A

HETEROTIC STANDARD MODEL

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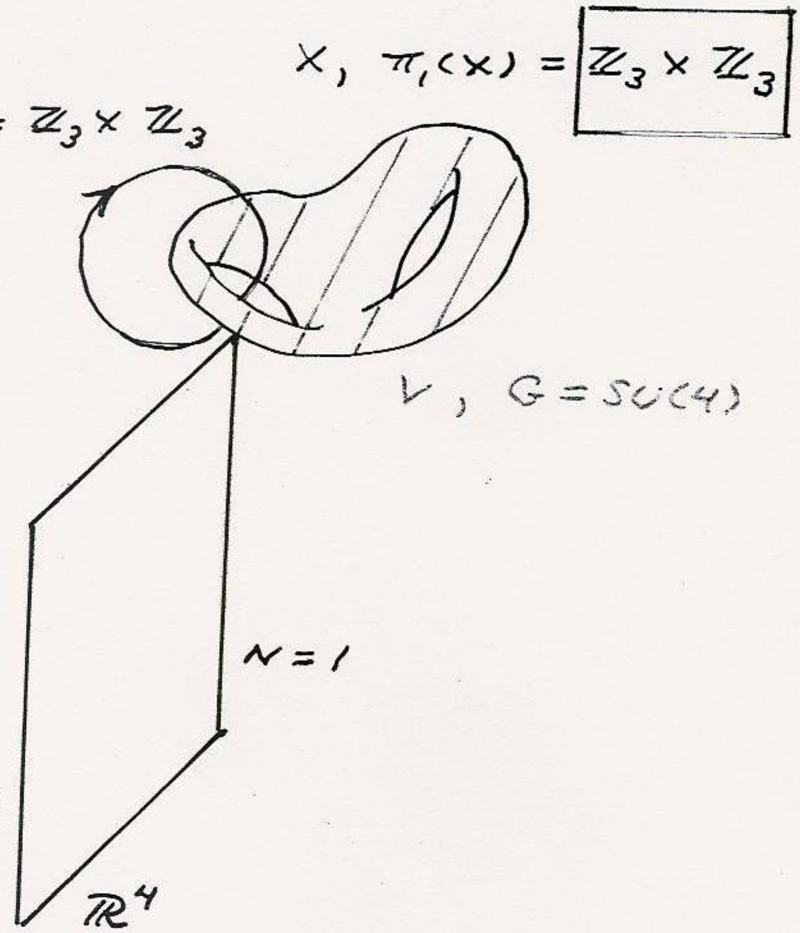
Observable Sector:

W, $hol(W) = \mathbb{Z}_3 \times \mathbb{Z}_3$



$$
\begin{array}{c}
E_8 \\
\Big\downarrow{\scriptstyle SU(4)} \\
Spin(10) \\
\Big\downarrow{\scriptstyle \mathbb{Z}_3 \times \mathbb{Z}_3} \\
SU(3)_C \times SU(2)_L \times U(1)_Y \times \boxed{U(1)_{B-L}}
\end{array}
$$

Spectrum:

2) $\underline{\underline{3}}$ families of quark/leptons. Each family is

$$(3,2,1,1),\ (\bar{3},1,-4,-1),\ (\bar{3},1,2,-1)$$

$$(1,2,-3,-3),\ (1,1,6,-3)$$

$$+$$

$$\boxed{(1,1,0,3)} \leftarrow \text{RH neutrino}$$

b) <u>2</u> pairs of Higgs / $\overline{\text{Higgs}}$ fields. Each pair is

$$(1, 2, 3, 0) \;,\; (1, \bar{2}, -3, 0)$$

c) <u>6</u> geometric moduli and a <u>small number</u> of vector bundle moduli.

d) <u>NO</u>

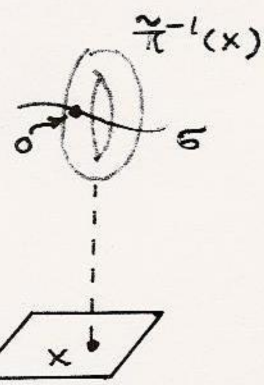
<u>EXOTIC MATTER</u> !

How does one construct this observable sector?

<u>X :</u>

Consider a simply connected Calabi-Yau 3-fold $\tilde{X}$ elliptically fibered as

$$\begin{array}{c} \tilde{X} \\ \downarrow \tilde{\pi} \\ d\tilde{\mathbb{P}}_9 \end{array} \qquad \simeq$$



One can show there is a free action

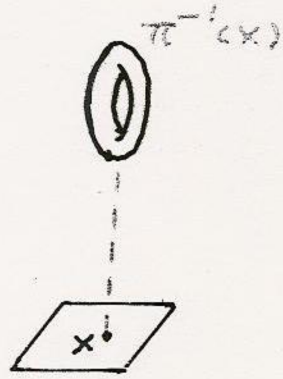
$$\mathbb{Z}_3 \times \mathbb{Z}_3 : \tilde{X} \longrightarrow \tilde{X}$$

Then

$$X = \tilde{X} / \mathbb{Z}_3 \times \mathbb{Z}_3$$

is a smooth CY 3-fold which is torus fibered

$$\begin{array}{c} X \\ \big\downarrow \pi \\ dP_9 \end{array} \qquad \simeq$$



and has non-trivial fundamental group

$$\pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3$$

One can show

$$h^{1,1}(X) = 3 \ , \ h^{2,1}(X) = 3$$

$\Rightarrow$ $\underline{\underline{3}}$ Kahler and $\underline{\underline{3}}$ complex structure moduli.

$\underline{\underline{V}}$:

Consider a stable, holomorphic vector bundle $\tilde{V}$ over $\tilde{X}$ with structure group

$$G = SU(4)$$

$\tilde{V}$ is constructed by "extension"

$$0 \longrightarrow V_2 \longrightarrow \tilde{V} \longrightarrow V_2' \longrightarrow 0$$

where V_2, V_2' have rank 2. The space of extensions, $Ext^1(V_2', V_2)$, carries a representation of $\mathbb{Z}_3 \times \mathbb{Z}_3$. $\tilde{V}$ is equivariant if

$$\tilde{V} \in Ext^1(V_2', V_2)^{\mathbb{Z}_3 \times \mathbb{Z}_3}$$

Choose $\tilde{V}$ to be equivariant under $\mathbb{Z}_3 \times \mathbb{Z}_3$. Then

$$V = \tilde{V}/_{\mathbb{Z}_3 \times \mathbb{Z}_3}$$

is a stable, holomorphic vector bundle on X

with structure group $G = SU(4)$.

<u>Spin (10) Spectrum:</u>

With respect to $SU(4) \times Spin(10)$

$$248 = (1,45) \oplus (15,1) \oplus (4,16) \oplus (\bar{4},\overline{16}) \oplus (6,10)$$

The zero mode spectrum is

$$\text{ker}\, \not{D}_{\tilde{V}} = \boxed{H^0(\tilde{X}, \mathcal{O}_{\tilde{X}}) \otimes 45} \oplus \boxed{H^1(\tilde{X}, ad\,\tilde{V}) \otimes 1} \oplus$$

$$H^1(\tilde{X}, \tilde{V}) \otimes 16 \oplus H^1(\tilde{X}, \tilde{V}^*) \otimes \overline{16} \oplus H^1(\tilde{X}, \wedge^2 \tilde{V}) \otimes 10$$

The number of 45 multiplets is

$$n_{45} = h^0(\tilde{X}, \mathcal{O}_{\tilde{X}}) = 1$$

For any other representation R

$$n_R = h^1(\tilde{X}, U_R(\tilde{V}))$$

where $U_R(\tilde{V})$ is the corresponding bundle. For example

$$n_{10} = h^1(\tilde{X}, \wedge^2 \tilde{V})$$

<u>Physical Constraints</u>:

Realistic particle physics $\Rightarrow$

1. $\underline{\underline{3}}$ generations

$$h^1(X, V) - h^1(X, V^*) = 3$$

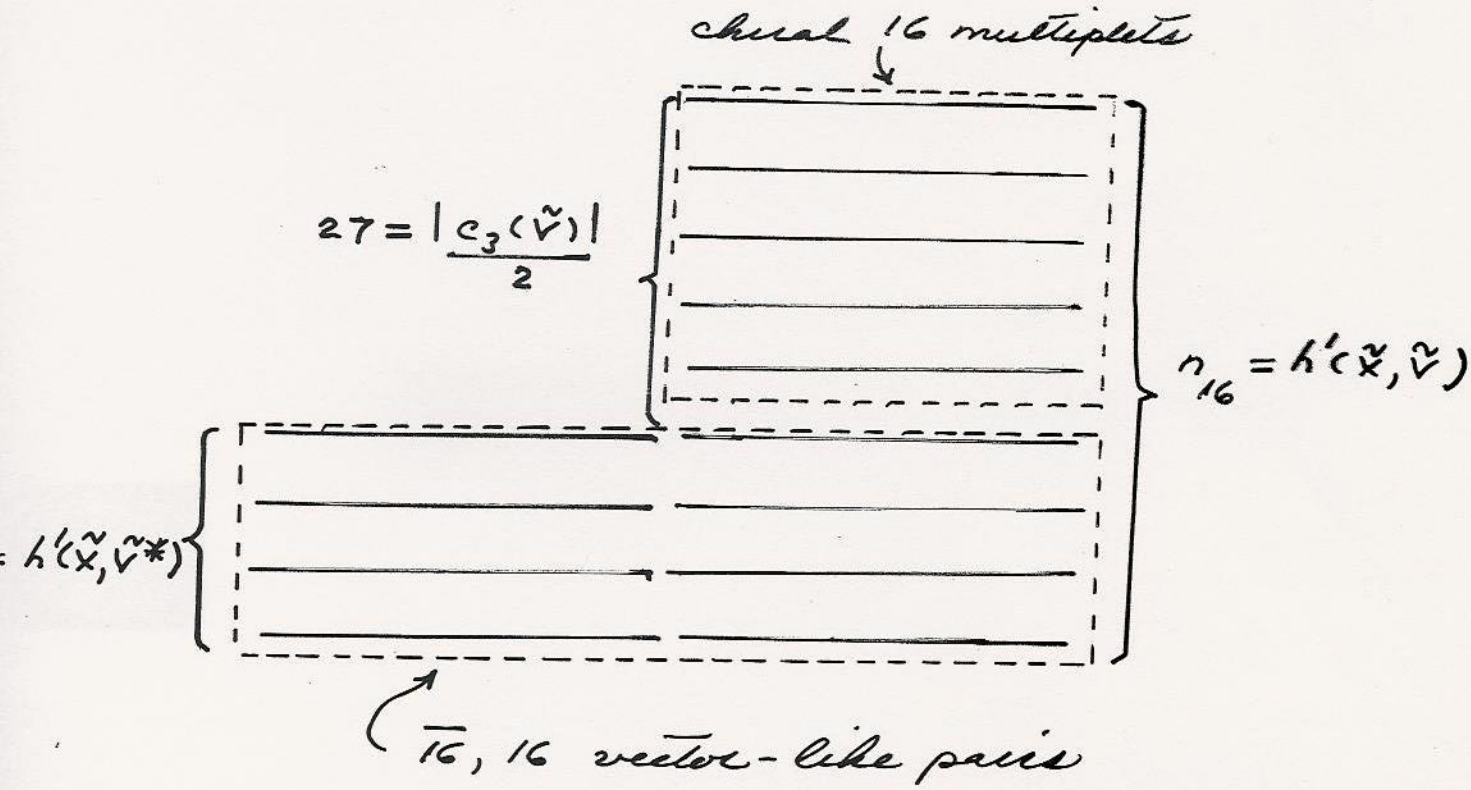
The Atiyah – Singer index theorem and Serre duality $\Rightarrow$

$$h^1(X,V) - h^1(X,V^*) = -\frac{1}{2}\int_X c_3(V)$$

Comparing $\Rightarrow$ must restrict $c_3(V) = -6$ $\Rightarrow$ choose

$$c_3(\tilde{V}) = -6 \times |\mathbb{Z}_3 \times \mathbb{Z}_3| = -54$$

Pictorially



2. <u>No</u> exotic matter

$\Rightarrow$ choose

$$n_{\overline{16}} = h^1(\tilde{X}, \tilde{V}^*) = 0$$

3. <u>Small</u> number of Higgs

The Higgs arise from the decomposition of 10.

$\Rightarrow$ choose

$$n_{10} = h^1(\tilde{X}, \wedge^2 \tilde{V}) \text{ minimal } (\neq 0)$$

<u>Result</u>:

Can construct bundles $\tilde{V}$ satisfying properties

1. and 2. with

$$n_{10} = h^1(\tilde{X}, \wedge^2 \tilde{V}) = 14$$

$SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ Spectrum:

The low energy spectrum is given by

$$\ker \not{\partial}_V = \left(H^0(\tilde{X}, \mathcal{O}_{\tilde{X}}) \otimes 45\right)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \oplus \left(H^1(\tilde{X}, \mathrm{ad}\,\tilde{V}) \otimes 1\right)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \oplus$$

$$\left(H^1(\tilde{X}, \tilde{V}) \otimes 16\right)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \oplus \left(H^1(\tilde{X}, \tilde{V}^*) \otimes \overline{16}\right)^{\mathbb{Z}_3 \times \mathbb{Z}_3} \oplus \left(H^1(\tilde{X}, \wedge^2 \tilde{V}) \otimes 10\right)^{\mathbb{Z}_3 \times \mathbb{Z}_3}$$

$\Rightarrow$ must find the representation of $\mathbb{Z}_3 \times \mathbb{Z}_3$ on both $H^1(\tilde{X}, U_R(\tilde{V}))$ and R, take the $\otimes$ product and choose the invariant subspace.

To proceed, note that two 1-dim representations of $\mathbb{Z}_3 \times \mathbb{Z}_3$ are

$$\chi_1(g_1) = \omega, \ \chi_1(g_2) = 1 \ ; \ \chi_2(g_1) = 1, \ \chi_2(g_2) = \omega$$

where g_1, g_2 are the generators of the two $\mathbb{Z}_3$ factors and $\omega = e^{\frac{2\pi i}{3}}$.

16 representation:

Using Leray spectral sequences we find that the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on $H^1(\tilde{X}, \tilde{V})$ is

$$H^1(\tilde{X},\tilde{V}) = Reg(\mathbb{Z}_3 \times \mathbb{Z}_3)^{\oplus 3}$$

where

$$Reg(\mathbb{Z}_3 \times \mathbb{Z}_3) = 1 \oplus \chi_1 \oplus \chi_2 \oplus \chi_1^2 \oplus \chi_1\chi_2 \oplus \chi_2^2 \oplus \chi_1^2\chi_2 \oplus \chi_1\chi_2^2 \oplus \chi_1^2\chi_2^2$$

Note this is consistent with $h^1(\tilde{X},\tilde{V}) = 27$. The Wilson line acts on 16 as

$$16 = \big(\chi_1^2\chi_2(3,2) \oplus \overset{\overset{10}{\downarrow}}{\chi_1^2\chi_2^2}(\bar{3},1) \oplus \chi_1^2(1,1)\big) \oplus$$

$$\big(\underset{\underset{5}{\uparrow}}{\chi_2^2(\bar{3},1)} \oplus (1,2)\big) \oplus \underset{\underset{1}{\uparrow}}{(1,1)}$$

Tensoring these actions $\Rightarrow$ the invariant subspace $(H^1(\tilde{X},\tilde{V}) \otimes 16)^{\mathbb{Z}_3 \times \mathbb{Z}_3}$ is spanned by $\underline{\underline{3}}$ copies of

$$q \to \quad (3,2,1,1),\ (\bar{3},1,-4,-1),\ (\bar{3},1,2,-1)$$

$$\ell \to \quad (1,2,-3,-3),\ (1,1,6,3)$$

$$\boxed{(1,1,0,3)} \leftarrow \text{rhn}$$

as desired.

<u>10 representation</u>

Using Leray spectral sequences we find that the $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on $H^1(\tilde{X}, \wedge^2 \tilde{V})$ is

$$H^1(\tilde{X}, \wedge^2 \tilde{V}) = 2 \oplus 2\chi_1 \oplus 2\chi_2 \oplus 2\chi_1^2 \oplus 2\chi_2^2 \oplus 2\chi_1\chi_2^2 \oplus 2\chi_1^2\chi_2$$

Note that this is consistent with $h^1(\tilde{X}, \wedge^2 \tilde{V}) = 14$.

The Wilson line acts on 10 as

$$10 = (\chi_1^2 (1,2) \overset{\overset{5}{\downarrow}}{\oplus} \chi_1^2\chi_2^2 (3,1)) \oplus (\chi_1 (1,\bar{2}) \overset{\overset{\bar{5}}{\downarrow}}{\oplus} \chi_1\chi_2 (\bar{3},1))$$

Tensoring these actions $\Rightarrow$ the invariant subspace

$(H^1(\tilde{X}, \Lambda^2\tilde{V}) \otimes 10)^{\mathbb{Z}_3\times\mathbb{Z}_3}$ is spanned by $\underline{\underline{2}}$ copies of

$$(1,2,3,0)\ ,\ (1,\bar{2},-3,0)$$

$\Rightarrow$ $\underline{\underline{2}}$ copies of $H, \bar{H}$ pairs. Note that the color triplets have been <u>projected out</u>!

This is an explicit mechanism for

<u>Doublet - Triplet</u>

<u>Splitting</u>

!

Finally,

$$h^1(\tilde{X}, ad\,\tilde{V}) \sim \mathcal{O}(10^2)$$

and $\mathbb{Z}_3\times\mathbb{Z}_3$ acts trivially on 1. Therefore

$$\dim(H^1(\tilde{X}, ad\,\tilde{V}) \otimes 1)^{\mathbb{Z}_3\times\mathbb{Z}_3} \sim \mathcal{O}\left(\frac{10^2}{9}\right) \sim 10$$

$\Rightarrow$ small number of vector bundle moduli.

Hidden Sector:

The condition for anomaly freedom is that

$$[W_5] = c_2(T\tilde{X}) - c_2(\tilde{V}) - c_2(\tilde{V}')$$

be an effective class. This $\Rightarrow$ a strong constraint on $\tilde{V}'$. The simplest possibility is that $\tilde{V}'$ is trivial. However, in this case

$$[W_5] \text{ not effective}$$

$\Rightarrow$ must choose $\tilde{V}'$ non-trivial. However, we always take

$$hol(V') = \mathbb{1}$$

Strong Coupling:

Choose $\tilde{V}'$ with structure group

$$G' = SU(2)$$

This breaks

$$E_8' \longrightarrow E_7$$

and under $SU(2) \times E_7$

$$248' = \boxed{(1, 133)} \oplus \boxed{(3, 1)} \oplus (2, 56)$$

$\Rightarrow$ for <u>no</u> exotic matter choose

$$n_{56} = h'(\tilde{X}, \tilde{V}') = 0$$

Such $\tilde{V}'$ exist but all have the property that

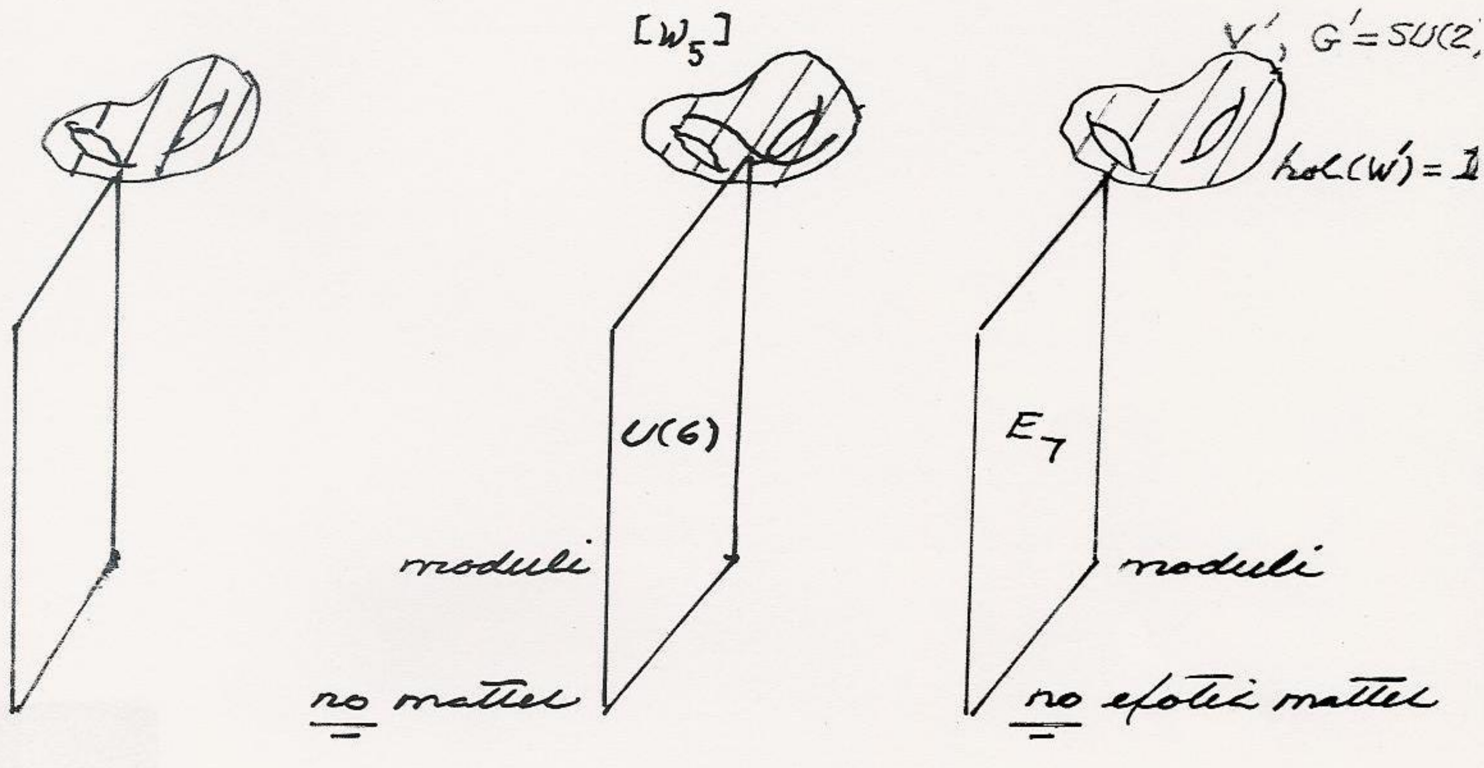
$$[W_5] \neq 0$$

$\Rightarrow$ strong coupling solution with 5-branes.

The 5-brane gauge group is

$$G_5' = U(6) \longrightarrow U(1)^6$$

Pictorially



<u>Weak Coupling</u>:

Want to find vacua with

$$[W_5] = 0$$

Choose $\tilde{V}' = \tilde{V}_1' \oplus \tilde{V}_2'$ with structure group

$$G' = SU(2) \times SU(2)$$

this breaks

$$E_8' \longrightarrow Spin(12)$$

and under $SU(2) \times SU(2) \times Spin(12)$

$$248' = \boxed{(3,1,1)} \oplus \boxed{(1,3,1)} \oplus \boxed{(1,1,66)} \oplus (1,2,32) \oplus (2,1,32) \oplus (2,2,12)$$

$\Rightarrow$ for <u>no</u> exotic matter try to choose

$$n_{32} = h'(\tilde{X}, \tilde{V}_1') = 0 \quad , \quad n'_{32} = h'(\tilde{X}, \tilde{V}_2') = 0$$

and

$$n_{12} = h'(\tilde{X}, \tilde{V}_1' \otimes \tilde{V}_2') = 0$$

However, we can only satisfy $h'(\tilde{X}, \tilde{V}_1') = 0$.

Then, minimally

$$h'(\tilde{X}, \tilde{V}_2') = 4 \quad , \quad h'(\tilde{X}, \tilde{V}_1' \otimes \tilde{V}_2') = 18$$

Be that as it may, we find

$$\dim\left(H'(\tilde{X}, \tilde{V}_2') \otimes 32\right)^{\mathbb{Z}_3 \times \mathbb{Z}_3} = \underline{\underline{0}}$$

$\Rightarrow$ 32 multiplets are projected out. Unfortunately

$$\dim\left(H'(\tilde{X}, \tilde{V}_1' \otimes \tilde{V}_2') \otimes 12\right)^{\mathbb{Z}_3 \times \mathbb{Z}_3} = \underline{\underline{2}}$$

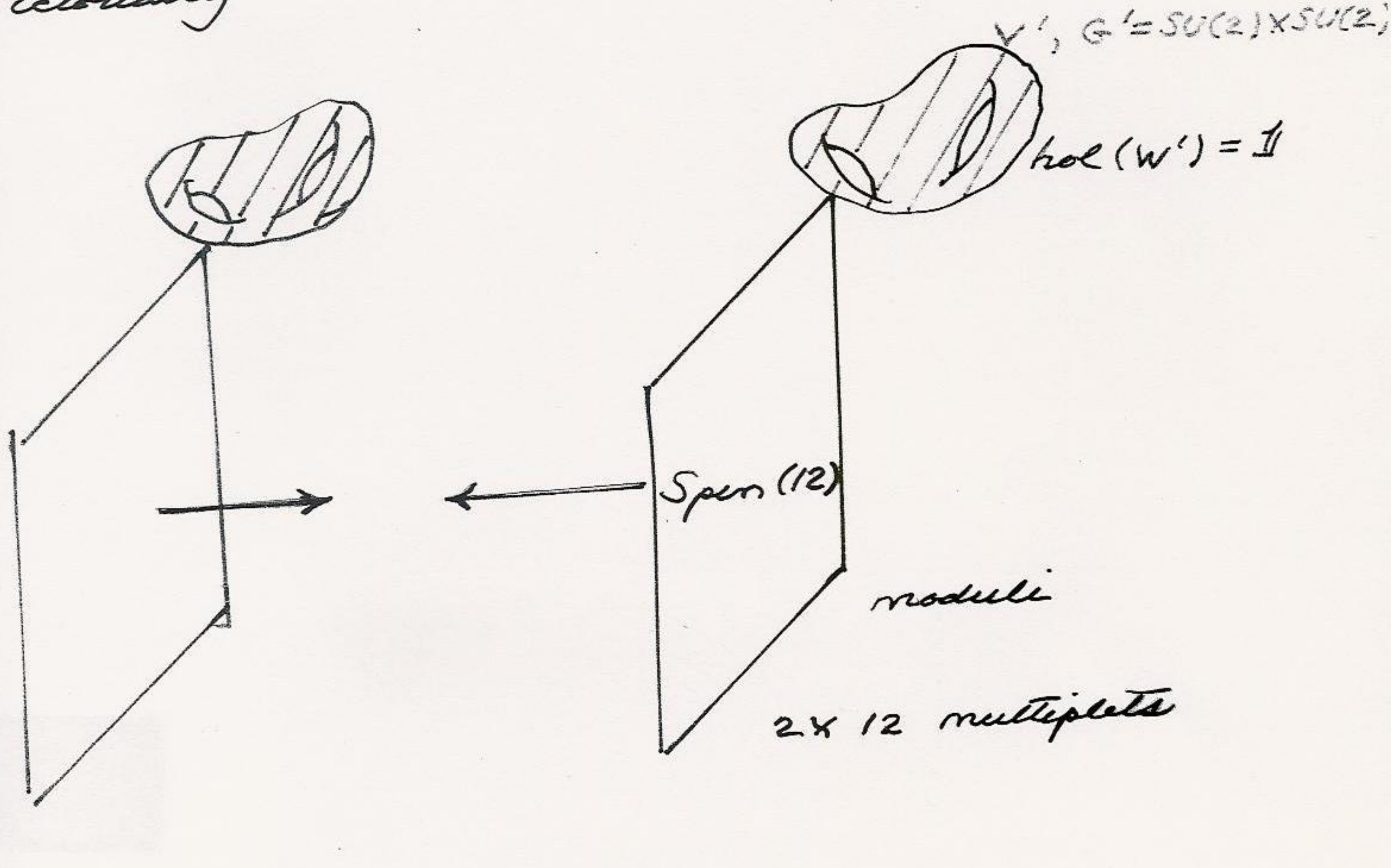
$\Rightarrow$ 2 copies of exotic 12 multiplets.

For such $\tilde{V}'$

$$[w_5] = 0$$

$\Rightarrow$ <u>weak coupling</u> solution (also a strong coupling solution) with <u>no</u> 5-branes.

Pictorially



Yukawa Couplings:

Compute the tri-linear maps

$$H^1(X,V)\times H^1(X,V)\times H^1(X,\wedge^2 V)\longrightarrow \mathbb{C}\ (\simeq H^0(X,\mathcal{O}_X))$$

Dualizing the third term, this is equivalent to

$$H^1(X,V)\times H^1(X,V)\longrightarrow H^1(X,\wedge^2 V)^* \simeq H^2(X,\wedge^2 V)$$

where we use Serre duality and $\wedge^2 V^* \simeq \wedge^2 V$ for $SU(4)$ bundles.

We do the analysis with $\tilde{X}, \tilde{V}$. Recall that

$$0\longrightarrow V_2\longrightarrow \tilde{V}\longrightarrow V_2'\longrightarrow 0$$

One can show that

$$H^*(\tilde{X},\wedge^2 V_2) = H^*(\tilde{X},\wedge^2 V_2') = 0$$

It follows that

$$H^2(\tilde{X}, \Lambda^2 V) \cong H^2(\tilde{X}, V_2 \otimes V_2')$$

Therefore, from $H^1(\tilde{X}, \tilde{V}) \times H^1(\tilde{X}, \tilde{V})$ we need only consider

$$H^1(\tilde{X}, V_2) \times H^1(\tilde{X}, V_2') \longrightarrow H^2(\tilde{X}, V_2 \otimes V_2')$$

To compute these maps, we push each of these terms on $\mathbb{P}^1$ using

$$\begin{tikzcd} \tilde{X} \arrow[d, "\pi"'] & \\ d\tilde{\mathbb{P}}_q \arrow[r, "\beta"'] & \mathbb{P}^1 \end{tikzcd}$$

and Leray spectral sequences.

The results are

1. $H^1(\tilde{X}, V_2) \simeq \mathrm{Reg}(\mathbb{Z}_3 \times \mathbb{Z}_3) H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1})$

. $H^1(\tilde{X}, V_2') \simeq \mathrm{Reg}(\mathbb{Z}_3 \times \mathbb{Z}_3) H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(1))$

. $H^2(\tilde{X}, V_2 \otimes V_2') \simeq \chi_1 H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(1)) \oplus \chi_1^2 H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(1)) \ldots$

These results $\Longrightarrow$ the following selection rules for the Yukawa couplings.

$$\alpha H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}) \times \beta H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(1)) \longrightarrow \chi_1 H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(1))$$

$$\gamma H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}) \times \lambda H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(1)) \longrightarrow \chi_1^2 H^0(\mathbb{P}^1, \mathcal{O}_{\mathbb{P}^1}(1))$$

where

$$\alpha\beta = \chi_1 \quad , \quad \gamma\lambda = \chi_1^2$$

Note that

$\bar{H}$ (down) $\Rightarrow$ $\boxed{\chi_1} H'(\tilde{x}, \Lambda^2\tilde{v}) \otimes \chi_1^2(1, \bar{2}, -3, 0)$

H (up) $\Rightarrow$ $\boxed{\chi_1^2} H'(\tilde{x}, \Lambda^2\tilde{v}) \otimes \chi_1 (1, 2, 3, 0)$

Wilson line

$\Rightarrow$ RHS of selection rule corresponds to H, $\bar{H}$

examples:

a)

l (lepton 2) $\Rightarrow$ $\overset{\alpha}{\boxed{1}} H'(\tilde{x}, \tilde{v}) \otimes 1\ (1, \bar{2}, -3, -3)$

e (lepton 1)$_{-1}$ $\Rightarrow$ $\overset{\beta}{\boxed{\chi_1}} H'(\tilde{x}, \tilde{v}) \otimes \chi_1^2 (1, 1, 6, 3)$

Wilson line

Now

$$\alpha\beta = \chi_1$$

$\Rightarrow$ Yukawa coupling

$$\boxed{l \bar{H} e}$$

(quark 2) $\Rightarrow \underbrace{\boxed{\chi_1 \chi_2^2}}^{\alpha} H'(\tilde{x}, \tilde{v}) \otimes \chi_1^2 \chi_2 (3, 2, 1, 1)$

(quark up 1) $\Rightarrow \underbrace{\boxed{\chi_1 \chi_2}}_{\beta} H'(\tilde{x}, \tilde{v}) \otimes \chi_1^2 \chi_2^2 (\bar{3}, 1, -4, -1)$

Wilson line

Now

$$\alpha\beta = \chi_1^2$$

$\Rightarrow$ Yukawa coupling

$$\boxed{q H u}$$

Clearly <u>all</u> Yukawa couplings are <u>non-zero</u>

but all <u>mix</u> with the <u>first family</u>.