of supersymmetric models (Scherk-Schwarz)

string-scale breaking

isolated vacua disconnected from supersymmetric ones (Brane Supersymmetry Breaking — BSB)

BRANE BREAKING SUPERSYMMETRY IN THE VACUUM

[Sugimoto 1999; Antoniadis, Dudas, Sagnotti 1999]

Supersymmetry is exact in the closed-string sector:

mutually BPS configurations of Orientifold planes

$O9_+ \qquad O9_+/O5_- \qquad O9_-/O5_+ \qquad O9_+/O5_+$

RR tadpole cancellation implies

Supersymmetry is explicitly broken in the open-string sector:

non-BPS combinations of D-branes and Orientifold planes

$\overline{D9} \qquad \overline{D9}/D5 \qquad D9/\overline{D5} \qquad \overline{D9}/\overline{D5}$

BRANE BREAKING SUPERSYMMETRY IN THE VACUUM

closed strings

$O9_-/O5_+$

$\mathscr{N} = (1,0)$ SUGRA with

$n_T = 17\,, \quad n_H = 4$

this model does not admit a supersymmetric version with the same fermionic spectrum

open strings

$D9/\overline{D5}$

$$G_{CP} = SO(16)_9 \times SO(16)_9 \times USp(16)_{\bar{5}} \times USp(16)_{\bar{5}}$$

$$\psi_L \sim (120,1;1,1) + (1,120;1,1) + (1,1;120,1) + (1,1;1,120)$$

$$\psi_R \sim (16,16;1,1) + (1,1;16,16)$$

$$4\phi \sim (16,16;1,1) + (1,1;16,16)$$

$$\psi_L^{SMW} \sim (16,1;16,1) + (1,16;1,16)$$

$$2\phi \sim (16,1;1,16) + (1,16;16,1)$$

BRANE BREAKING SUPERSYMMETRY IN THE VACUUM

$$\begin{aligned} I_8 = {} & \tfrac{1}{2}\left(\mathrm{tr}\,R^2 + \tfrac{1}{2}\mathrm{tr}\,F_2^2 - \tfrac{1}{2}\mathrm{tr}\,G_1^2 - \tfrac{1}{2}\mathrm{tr}\,G_2^2\right)^2 - \tfrac{1}{2}\left(\mathrm{tr}\,R^2 - \tfrac{1}{2}\mathrm{tr}\,G_2^2\right)^2 \\ & - \tfrac{1}{2}\left(\tfrac{1}{2}\mathrm{tr}\,R^2 + \tfrac{1}{4}\mathrm{tr}\,F_2^2 - \tfrac{1}{2}\mathrm{tr}\,G_2^2\right)^2 - \tfrac{1}{2}\left(\tfrac{1}{2}\mathrm{tr}\,R^2 + \tfrac{1}{4}\mathrm{tr}\,F_2^2 - \tfrac{1}{2}\mathrm{tr}\,G_1^2\right)^2 \\ & - \left(\tfrac{1}{2}\mathrm{tr}\,R^2 - \tfrac{1}{4}\mathrm{tr}\,F_2^2\right)^2 - \tfrac{1}{2}\left(\mathrm{tr}\,R^2 - \tfrac{1}{4}\mathrm{tr}\,F_1^2 + \tfrac{1}{4}\mathrm{tr}\,F_2^2 - \tfrac{1}{2}\mathrm{tr}\,G_1^2\right)^2 - \tfrac{3}{2}\left(\tfrac{1}{4}\mathrm{tr}\,F_1^2 - \tfrac{1}{4}\mathrm{tr}\,F_2^2\right)^2 \end{aligned}$$

$$a \cdot a = -8\,, \qquad a \cdot b_i = \begin{pmatrix} 2 \\ 2 \\ 1 \\ 1 \end{pmatrix},$$

$$b_i \cdot b_j = \begin{pmatrix} -4 & 4 & -1 & 0 \\ 4 & -4 & 0 & -1 \\ -1 & 0 & -1 & 1 \\ 0 & -1 & 1 & -1 \end{pmatrix}$$

An integral basis

$$a = (2,-2,0,0,0,-1^3,1,0,2,0^7)\,,$$

$$b_1 = (0,1^4,0^{13})\,,$$

$$b_2 = (2,-1^4,1,-1,1^2,0^9)\,,$$

$$b_3 = (-1,1,0^4,1,0^{11})\,,$$

$$b_4 = (-1,0^7,-1,0,-1,0^7)$$

Existence of a Kähler form

$$J \cdot J > 0\,, \qquad J \cdot a < 0\,, \qquad J \cdot b_i > 0$$

$$|J_0| > |J_1|\,, \quad J_1 < 0\,, \quad J_0 - J_1 > 0\,, \quad J_0 + J_1 < 0\,,$$

NO SOLUTION

BRANE BREAKING SUPERSYMMETRY IN THE VACUUM

D1 branes at the orbifold fixed point with $G_{\mathrm{CP}} = \mathrm{SO}(r)$

representation	$\mathrm{SO}(1,1) \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{SO}(4)$
$\frac{1}{2}r(r-1)$	$(0,1,1,1) + (\frac{1}{2},1,2,2')_{\mathrm{L}}$
$\frac{1}{2}r(r+1)$	$(1,2,2,1) + (\frac{1}{2},2,1,2')_{\mathrm{R}}$
$(r;16,1;1,1)$	$(\frac{1}{2},1,1,1)_{\mathrm{L}}$
$(r;1,1;16,1)$	$(\frac{1}{2},1,1,2')_{\mathrm{L}}$
$(r;1,1;1,16)$	$(\frac{1}{2},1,1,2)_{\mathrm{R}}$

$$c_{\mathrm{L}} = 4_{\mathrm{CM}} + 8 + 16_{\overline{\mathrm{D5}}}, \qquad c_{\mathrm{R}} = 6_{\mathrm{CM}} + 0 + 16_{\overline{\mathrm{D5}}} \qquad k_i = Q \cdot b_i = (1,0,1,-1)$$

$$c_{\mathrm{L}} = 3Q \cdot Q - 9Q \cdot a + 2,$$
$$c_{\mathrm{R}} = 3Q \cdot Q - 3Q \cdot a.$$
$$Q \cdot Q = Q \cdot a = -1$$

BRANE BREAKING SUPERSYMMETRY IN THE VACUUM

What does it means that one cannot find J?

Re-think the anomaly polynomial

$$I_8 = \tfrac{1}{64}\left(\operatorname{tr} F_1^2 + \operatorname{tr} F_2^2 - \operatorname{tr} G_1^2 - \operatorname{tr} G_2^2\right)^2 - \tfrac{1}{64}\left(8 \operatorname{tr} R^2 - \operatorname{tr} F_1^2 - \operatorname{tr} F_2^2 - \operatorname{tr} G_1^2 - \operatorname{tr} G_2^2\right)^2$$
$$- \tfrac{1}{128}\left(\operatorname{tr} F_1^2 - \operatorname{tr} F_2^2 + 4 \operatorname{tr} G_1^2 - 4 \operatorname{tr} G_2^2\right)^2 - \tfrac{15}{128}\left(\operatorname{tr} F_1^2 - \operatorname{tr} F_2^2\right)^2$$

in the *string basis*

$$I_8 \sim \left((Q_{O9} + Q_{O5})\mathrm{tr}R^2 - Q_{D9}\mathrm{tr}F_9^2 - Q_{D5}\mathrm{tr}F_5^2\right)^2 - \left((Q_{O9} - Q_{O5})\mathrm{tr}R^2 - Q_{D9}\mathrm{tr}F_9^2 + Q_{D5}\mathrm{tr}F_5^2\right)^2$$
$$- \sum_{a\in\{\text{fixed points}\}} \left((Q^a_{O9} + Q^a_{o5})\,\mathrm{tr}R^2 - Q^a_{D9}\mathrm{tr}F_9^2 - Q^a_{D5}\mathrm{tr}F_5^2\right)^2$$

reflects the structure of RR tadpoles

BRANE BREAKING SUPERSYMMETRY IN THE VACUUM

The gauge kinetic terms should not be determined by the RR tadpoles

$$S \supset \int \sum_i J(\phi) \cdot b_i \, \mathrm{tr}(F_i^2)$$

On the contrary, they are determined by the NS-NS tadpoles

$$\big((T_{O9} + T_{O5})\mathrm{tr}R^2 - T_{D9}\mathrm{tr}F_9^2 - T_{D5}\mathrm{tr}F_5^2\big)^2 - \big((T_{O9} - T_{O5})\mathrm{tr}R^2 - T_{D9}\mathrm{tr}F_9^2 + T_{D5}\mathrm{tr}F_5^2\big)^2$$
$$- \sum_{a \in \{\text{fixed points}\}} \big((T^a_{O9} + T^a_{O5})\,\mathrm{tr}R^2 - T^a_{D9}\mathrm{tr}F_9^2 - T^a_{D5}\mathrm{tr}F_5^2\big)^2$$

$$\to \tfrac{1}{64}\left(\mathrm{tr}\,F_1^2 + \mathrm{tr}\,F_2^2 + \mathrm{tr}\,G_1^2 + \mathrm{tr}\,G_2^2\right)^2 - \tfrac{1}{64}\left(8\,\mathrm{tr}\,R^2 - \mathrm{tr}\,F_1^2 - \mathrm{tr}\,F_2^2 + \mathrm{tr}\,G_1^2 + \mathrm{tr}\,G_2^2\right)^2$$
$$- \tfrac{1}{128}\left(\mathrm{tr}\,F_1^2 - \mathrm{tr}\,F_2^2 - 4\,\mathrm{tr}\,G_1^2 + 4\,\mathrm{tr}\,G_2^2\right)^2 - \tfrac{15}{128}\left(\mathrm{tr}\,F_1^2 - \mathrm{tr}\,F_2^2\right)^2$$

extract

$\tilde{a}, \quad \tilde{b}_i$

BRANE BREAKING SUPERSYMMETRY IN THE VACUUM

Tadpole conditions in BSB: $\mathrm{SO}(N_1) \times \mathrm{SO}(N_2)\Big|_9 \times \mathrm{USp}(D_1) \times \mathrm{USp}(D_2)\Big|_{\bar{5}}$

$$V_4 O_4 \, (O_4 V_4) \; : \qquad (N_1 + N_2 - 32)\sqrt{v} \pm (D_1 + D_2 + 32)\frac{1}{\sqrt{v}} = 0$$

$$C_4 C_4 \, (S_4 S_4) \; : \qquad (N_1 + N_2 - 32)\sqrt{v} \mp (D_1 + D_2 - 32)\frac{1}{\sqrt{v}} = 0$$

$C^+_{\mu\nu}$ $\quad$ $C^-_{\mu\nu}$

untwisted tadpoles

$$O_4 C_4 \; : \qquad (N_1 - N_2) - 4(D_1 - D_2) = 0\Big|_{\mathrm{fp}=1} , \qquad (N_1 - N_2) = 0\Big|_{\mathrm{fp}=2,\ldots,16} = 0$$

twisted tadpoles

$$S_4 O_4 \; : \qquad (N_1 - N_2) + 4(D_1 - D_2) = 0\Big|_{\mathrm{fp}=1} , \qquad (N_1 - N_2) = 0\Big|_{\mathrm{fp}=2,\ldots,16} = 0$$

$C^-_{\mu\nu}$

BSB: NEW CONSISTENCY CONDITIONS(?)

$J \cdot J > 0$

tensor metric
positivity

$J \cdot \tilde{a} < 0$

Gauss-Bonnet
positivity

$J \cdot \tilde{b}_i > 0$

vector metric
positivity

$Q \cdot J \geq 0$

positivity
D1 tension

$Q \cdot Q \geq -1$

$Q \cdot Q + Q \cdot a \geq -2$

$k_i = Q \cdot b_i \geq 0$

positivity of central charges (c_R and k_l)

unitarity

$$\sum_i \frac{k_i \dim G_i}{k_i + h_i^\vee} \leq c_\mathrm{L}$$

OUTLOOK

Which are the consistency conditions when supersymmetry is broken?

What determines the sign of k_i?

How to generalise these consistency conditions to four dimensions?

THANK YOU