INFLATION IN M-THEORY

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work in progress,

K. Becker, M. Becker, A.K: hep-th/0501130, NPB

plus related work w/ M. Becker, G. Curio, D. Lüst

Overview

- ➢ Single Field Inflation in String-Theory?
- ➢ Power-Law and Assisted Inflation
- ➢ Realizing Assisted Inflation in M-Theory
- ➢ The Whole Evolution: Cascade Inflation
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- ➢ Conclusion

Single-Field Inflation in String-Theory?

To embed single-field slow-roll inflation into string- or M-theory, take your favorite model, use the available fluxes and non-perturbative effects and stabilize all moduli except for one, φ.

Since the aim is to derive a single extremely flat φ direction, the masses of the stabilized moduli should be heavier than the one for φ, such that we can integrate these moduli out and remain with an effective potential $U(\varphi)$.

To see whether the resulting $U(\varphi)$ is flat enough to give rise to a period of inflation which is long enough (50-60 e-foldings usually), you have to check that both

$$\epsilon = \frac{M_{Pl}^2}{2}\left(\frac{U'(\varphi)}{U(\varphi)}\right)^2 \ll 1$$

$$|\eta| = \left|M_{Pl}^2 \frac{U''(\varphi)}{U(\varphi)}\right| \ll 1$$

While the ϵ condition is typically easy to satisfy, the contrary is true for the η constraint and hence developed into the

Eta-Problem = Inflaton-Mass Problem

Various proposals for its solution were made (warp factors, shift symmetries, etc.) but when analyzed carefully the problem persists [see e.g. McAllister hep-th/0502001].

$\Rightarrow$ **Let's try to embed inflation into string- and M-theory through multi-field inflation (more natural in view of the multitude of scalars)?**

Power-Law and Assisted Inflation

To this end, let us emphasize that there are are various different types of inflation through which one might try to embed inflation into string-theory:

New Inflation: is based on a vacuum energy dominated de Sitter expansion for which

$$a(\mathrm{t}) = a_0 e^{H\mathrm{t}}$$

Many efforts concentrated on embedding inflation into string-theory via new inflation (or hybrid inflation)

Power-Law Inflation: [Lucchin, Matarrese 1985]
relies on an exponential potential

$$U(\varphi) = U_0 e^{-\sqrt{\frac{2}{p}}\frac{\varphi}{M_{Pl}}}$$

with parameter $p > 1$ leading to a scale-factor

$$a(\mathrm{t}) = a_0 \mathrm{t}^p$$

and an evolution of the inflaton

$$\varphi(\mathrm{t}) = \sqrt{2p} M_{Pl} \ln \left(\sqrt{\frac{U_0}{p(3p-1)}} \frac{\mathrm{t}}{M_{Pl}} \right)$$

(solution is valid for $p > 1/3$ but inflation arises only if $p > 1 \Leftrightarrow \ddot{a} > 0$)

power-law inflation implies very simple constant slow-roll parameters

$$\epsilon = \frac{1}{p} \qquad \eta = \frac{2}{p}$$

constant slow-roll parameters mean there is no exit from power-law inflation. When embedded into M/string theory this presents, however, no problem as additional contributions will eventually modify the simple exponential potential causing inflation to end.

exponential potentials arise naturally from various non-perturbative effects which need to be included anyway for moduli stabilization (and spontaneous supersymmetry breaking)

Single D-brane or Membrane instantons lead however to $p = \mathcal{O}(1)$ which is not sufficient for a sustained period of inflation!

So what can be done to obtain inflation nevertheless?

Assisted Inflation: [Liddle, Mazumdar, Schunck 1998]
consider instead a multi-inflaton extension of the single scalar power-law inflation scenario, termed assisted inflation

each of the N scalar fields φ_i, $i = 1, \ldots, N$, has a potential

$$U = U_0 e^{-\sqrt{\frac{2}{p}}\frac{\varphi_i}{M_{Pl}}} , \qquad \forall i = 1, \ldots, N .$$

(for $p = \mathcal{O}(1)$ as in M/string theory, individual potentials too steep to give power-law inflation) Since all scalars obey identical dynamics, one can map this multi-field problem by rescaling to the single field power-law problem and show that it gives again a power-law solution

$$a(\mathrm{t}) = a_0 \mathrm{t}^{p(N)}$$

but this time with

$$p(N) = Np$$

⇒ even though single exponential contributions are too steep to support inflation individually, it is easy to obtain large $p(N) \gg 1$ by increasing N

$\Rightarrow$ this mechanism gives inflation with small ϵ, η by using many exponential potentials which would usually be discarded as too steep when considered individually (increased Hubble-friction experienced by every individual scalar)!

Is it possible to embed inflation via the method of assisted inflation into M/string-theory, thereby using the naturally available "too steep" potentials?

Realizing Assisted Inflation in M-Theory

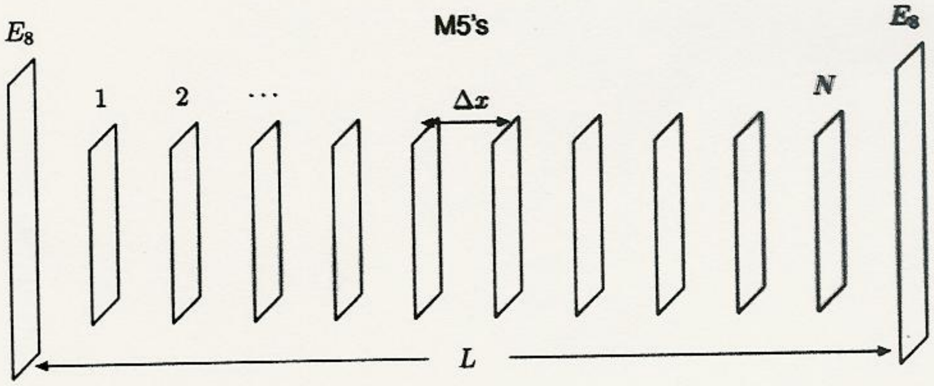
[K.& M. Becker, AK '05]

1) The Multi M5-Brane Potential

For definiteness and because of its direct contact with a realistic GUT or MSSM sector [see e.g. B. Ovrut et al. '04] – which is important for the issue of reheating – we will focus subsequently on heterotic M-theory.

When compactified down to 4d on a 6-dim. manifold preserving $N = 1$ supersymmetry, background is given either by a warped Calabi-Yau threefold or a warped non-Kähler manifold depending on type of flux. But warp-factor along S^1/Z_2 stays the same [Curio, AK 2000, 2003]

Let's focus on the CY case and consider the following setup of N parallel M5-branes distributed along the $\mathbf{S}^1/\mathbf{Z}_2$ interval (all M5-branes fill 4d spacetime and wrap same genus zero holomorphic 2-cycle on the CY; for simplicity $h^{1,1}=1$, i.e. one Kähler modulus T only.



Moduli and Kähler-Potential

effective 4d N=1 supergravity is described in terms of

- $h^{2,1}$ complex structure moduli Z^α
- CY volume modulus S
- T modulus measuring $\mathbf{S}^1/\mathbf{Z}_2$ length
- M5-brane position fields Y_i

defined as

$$
\begin{aligned}
S &= \mathcal{V} + \mathcal{V}_{OM} \sum_{i=1}^{N} \left(\frac{x_i^{11}}{L} \right)^2 + i\sigma_S \\
T &= \mathcal{V}_{OM} + i\sigma_T \\
Y_i &= \mathcal{V}_{OM} \left(\frac{x_i^{11}}{L} \right) + i\sigma_i \,, \qquad i = 1, \ldots, N
\end{aligned}
$$

where

- $\mathcal{V}$ = average CY volume over $\mathbf{S}^1/\mathbf{Z}_2$
- $\mathcal{V}_{OM}$ = average volume of 3-cycle $\Sigma_2 \times \mathbf{S}^1/\mathbf{Z}_2$
- L = length of $\mathbf{S}^1/\mathbf{Z}_2$ interval
- $0 \leq x_i^{11} \leq L$: position of the ith M5-brane

in addition it's useful to define

$$
s = S + \overline{S}, \qquad t = T + \overline{T}, \qquad y_i = Y_i + \overline{Y}_i,
$$

$$
y = \left(\sum_{i=1}^{N} y_i^2 \right)^{1/2}
$$

plus

$$
\begin{aligned}
Q &= s - \frac{y^2}{t} \\
R &= 3Q^2 - 2\frac{y^4}{t^2}
\end{aligned}
$$

such that the Kähler-potential becomes

$$K = K_{(S)} + K_{(T)} + K_{(Y)} + K_{(Z)}$$

where

$$\begin{aligned} K_{(S)} + K_{(Y)} &= -\ln Q \\ K_{(T)} &= -\ln\left(\frac{d}{6}t^3\right) \\ K_{(Z)} &= -\ln\left(i\int_{CY}\Omega\wedge\overline{\Omega}\right), \end{aligned}$$

(d = CY intersection number)

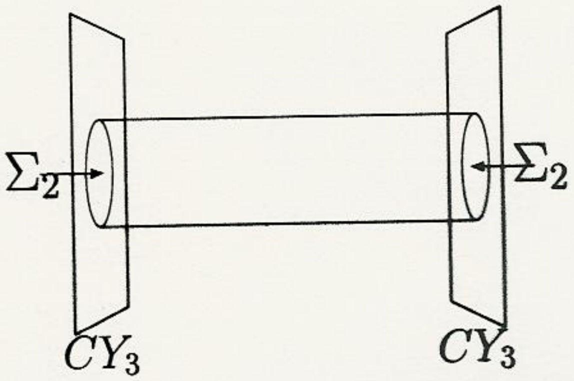
Obviously $Q = 2\mathcal{V} > 0$; likewise $R > 0$ to ensure a positive definite Kähler metric $K_{I\bar{J}}$ for which one finds ($I, J, \ldots$ run over all complex moduli, $G_{\alpha\bar{\beta}}$ = metric on the complex structure moduli space)

$$\det K_{I\bar{J}} = \frac{16R}{Q^{2N}t^6}\det G_{\alpha\bar{\beta}}$$

Which Superpotentials Need to be Considered?

a priori contributions to superpotential come from open membrane instantons (wrapping same genus zero curve as M5's) stretching between:

- both boundaries (99),
- between two of the M5-branes (55),
- between the visible boundary and an M5-brane (95)
- or between an M5-brane and the hidden boundary (59)



$$W_{OM} = W_{99} + W_{55} + W_{95} + W_{59}$$

where

$$W_{99} = he^{-T}, \quad W_{95} = h\sum_{i=1}^{N} e^{-Y_i}, \quad W_{59} = h\sum_{i=1}^{N} e^{-(T-Y_i)}$$

$$W_{55} = h\sum_{i<j} e^{-Y_{ji}}$$

with

$$Y_{ji} = Y_j - Y_i$$

describing the distance between the jth and the ith **M5**-brane

When T would have been stabilized at the critical length [Curio, AK 2001; M. Becker, Curio, AK 2004] where volume (gauge coupling) of hidden boundary becomes small (large), then one would also consider gaugino condensation on hidden boundary

$$W_{GC} = -C_H \mu^3 e^{-\frac{1}{C_H} f_h}, \qquad f_h = S + \gamma_h T + \frac{\sum_i \gamma_i Y_i^2}{T},$$

(C_H = dual Coxeter number of hidden gauge group, μ = ultraviolet cut-off scale, $\gamma_{h,i} = \beta_{h,i} \frac{\pi L}{V_v} \left(\frac{\kappa}{4\pi}\right)^{2/3} \int_{\Sigma_2} \omega$, where ω = CY Kähler form and V_v CY volume of visible boundary; $\beta_i = 1$ and $\beta_h \in \mathbf{Z}$ obtained as expansion coefficients of 2nd Chern classes of hidden boundary vector F_h and tangent bundle TX: $c_2(F_h) - \frac{1}{2} c_2(TX) = \beta_h [\Sigma_2]$)

Via a perfect square structure within the heterotic M-theory action, gaugino condensation implies a non-vanishing NS 3-form flux H of type $(3,0) + (0,3)$ on the hidden boundary [Horava 1996]. Hence, flux superpotential [Gukov, Vafa, Witten 1999; Gukov 1999]

$$W_H = \int_{CY_h} H \wedge \Omega$$

is induced on hidden boundary

Here, however we will start at subcritical distances! ($\alpha_h < 1/2$ if $L < 0.72 L_c$) Only towards the end of inflation T will grow towards T_c and at this time gaugino condensation and the induced H flux need to be included and will stabilize S and T moduli.

Among the open membrane contributions, we can focus on the dominant nearest neighbor 55 contributions. This is correct as long as nearest M5 brane distances are smaller than the orbifold size implying that the neglected OM instantons have to stretch over longer distances. Hence

$$W = W_{55}$$

M5-brane Interaction Potential

Potential follows from standard F-term expression

$$U = M_{Pl}^4 e^K \left(\sum K^{\bar{I}J} D_{\bar{I}} \overline{W}_{55} D_J W_{55} - 3|W_{55}|^2 \right) ,$$

which leads to

$$\begin{aligned} \frac{U}{M_{Pl}^4 e^K} &= G^{\bar{\alpha}\beta} D_{\bar{\alpha}} W_{55} D_\beta W_{55} \\ &+ Qt \sum_{i,j=1}^{N} \left(\frac{1}{2}\delta_{ij} + \frac{Q}{Rt} y_i y_j \right) \overline{D_i W_{55}} D_j W_{55} \\ &+ \left(\frac{3Q^2}{R} - \frac{2y^2}{Qt} \right) |W_{55}|^2 \end{aligned}$$

with Kähler factor $e^K = 6/(i \int \Omega \wedge \overline{\Omega}) Q t^3 d$

the only term which in principle could become negative is the last term which arises from

$$\sum K^{\bar{I}J} K_{\bar{I}} K_J |W_{55}|^2 - 3|W_{55}|^2$$

It is however easy to check that $3Q^2/R > 2y^2/Qt$ when $Q > 0$ and $R > 0$, as required for a positive definite Kähler metric. Hence this term and therefore the whole potential will be positive – an important requisite for the derivation of assisted inflation!

2) Mapping M5-Brane Dynamics to Assisted Inflation Dynamics

Since M5-brane interaction potential is positive we can partially minimize it by demanding

$$D_\alpha W_{55} = 0$$
$$D_i W_{55} = 0$$

Let us see what they imply. The first equation is equivalent to

$$\frac{\partial \ln h}{\partial Z^\alpha} = -\frac{\partial K_{(Z)}}{\partial Z^\alpha}$$

and implies

$$h = i \int_{CY} \Omega \wedge \overline{\Omega}$$

It will fix the $h^{2,1}$ complex structure moduli

The second equation has a simple geometric meaning: in the large volume regime, where we can trust the supergravity analysis, we have $Qt \simeq st \gg t > y_i$ and therefore

$$0 = D_i W_{55} = W_{55,i} + \frac{2y_i}{Qt} W_{55} \to W_{55,i}$$

With

$$W_{55} = h \sum_{i=1}^{N-1} e^{-Y_{i+1,i}}$$

this implies

$$Y_{i+1,i} \equiv \Delta Y \quad \forall i$$

Hence partially minimizing the energy through setting $D_i W_{55} = 0$ forces the inter M5-brane distances to be equidistant

Specifying the Regime where Inflation Occurs

So far, by partially minimizing the energy, we have arrived at

$$U \propto \left(\frac{3Q}{Rt^3} - \frac{2y^2}{Q^2t^4}\right)|W_{55}|^2$$

If this potential is to be mapped to an assisted inflation dynamics with the inflatons arising from the M5-brane position differences $y_{i+1} - y_i$, we have to make sure that there is a y_i dependence only in the exponentials of W_{55}

can be achieved by working in the regime where

$$Qt \gg y^2$$

which is consistent with large volume $Q \simeq s \gg 1$ and implies $3Q/Rt^3 \gg 2y^2/Q^2t^4$. In this regime we simply have

$$U \propto \frac{1}{st^3}|W_{55}|^2$$

and therefore finally

$$\frac{Ud}{6M_{Pl}^4(i\int \Omega \wedge \overline{\Omega})} = \frac{(N-1)^2}{st^3}e^{-\Delta y}\,, \quad \Delta y = \Delta Y + \overline{\Delta Y}$$

The Mapping

1) Transformation to canonically normalized scalars:

Y_i kinetic term

$$S_{kin} = -M_{Pl}^2 \int d^4x\sqrt{-g}K_{i\bar{j}}\partial_\mu Y_i \partial^\mu \overline{Y}_{\bar{j}}\,,$$

where

$$K_{i\bar{j}} = \frac{4y_i y_j + 2Qt\delta_{ij}}{Q^2t^2}\,.$$

In the regime which we had just specified, we have $Qt \gg y^2 = \sum y_i^2 > y_i y_j$. Thus (under the sum) we can neglect the first piece and obtain $K_{i\bar{j}} = 2\delta_{ij}/Qt$ which leads to the following canonically normalized real M5-brane position and difference fields

$$\phi_i = \frac{2M_{Pl}}{\sqrt{Qt}} y_i, \qquad \Delta\phi = \frac{2M_{Pl}}{\sqrt{Qt}} \Delta y$$

2) Switching to COM and Relative Coordinates:

A potential is only generated for distances between adjacent M5-branes but not for their combined com position. Let us therefore switch from the N position fields ϕ_i to the more adequate description in terms of the M5-brane com field

$$\phi_{com} = \frac{1}{N}(\phi_1 + \ldots + \phi_N) ,$$

and the difference field $\Delta\phi$. The relation between the two sets of fields is provided by the relation

$$\phi_i = \phi_{com} + \left(i - \frac{N+1}{2}\right)\Delta\phi$$

Since there is no potential for ϕ_{com}, its value will stay constant and its kinetic term vanishes. The sum of the

ϕ_i kinetic terms then becomes

$$\frac{1}{2}\sum_{i=1}^{N}\partial_\mu\phi_i\partial^\mu\phi_i = \partial_\mu\Delta\phi\partial^\mu\Delta\phi\sum_{i=1}^{N}\left(i-\frac{N+1}{2}\right)^2$$
$$= \frac{N(N^2-1)}{12}\partial_\mu\Delta\phi\partial^\mu\Delta\phi$$

requiring us to perform a second renormalization

Eventually, we arrive at the final canonically normalized difference field φ

$$\varphi = \sqrt{\frac{N(N^2-1)}{6}}\Delta\phi = M_{Pl}\sqrt{\frac{2N(N^2-1)}{3Qt}}\Delta y$$

in terms of which the potential reads

$$U(\varphi) = \tilde{U}_0(N-1)^2 e^{-\sqrt{\frac{3Qt}{2N(N^2-1)}}\frac{\varphi}{M_{Pl}}}$$

with $\tilde{U}_0 = 6M_{Pl}^4(i\int\Omega\wedge\overline{\Omega})/st^3d$ (the approximate constancy of s, t will be discussed shortly)

For a spatially flat 4d FRW universe we then have a Hubble parameter

$$H^2 = \frac{1}{3M_{Pl}^2}\left(U(\varphi)+\frac{1}{2}\dot{\varphi}^2\right),$$

and the dynamics of φ is determined by

$$\ddot{\varphi}+3H\dot{\varphi}+\frac{dU}{d\varphi} = 0\,.$$

This is precisely the dynamics which gives power-law inflation once we identify the parameters

$$p = \frac{4N(N^2-1)}{3Qt}$$
$$U_0 = \tilde{U}_0(N-1)^2$$

This completes the mapping and therefore embedding of assisted inflation into M-theory.

Bounds on N

Contrary to what one might think, the value for N is rather constrained

1) We had the condition

$$Qt \gg y^2$$

which implies an upper bound on N as y grows with N. For typical values $\mathcal{V} = 341, \mathcal{V}_{OM} = 7$ and $x_i^{11}/L = \mathcal{O}(1/2)$ we have $s = 682 + 3.5N, t = 14, y^2 \simeq 49N$ which leads to

$$N \ll 195$$

2) to obtain inflation we need $p > 1$ which implies a lower bound on N

$$p > 1 \qquad \Leftrightarrow \qquad 4N(N^2-1) > 3Qt$$

For the same s, t values as before we get

$$19 < N$$

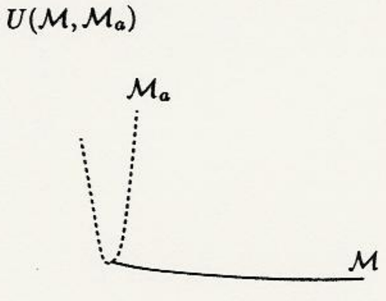
There is thus a non-empty set of N's satisfying both constraints. Moreover also observational constraints coming from the number of e-foldings or the scalar spectral index require values for N within same range!

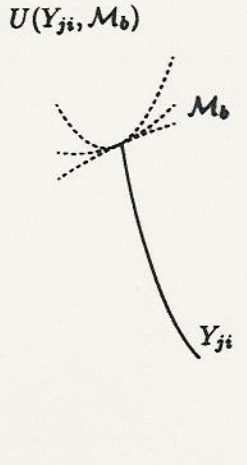
Moduli Stabilization

There is a crucial difference between new inflaton models and this assisted inflation model for the embedding into M/string-theory

New inflaton models have to select a single very flat direction. Necessarily one has to stabilize all other moduli before inflation.

Here, we have made use of the very steepest directions available. The universe is rolling down exponentially steep directions during inflation





Hence, this alleviates considerably task to stabilize all (up to one) moduli before inflation. Mild, i.e. power-law runaways might be tolerable (s, t)

The Whole Evolution: Cascade Inflation

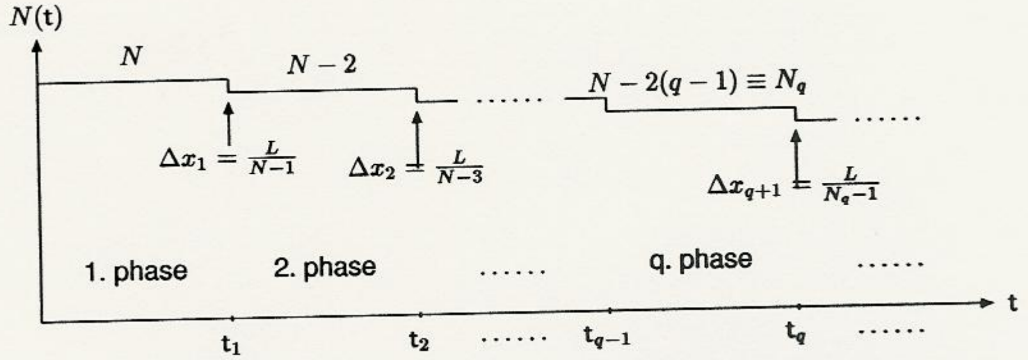
So far we have kept N = # of M5-branes fixed and analyzed therefore only part of the full process. Since the M5-brane distances are growing N will jump to $N-2$ as soon as the two outermost M5-branes hit the boundaries

This can also be seen from the anomaly cancellation equation for the G flux

$$\beta_v + \beta_h + N = 0$$

($\beta_{v,h}$ are integers characterizing the boundary's 2nd Chern classes). When the M5-branes coalesce with the boundaries they change the boundaries topological data via small instanton transitions which has to be compensated for by a change in N

Hence, we have the following "cascade-like" evolution:



For each interval we obtain power-law inflation via the same derivation as before, as long as

$$p(N_q) > 1 \qquad \text{(Exit Condition : } p(N_q) = 1)$$

(2nd constraint $Qt \gg y^2$ always remains satisfied as y decreases with decreasing N)

The full "cascade inflation" process has the following structure

$$\begin{aligned} a_1(\mathrm{t}) &= a_1 \mathrm{t}^{p_1}\,, \quad \mathrm{t}_0 \le \mathrm{t} \le \mathrm{t}_1 \\ a_2(\mathrm{t}) &= a_2 \mathrm{t}^{p_2}\,, \quad \mathrm{t}_1 \le \mathrm{t} \le \mathrm{t}_2 \\ &\vdots \\ a_q(\mathrm{t}) &= a_q \mathrm{t}^{p_q}\,, \quad \mathrm{t}_{q-1} \le \mathrm{t} \le \mathrm{t}_q \\ &\vdots \end{aligned}$$

Matching them at the transition times t_q determines the constant prefactors

$$a_q = a_1 \mathrm{t}_1^{p_1} \left(\frac{\mathrm{t}_2}{\mathrm{t}_1}\right)^{p_2} \left(\frac{\mathrm{t}_3}{\mathrm{t}_2}\right)^{p_3} \cdots \left(\frac{\mathrm{t}_{q-1}}{\mathrm{t}_{q-2}}\right)^{p_{q-1}} \frac{1}{\mathrm{t}_{q-1}^{p_q}}$$

Number of E-Foldings

$$\mathsf{N}_e \equiv \ln\left(\frac{a(\mathsf{t_f})}{a(\mathsf{t_0})}\right) = \sum_{q=1}^{q_f} p_q \ln\left(\frac{\mathsf{t}_q}{\mathsf{t}_{q-1}}\right)$$

with

$$p_q = \frac{4N_q(N_q^2-1)}{3st}$$

the constant ratios $\mathsf{t}_q/\mathsf{t}_{q-1}$ can be determined by using the exact solution for the inflaton (distances between adjacent M5-branes) evolution

$$\frac{\mathsf{t}_q}{M_{Pl}} \simeq \frac{\mathsf{t}_q - \mathsf{t}_0}{M_{Pl}} = \frac{1}{\sqrt{\tilde{U}_0}} \sum_{a=1}^{q} \frac{p_a(3p_a-1)}{N_a-1} e^{t\left(\frac{\Delta x_a}{L} - \frac{\Delta x_{a-1}}{L}\right)}$$
$$= \frac{1}{\sqrt{\tilde{U}_0}} \sum_{a=1}^{q} \frac{p_a(3p_a-1)}{N_a-1} e^{t\left(\frac{1}{N_a-1} - \frac{1}{N_{a-1}-1}\right)}$$

Taking typical values for s, t as before, this series can be summed numerically.

Exit from Inflation: $p(N_{q_f}) \stackrel{!}{=} 1 \quad \Rightarrow \quad N_{q_f} = 19$

Knowing N_{q_f}, we can carry out the summation and obtain

$$N = 40 \quad \Rightarrow \mathsf{N}_e = 13.3$$
$$N = 50 \quad \Rightarrow \mathsf{N}_e = 28.6$$
$$N = 60 \quad \Rightarrow \mathsf{N}_e = 53.2$$
$$N = 70 \quad \Rightarrow \mathsf{N}_e = 89.7$$

$\Rightarrow$ Since $19 \leq N \ll 195$, we see that a realistic $\mathsf{N}_e = 50 - 60$ can be obtained within required regime!

Conclusion

- Cascade Inflation offers a direct way to obtain a small η (for power-law inflation $\epsilon = \frac{1}{p_q}$, $\eta = \frac{2}{p_q}$)
- able to account for a realistic number of e-foldings in the regime where derivation is valid and inflation takes place
- study of small instanton phase transitions important for reheating (cosmic strings?)