

# 4D Heterotic Models with U(1) Bundles

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hep-th/0504232 and work in progress

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# Motivation

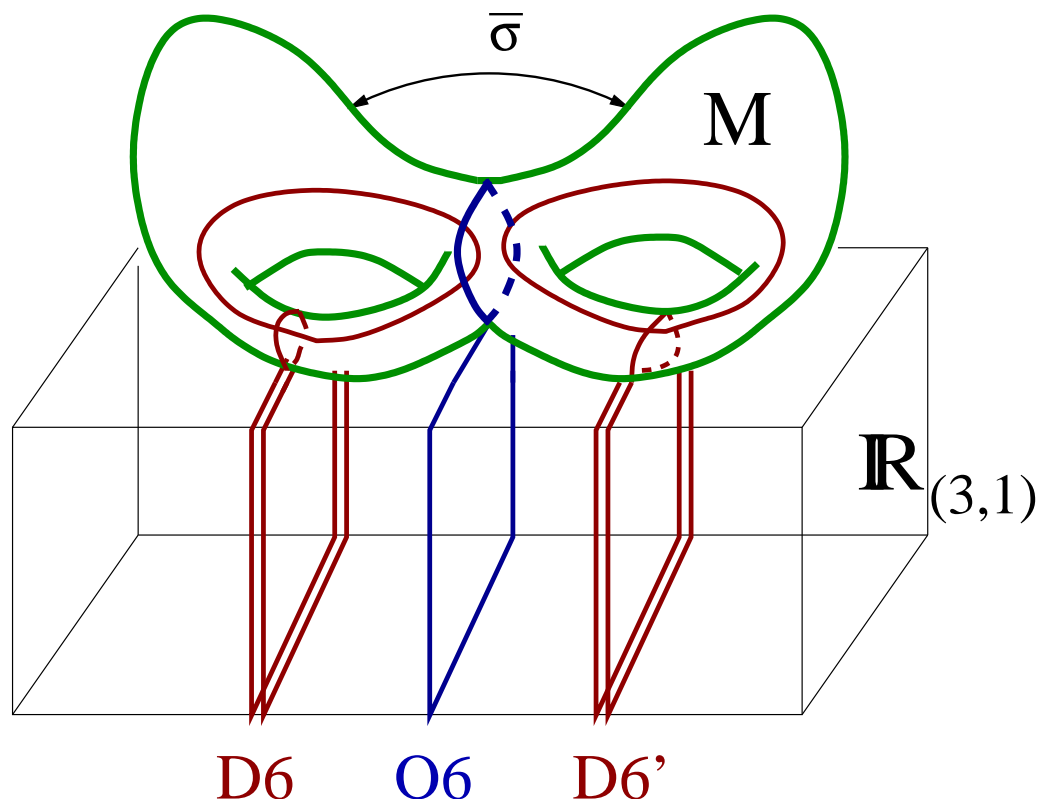
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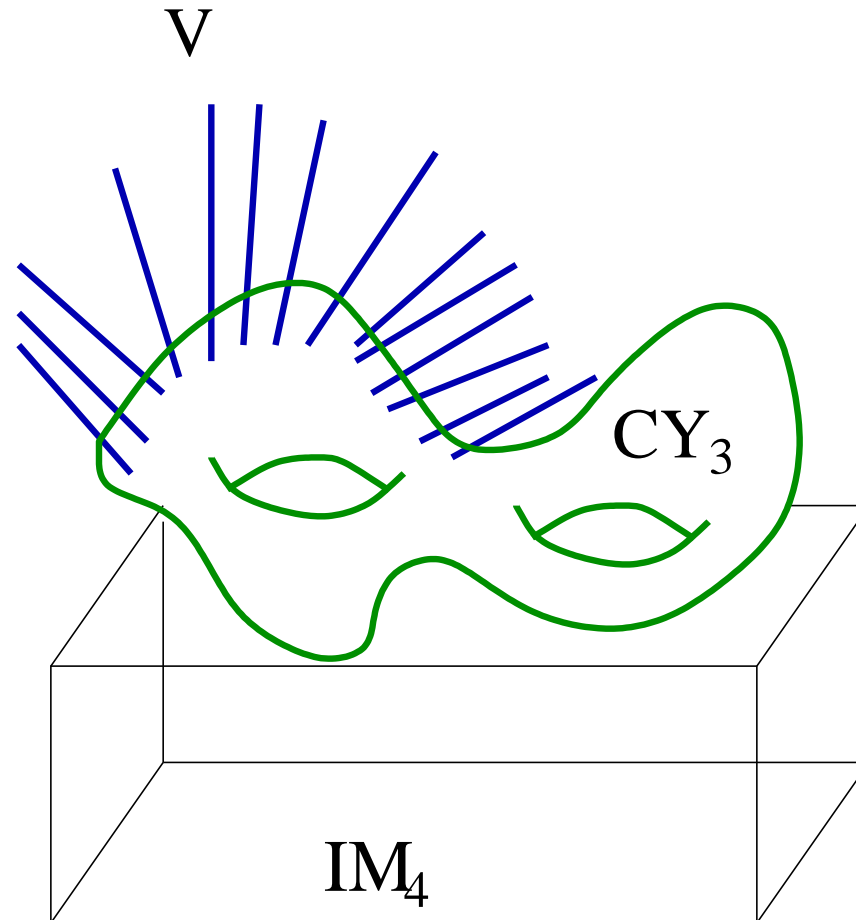


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- Heterotic strings: Mostly non-abelian  $SU(N)$  bundles considered, at most a **single** anomalous  $U(1)$ , GS mechanism involves just the universal dilaton-axion multiplet, **tree level** universal gauge couplings
- Conflict with S-duality  $\rightarrow$  systematically study heterotic string with abelian (line) bundles and Type I vacua with non-abelian bundles.





## Heterotic Compactifications:

M. Green/J. Schwarz/P. West '85, E. Witten '85, J.-P. Derendinger/L. Ibanez/H.-P. Nilles '85, K. Choi/J. Kim '85, J. Distler/B. Greene '88, A. Lukas/K. Stelle '99, B. Andreas, R. Blumenhagen, V. Braun, G. Cardoso, G. Curio, M. Cvetič, K. Dienes, J. Erler, A. Faraggi, S. Förste, J. Gray, M. Hillenbach, G.H., J. Kim, T. Kobayashi, J. Louis, D. Lüst, P. Mayr, S. Groot Nibbelink, B. Ovrut, J. Polchinski, F. Quevedo, E. Rabinovici, S. Raby, E. Sharpe, S. Stieberger, T. Weigand, A. Wingerter + others

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## Intersecting/magnetised branes & fluxes:

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## Stability:

M. Douglas/B. Fiol/C. Römelsberger '00, H. Enger/C. Lutken '03, M. Marino/R. Minasian/G. Moore/A. Strominger '99, P. Aspinwall '04 + others

# Compactifications of heterotic string

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$E_8 \times E_8$  HS with **vector bundles** of the following form

$$W = \bigoplus_{i=1}^K V_{n_i} \oplus \bigoplus_{m=1}^M L_m,$$

where the  $V_{n_i}$  are  $SU(n_i)$  or  $U(n_i)$  bundles and the  $L_m$  denote some **complex line bundles** with structure group  $U(1)_m$ .

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- The vector bundle  $W$  has to admit **spinors**

$$c_1(W) \in H^2(\mathcal{M}, 2\mathbb{Z}).$$

# Hermitian Yang-Mills equation



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- At **string tree level**, the field strength of the vector bundle has to satisfy the **hermitian Yang-Mills** equations

$$F_{ab} = F_{\bar{a}\bar{b}} = 0, \quad g^{a\bar{b}} F_{a\bar{b}} = 0.$$

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$W$  has to be a holomorphic vector bundle, satisfying  $J \wedge J \wedge F = 0$ .

- A necessary condition is the so-called [Donaldson-Uhlenbeck-Yau](#) (DUY) condition,

$$\int_{\mathcal{M}} J \wedge J \wedge c_1(V_{n_i}) = 0, \quad \int_{\mathcal{M}} J \wedge J \wedge c_1(L_m) = 0,$$

to be satisfied for all  $n_i, m$ . If so, a theorem by Uhlenbeck-Yau guarantees a unique solution provided each bundle is *stable*.

# Tadpole cancellation

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- The Bianchi identity for  $H = dB - \frac{\alpha'}{4}(\omega_Y - \omega_L)$ ,

$$dH = \frac{\alpha'}{4} (\text{tr}(R^2) - \text{tr}(F_1^2) - \text{tr}(F_2^2)) ,$$

imposes the so-called **tadpole condition** in cohomology

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- The **structure group**  $G$  of the bundle  $W$  has to be embedded into  $E_8 \times E_8$ . The **observable** gauge group in four dimensions  $H$  is the **commutant** of  $G$  in  $E_8 \times E_8$ .

# Massless spectrum

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- The massless spectrum is determined by various **cohomology** classes

$$H^*(\mathcal{M}, \mathcal{W}) \quad \mathcal{W} = \bigotimes_{i=1}^K \wedge^{p_i} V_{n_i} \otimes \bigotimes_{m=1}^M L_m^{q_m},$$

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- The net-number of **chiral matter** multiplets is given by the Euler characteristic of the respective bundle  $\mathcal{W}$

$$\begin{aligned} \chi(\mathcal{M}, \mathcal{W}) &= \sum_{i=0}^3 (-1)^i \dim(H^i(\mathcal{M}, \mathcal{W})) \\ &= \int_{\mathcal{M}} \left[ \text{ch}_3(\mathcal{W}) + \frac{1}{12} c_2(T) c_1(\mathcal{W}) \right]. \end{aligned}$$



# The Green-Schwarz mechanism

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- All **non-abelian** cubic gauge anomalies do cancel, whereas the **mixed** abelian-nonabelian, the mixed abelian-gravitational and the cubic abelian ones do not:

$$A_{U(1)_m - SU(N)^2} \sim f_m \text{tr} F_1^2 \int_{\mathcal{M}} \bar{f}^m \left( \text{tr} \bar{F}_1^2 - \frac{1}{2} \text{tr} \bar{R}^2 \right),$$

$$A_{U(1)_m - G_{\mu\nu}^2} \sim f_m \text{tr} R^2 \int_{\mathcal{M}} \bar{f}^m \left( \text{tr} \bar{F}_1^2 - \frac{5}{12} \text{tr} \bar{R}^2 \right)$$

$$A_{U(1)_{mnp}} \sim f_m f_n f_p \left[ \int_{\mathcal{M}} \delta_{np} \bar{f}^m \left( \text{tr} \bar{F}_1^2 - \frac{1}{2} \text{tr} \bar{R}^2 \right) + c_{mnp} \bar{f}^m \bar{f}^n \bar{f}^p \right]$$

Need to be cancelled by a generalized **Green-Schwarz mechanism**.

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Dimensionally reducing the 10D 1-loop counter term

$$S_{GS} = \frac{1}{48 (2\pi)^5 \alpha'} \int B \wedge X_8, \quad X_8 = \frac{1}{4} (\text{tr} F_1^2)^2 + \dots$$

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leads to **vertex couplings** for the **internal axions**

$$S_{GS} = \frac{1}{64 (2\pi)} \int_{\mathbb{R}_{1,3}} \sum_{k=1}^{h_{11}} \left( b_k^{(0)} \text{tr} F_1^2 \right) \left( \text{tr} \overline{F}_1^2 - \frac{1}{2} \text{tr} \overline{R}^2 \right)_k$$

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and **mass terms** for the **external (universal) axions**:

$$S_{mass}^0 = \sum_{m=1}^M \frac{Q_0^m}{2\pi\alpha'} \int_{\mathbb{R}_{1,3}} \left( b_0^{(2)} \wedge f_m \right),$$

$$Q_0^m = \frac{\text{tr}_{E_8}(Q_m^2)}{32 (2\pi)^4} \int_{\mathcal{M}} \overline{f}^m \wedge \left( \text{tr} \overline{F}_1^2 - \frac{1}{2} \text{tr} \overline{R}^2 \right),$$

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Additional terms arise from the [tree-level kinetic](#) term

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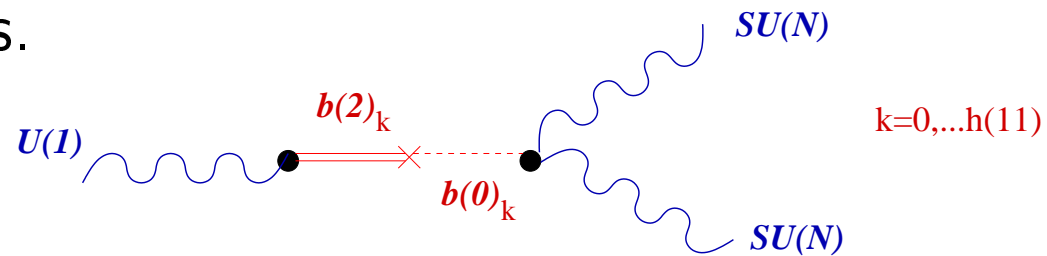
$$S_{\text{mass}} = \sum_{k=1}^{h_{11}} \sum_{m=1}^M \frac{Q_k^m}{2\pi\alpha'} \int_{\mathbb{R}_{1,3}} \left( b_k^{(2)} \wedge f_m \right),$$

$$Q_k^m = \frac{\text{tr}_{E_8}(Q_m^2)}{4\pi} \bar{f}_k^m$$

# Massive gauge fields + axions

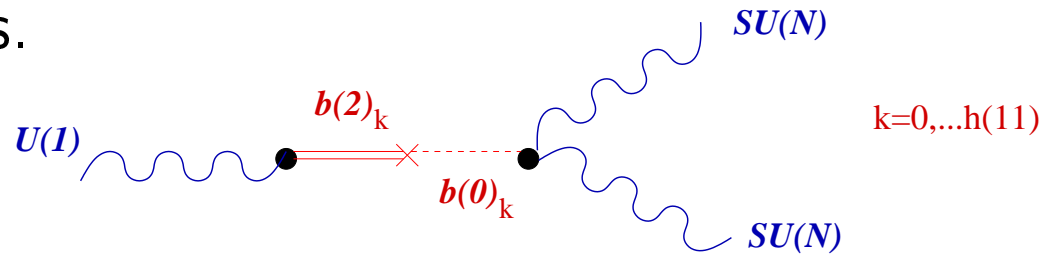
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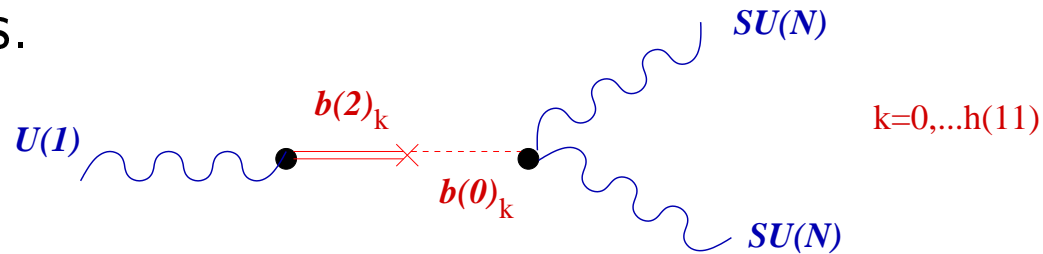
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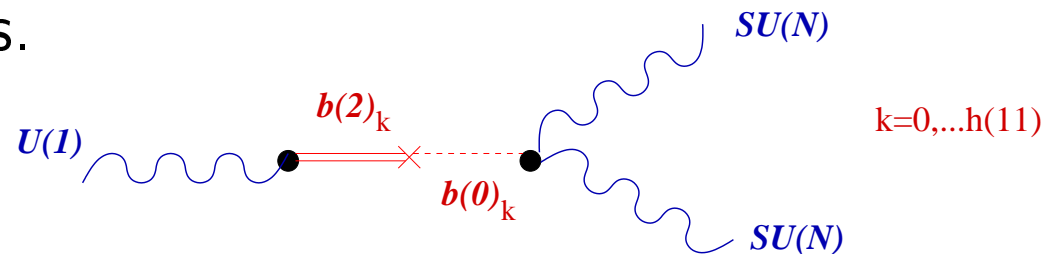
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- The **# massive  $U(1)$**  gauge fields is given by  $\text{rank}(\mathcal{Q})$ .
- All mass terms are of the **same order** in both string and sigma model perturbation theory.
- Massive axions  $b_k^{(0)}$  are longitudinal modes of  $U(1)$ s. Since axions complexify the Kähler moduli and the dilaton, **supersymmetry** dictates that the same **#** is massive.  
Only source for mass terms is the **DUY** equation.

# Gauge kinetic function



# Gauge kinetic function

- Defining the **complexified dilaton**

$$S = \frac{1}{2\pi} \left[ e^{-2\phi_{10}} \frac{\text{Vol}(\mathcal{M})}{\ell_s^6} + i b_0^{(0)} \right].$$

and the **complexified Kähler moduli**

$$T_k = \frac{1}{2\pi} \left[ -\alpha_k + i b_k^{(0)} \right],$$

the GS terms induce the axionic part of the **gauge kinetic function**, e.g.

$$S_{kin} = \frac{1}{8\pi} \int_{\mathbb{R}_{1,3}} b_0^{(0)} \wedge \text{tr} F_1^2$$

# Gauge kinetic function

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- The 1-loop corrected gauge kinetic function for the non-abelian gauge fields are

$$f = S + \frac{1}{16} \sum_k T_k \left( \text{tr} \overline{F}_1^2 - \frac{1}{2} \text{tr} \overline{R}^2 \right)_k.$$

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- For the **abelian** ones we find

$$\begin{aligned} f_{mn} = & \text{tr}_{E_8}(Q_m^2) \delta_{mn} \times \\ & \left( S + \frac{1}{16} \sum_k T_k \left[ \left( \text{tr} \overline{F}_1^2 - \frac{1}{2} \text{tr} \overline{R}^2 \right)_k \right] \right. \\ & \left. + \frac{\text{tr}_{E_8}(Q_m^2) \text{tr}_{E_8}(Q_n^2)}{12} \sum_{i,j} T_k d_{ijk} \overline{f}_i^m \overline{f}_j^n \right) \end{aligned}$$

# Fayet-Iliopoulos terms

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- Since we are dealing with anomalous  $U(1)$  gauge factors, there are potential **Fayet-Iliopoulos** (FI) terms generated. Employing the standard supersymmetric field theory formula

$$\frac{\xi_m}{g_m^2} = \left. \frac{\partial \mathcal{K}}{\partial V_m} \right|_{V=0},$$

the FI parameters  $\xi_m$  can be computed from the **Kähler potential**  $\mathcal{K}$ , which in our case takes the following gauge invariant form.

$V_m$  denotes the vector superfields.

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$$\mathcal{K} \sim -\ln\left(S + S^* - \sum_m Q_0^m V_m\right) - \ln\left(\sum_{i,j,k=1}^{h_{11}} \frac{d_{ijk}}{6} (T_i + T_i^* - \sum_m Q_i^m V_m) (T_j + \dots) (T_k + \dots)\right).$$



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For the FI term one gets

$$\frac{\xi_m}{g_m^2} = -\frac{1}{2\ell_s^6} \text{tr}_{E_8}(Q_m^2) \left[ e^{-2\phi_{10}} \frac{1}{2} \int_{\mathcal{M}} J \wedge J \wedge \bar{f}_m - \frac{(2\pi\alpha')^2}{16} \int_{\mathcal{M}} \bar{f}_m \wedge \left( \text{tr} \bar{F}_1^2 - \frac{1}{2} \text{tr} \bar{R}^2 \right) \right].$$

Apparently, the first term arises at string tree-level, whereas the second term is a **1-loop** term.

# 1-loop DUY equation

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- Interpretation:  $\xi_m = 0$  contains a 1-loop correction to the DUY condition. In contrast to earlier claims that abelian gauge fluxes freeze some combinations of the Kähler moduli, we now realize that actually combinations of the dilaton and the Kähler moduli are frozen.

# 1-loop DUY equation

- Interpretation:  $\xi_m = 0$  contains a **1-loop correction** to the **DUY** condition. In contrast to earlier claims that abelian gauge fluxes freeze some combinations of the Kähler moduli, we now realize that actually combinations of the **dilaton and the Kähler moduli** are frozen.
- Support from **Type I - HS** duality. It is known that a D9-brane wrapping the Calabi-Yau  $\mathcal{M}$  with Kähler form  $J^I$  and a  $U(1)$  bundle with field strength  $F$  is supersymmetric if it satisfies the **MMMS condition**

$$\frac{1}{2} \int_{\mathcal{M}} J^I \wedge J^I \wedge F - \frac{(2\pi\alpha')^2}{3!} \int_{\mathcal{M}} F \wedge F \wedge F = 0$$

# 1-loop DUY equation

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The MMMS condition is at string tree level and the  $F^3$  term is an  $\alpha'$  correction. Applying the heterotic-Type I string duality relations

$$\begin{aligned} e^{\phi_{10}^I} &= e^{-\phi_{10}^H}, \\ J^I &= J^H e^{-\phi_{10}^H}, \end{aligned}$$

precisely leads to the form of the HS-DUY equation.

# Non-abelian MMMS equation

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Similar analysis for the  $SO(32)$  string leads after applying S-duality to the non-abelian generalization of the integrability condition for perturbative  $\Pi$  stability for  $\varphi = 0$

$$\begin{aligned} & \cos \varphi \left[ \frac{1}{2} \int_{\mathcal{M}} J^2 \operatorname{tr} \overline{\mathcal{F}}_x - \frac{1}{3!} \int_{\mathcal{M}} \left( \operatorname{tr} \overline{\mathcal{F}}_x^3 - \frac{1}{16} \operatorname{tr} \overline{\mathcal{F}}_x \operatorname{tr} \overline{R}^2 \right) \right] \\ & + \sin \varphi \left[ \frac{N_x}{3!} \int_{\mathcal{M}} J^3 - \frac{1}{2} \int_{\mathcal{M}} J \left( \operatorname{tr} \overline{\mathcal{F}}_x^2 - \frac{N_x}{48} \operatorname{tr} \overline{R}^2 \right) \right] = 0 \end{aligned}$$

This equation also includes the case of curved backgrounds.



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$$\begin{aligned} & \cos \varphi \left[ \frac{1}{2} \int_{\mathcal{M}} J^2 \operatorname{tr} \overline{\mathcal{F}}_x - \frac{1}{3!} \int_{\mathcal{M}} \left( \operatorname{tr} \overline{\mathcal{F}}_x^3 - \frac{1}{16} \operatorname{tr} \overline{\mathcal{F}}_x \operatorname{tr} \overline{R}^2 \right) \right] \\ & + \sin \varphi \left[ \frac{N_x}{3!} \int_{\mathcal{M}} J^3 - \frac{1}{2} \int_{\mathcal{M}} J \left( \operatorname{tr} \overline{\mathcal{F}}_x^2 - \frac{N_x}{48} \operatorname{tr} \overline{R}^2 \right) \right] = 0 \end{aligned}$$

This equation also includes the case of curved backgrounds. The second line is obtained from gauge kinetic function. It can be compactly written as

$$\int_{\mathcal{M}} \operatorname{tr}_{U(N)} \left[ \operatorname{Im} \left( e^{i\varphi} e^{J+i\mathcal{F}} \sqrt{\hat{A}(\mathcal{M})} \right) \right] = 0$$

# Vector bundles via exact sequences

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- CY manifold  $\mathcal{M}$  given by a **complete intersection in some toric variety**, e.g.  $\mathbb{P}_4[5]$ , and has  $k = h_{11}$  Kähler parameters.

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- CY manifold  $\mathcal{M}$  given by a **complete intersection in some toric variety**, e.g.  $\mathbb{P}_4[5]$ , and has  $k = h_{11}$  Kähler parameters.
- A line bundle  $L$  on  $\mathcal{M}$  is specified completely by its **first Chern class** which takes values in  $H^2(\mathcal{M}, \mathbb{Z})$  and can be expanded in terms of a basis  $\omega_i$  with  $n_i \in \mathbb{Z}$

$$c_1(L) = \sum_{i=1}^{h_{11}} n_i \omega_i.$$

One also denotes such a line bundle as  $\mathcal{O}(n_1, \dots, n_k)$ .

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- Then a vector bundle  $V$  of rank  $r$  is defined by the cohomology of the monad ( $i = 1, \dots, h_{11}$ )

$$0 \rightarrow \mathcal{O}|_{\mathcal{M}} \xrightarrow{g} \bigoplus_{a=1}^{r+2} \mathcal{O}(n_{a,i})|_{\mathcal{M}} \xrightarrow{f} \mathcal{O}(m_i)|_{\mathcal{M}} \rightarrow 0,$$

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- The total Chern class of the vector bundle  $V$  is then given by

$$c(V) = \frac{\prod_{a=1}^{r+1} (1 + \sum_i n_{a,i} \omega_i)}{(1 + \sum_i m_i \omega_i)} = 1 + \sum_{i=1}^{h_{11}} \left( \sum_{a=1}^{r+1} n_{a,i} - m_i \right) \omega_i + \dots$$



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- For  $SU(N)$  bundles one has  $c_1(V) = 0$ , whereas for  $U(N)$  bundles at first sight no condition.

# Example: $SU(4) \times U(1)$ bundle

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- $E_8 \times E_8$  heterotic string compactified on a Calabi-Yau manifold  $\mathcal{M}$  equipped with the specific class of bundles

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with **structure group**  $G = SU(4) \times U(1)$ .

- Embedding this structure group into one of the  $E_8$  factors leads to the breaking to **gauge group**  $H = SU(5) \times U(1)$ , where the adjoint of  $E_8$  decomposes as follows into  $G \times H$  representations.

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$$248 \xrightarrow{SU(4) \times SU(5) \times U(1)} \left\{ \begin{array}{l} (\mathbf{15}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{24})_0 \\ (\mathbf{1}, \mathbf{10})_4 + (\mathbf{6}, \bar{\mathbf{5}})_{-2} + h.c. \\ (\mathbf{4}, \mathbf{1})_{-5} + (\mathbf{4}, \bar{\mathbf{5}})_3 + (\mathbf{4}, \mathbf{10})_{-1} + h.c. \end{array} \right\} .$$

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$$248 \xrightarrow{SU(4) \times SU(5) \times U(1)} \left\{ \begin{array}{l} (15, 1)_0 + (1, 1)_0 + (1, 24)_0 \\ (1, 10)_4 + (6, \bar{5})_{-2} + h.c. \\ (4, 1)_{-5} + (4, \bar{5})_3 + (4, 10)_{-1} + h.c. \end{array} \right\}.$$

$SU(5) \times U(1)$	Cohomology
$10_{-1}$	$H^*(\mathcal{M}, V \otimes L^{-1})$
$10_4$	$H^*(\mathcal{M}, L^4)$
$\bar{5}_3$	$H^*(\mathcal{M}, V \otimes L^3)$
$\bar{5}_{-2}$	$H^*(\mathcal{M}, \wedge^2 V \otimes L^{-2})$
$1_{-5}$	$H^*(\mathcal{M}, V \otimes L^{-5})$

# **n** $SU(4) \times U(1)$ bundle on the Quintic



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Consider the **Quintic**  $\mathbb{P}_4[5]$  with Hodge numbers  
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- Choose a bundle of the form

$$W = V_1 \oplus V_2 \oplus L,$$

where the  $SU(4)$  bundle  $V_1$  and the line bundle  $L$  are embedded into the **visible**  $E_8$  factor and the  $SU(4)$  bundle  $V_2$  into the **hidden**  $E_8$  bundle.

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- Concretely, we define both **vector bundles** as the cohomology of the monad

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$$c_2(V_1) + c_2(V_2) - 10 c_1^2(L) = c_2(T).$$

- Defining the Kähler form  $J = \ell_s^2 r \eta$ , the **DUY** equation reads

$$r^2 = 10 g_s^2,$$

Choosing for instance  $g_s = 0.3$  yields  $r = 0.95$ .



# Massless spectrum

# Massless spectrum

The **chiral**  $SU(5) \times U(1)$  spectrum reads, where the **anomalous**  $U(1)$  only survives as a global symmetry:

reps.	$\chi$
$\mathbf{10}_{-1}$	290
$\mathbf{10}_4$	460
$\bar{\mathbf{5}}_3$	170
$\bar{\mathbf{5}}_{-2}$	580
$\mathbf{1}_{-5}$	-2150

# Gauge symmetry enhancement

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# Gauge symmetry enhancement

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- Whenever cohomology classes

$$H^*(M, \otimes_i L_i^{q_i})$$

are to be computed, **non-abelian gauge symmetry enhancement** occurs when

$$\sum_i q_i c_1(L_i) = 0.$$

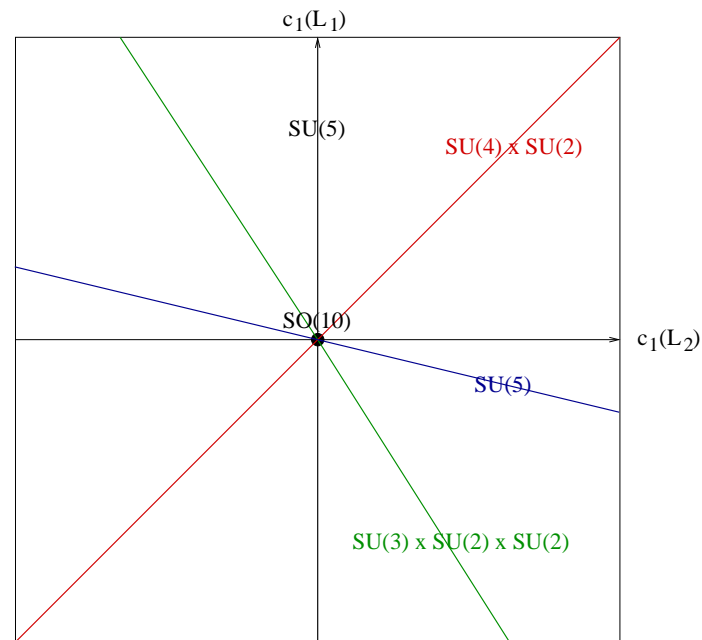
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- For an  $SU(4) \times U(1) \times U(1)$  bundle in  $E_8$

$$W = V \oplus L_1 \oplus L_2$$

we obtained the following **enhancements** of  $SU(3) \times SU(2)(\times U(1))$



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- There appears a **1-loop** correction to the **DUY** supersymmetry condition, which “freezes” combinations of Kähler moduli and the dilaton.
- As on the Type I side, one finds **non-universal** gauge threshold corrections.
- Does the non-abelian version of the MMMS equation help to uncover the **non-abelian Born-Infeld** action?
- What about the heterotic string **landscape**?