Fixing all moduli in F-theory and type II strings

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- “Flux” compactifications are important in many constructions of string “vacua”

- Crucial feature is that supersymmetry is spontaneously broken (=: “flux”)

- It appeared that only a subset of moduli is fixed by “flux”

- I’ll describe 2 generalizations of the superpotential in type II strings where

  - all moduli get a potential from “fluxes”
  - the same “flux” induces also a non-perturbative superpotential

- The first possibility involves open strings (F-theory on K3xK3 -> KKLT)

- The second is type IIB without branes on “generalized CY manifolds” (non CY)

The two are in fact related by an open/closed string duality …

full non-perturbative potential can be computed

(<-> tension of new BPS domain walls)

- Starting point for generalization: Type IIB compactifications on CY3 Z

$$\boxed{W_{GVW} = \int \Omega^{3,0} \wedge G_3}$$

$\Omega^{3,0}(z_i)$: Hol. 3,0 form

$G_3 = F_3 - \tau H_3$: 3-form flux

Gukov, Vafa, W

(' BPS tension of NS5/D5 domain walls)

Taylor, Vafa; P.M.

- Geometry computes exact non-perturbative potential

$$V = V_{pert} + \sum_k A_k e^{-k/g^2}$$

(F- and D-terms)

- "Flux" generates also instanton corrections:

N= 2 Parent: No Flux

N< 2: SSB by Flux

$$\int d\theta^4 \leftarrow \langle \prod O_i \rangle \quad \longleftrightarrow \quad \langle \prod O_i' \rangle \rightarrow \int d\theta^2$$

Same instantons, different amplitudes

(lift of fermionic 0-modes)

-> Instanton corrections can often be computed exactly

in (simpler) parent theory without flux

1) Adding Open Strings …

- type IIB + 7 branes -> F-theory

$$W_{GKP} = \int \Omega^{3,0} \wedge G_3$$

DR of 10d supergravity (!)

Giddings, Kachru, Polchinski; Dasgupta, Rajesh, Sethi;

Not (at all) an exact expression

(no exact BPS tension)

1) Open string sector ? (W_{GKP} depends only on bulk fields)

2) Quantum corrections ? (Flux does generate instanton corrections !)

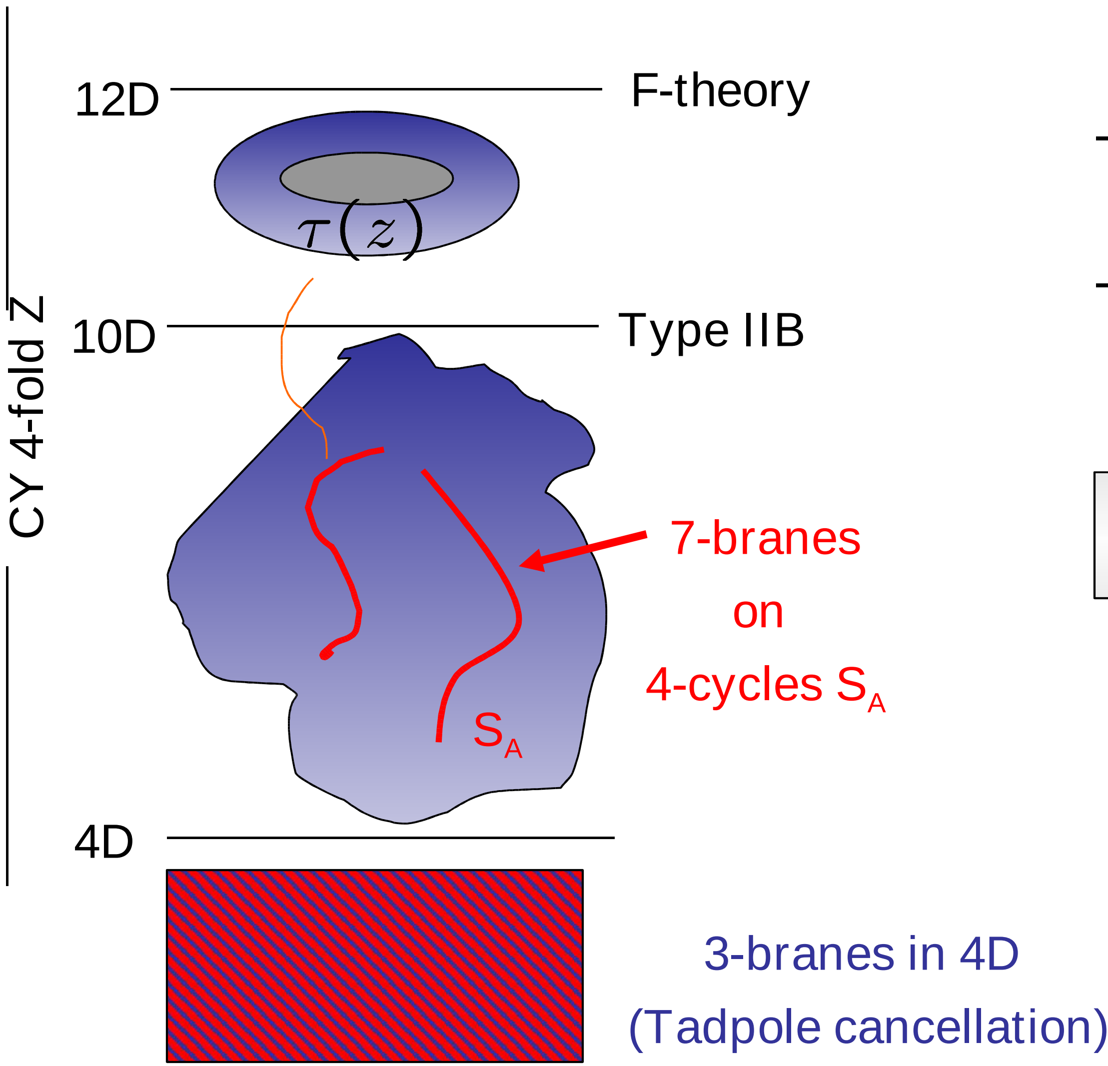
Expect:

$$W_{eff} = (W_{GKP} + W_{open}) \ + \ W_{n.p.}$$

Understand open string sector and new quantum corrections
Lift to a BPS tension of domain walls ?
Will also learn something new about closed type II strings…

F-theory: Branes & Fluxes

GEOMETRY



FLUX

- No 3-form Flux (i.g.)

- Worldvolume gauge fields $\langle F \rangle \neq 0$
  U(1) on A-th 7-brane

$$F_{AI} = \int_{\gamma^I} F_A \qquad \gamma^I \in H_2(S_A)$$

Potential ?

Superpotential on 7-branes

- World-volume theory on D-branes = "Holomorphic Chern-Simons" (+top. twist)

Witten;
Bershadsky,
Sadov,Vafa

DR
+
top. twist

$$W = C\int_{S_A} \omega^{2,0} \wedge F$$

7-brane position

Res(Ω)

2-form flux

LMRS
Jockers, Louis

cpw
$$W = \int \Omega^{3,0} \wedge G_3$$

- Local D7-branes -> sum over D7-branes

- Not all 7-branes are local: (p,q) 7-branes -> compute global contribution from CY 4-fold

Ex: F-theory on K3 x K3 and orientifolds

Tripathy, Trivedi

LMRS

Jockers, Louis;

- Generic F-theory moduli:

$$\boxed{W_{open} = \sum_{A,I} \Pi^A \ F_{AI} \ \tilde{\Pi}^I}$$

2-form flux $F_{AI} = \int_{\gamma^I} F_A$

Bi-linear in K3 periods $\Pi^A = \int_{\gamma_A} \omega^{2,0}$

- The **same** flux also generates a D-term potential

$$\boxed{D_A \sim \int j \wedge F_A}$$

j: Kaehler form

-> positive energy contribution V_D of KKLT (AdS->dS)

Perturbative potential depends on **all moduli** * :

- complex structure
- Kaehler moduli
- 7 brane positions

* Certain restrictions apply:
- 2 volume moduli unfixed
- D3 brane moduli unfixed -> will be fixed by non-perturbative effects…

Orientifold limit I

1) Geometry: Orientifold = special configuration (location) of 7-branes:
4 O-planes and 16 D7-branes

2) Flux: F_{AI} on 7-branes splits into
- "3-form fluxes" $G_3 = F_3 - \tau H_3$
- 16 2-form fluxes on D7-branes

LMRS

$$W_{\text{"open"}} = \Pi^A F_{AI} \tilde{\Pi}^I \longrightarrow \boxed{\int \Omega \wedge G_3 + m(G_3) \sum C_A^2 + \sum_{A=1}^{16} C_A \int \omega \wedge F}$$

Bulk:
GKP superpotential

Brane:
- C_A: tadpoles for D7 moduli

Bulk:- C_A^2 universal mass for D7 brane moduli
only 1 flux component contributes

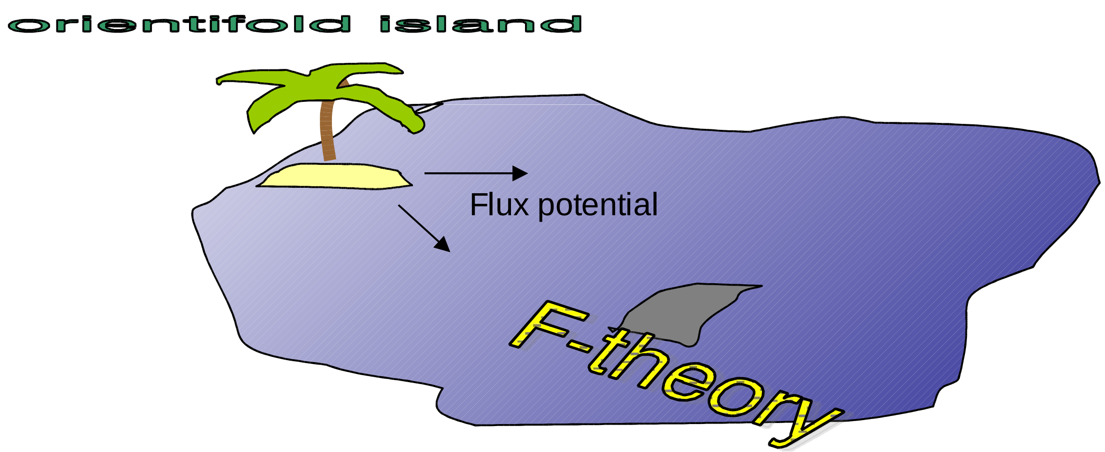
-> *(cf. Louis et al, Goerlich et al; Ibanez et al; Luest et al)*

For generic flux

- D7 branes driven away from orientifold $\langle C_A \rangle \neq 0$

- Orientifold is NOT a preferred configuration

Destabilization of Orientifold by flux



-> use orientifold preserving fluxes

Classified in
LMRS
Aspinwall, Kallosh

- F-theory flux gives a F- and a D-term Potential, that depends on almost all open and closed string moduli

- However D3-brane moduli and 2 volume moduli are still unfixed

- So far the discussion has been perturbative in the sense that D3-instanton corrections are neglected *

$$\boxed{W_{eff} = (W_{GKP} + W_{open}) \;+\; W_{n.p.}}$$

✓ ✓ ?

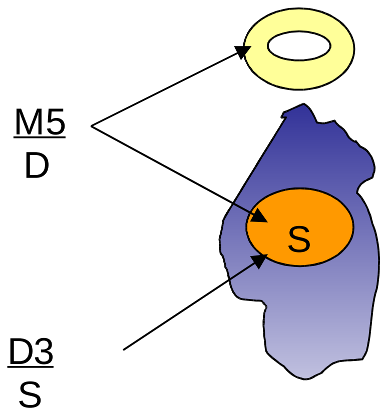
"Flux"

I'll now discuss the non-perturbative part. It turns out that the same flux generates instanton corrections to the potential that fix the remaining moduli …

* However, perturbative F-theory takes care of certain non-perturbative corrections to the metric and gauge couplings

D3-instanton computation

Witten



- D3-Instanton correction to superpotential from M5-Instanton in M-theory (=F-theory on X x S^1)

Rare …

- Necessary condition: $\chi(D)=1$

$$\boxed{W_D = \Delta(\phi) e^{-Vol(D)+i\varphi}}$$

- $\Delta(\phi)$: 1-loop determinant on M5-brane

$$W_{n.p.} = \sum_{all\ D} W_D$$

Sum over all such instantons

Difficult …

- Interplay Flux <-> D-instantons ?

$$\boxed{W_{eff} = (W_{GKP} + W_{open}) + W_{n.p.}}$$

" Flux" induces instanton corrections

$$\chi = 1?$$

(Goerlich et al)

INSTANTONS & FLUX

- Flux = lift of 0-modes. Same instantons contribute to new amplitudes in theory with SSB

Instantons in N= 2 parent Superpotential (SSB)

$$\langle \prod O_i \rangle \longrightarrow \langle \prod O_i' \rangle$$

Non-perturbative N= 1 Superpotential from instantons of parent theory (Taylor,Vafa;PM)

- World-volume Mechanism: lift of fermionic 0-modes ψ_0

$$\langle F \rangle \to m\Psi_0^2$$

(flux can also obstruct instantons (world-volume tadpole))

- For D3/M5 branes this was computed in the world-volume theory
- However world-volume fields are no physical observables -> necessary conditions

Tripathy,Trivedi; Saulina; Kallosh et al;

- Physical instanton correction to the superpotential < -> N+ 1 physical observables in a theory with N "moduli"

String duality: N physical observables = coordinates on n.p. field space

= BPS tensions of "electric" domain walls

N+ k-th observables = n.p. superpotential, gauge couplings …

Open/closed string Duality

F-theory	Type II
K3 x K3_	CY_3 X

- D-brane Geometry, Orientifold, Tadpoles
- Pure geometry !
- D3-instantons
- F1 instantons
- F-theory fluxes h F i_7
- "Generalized CY" compactification

Link: Effective 4d field theory = **N= 2 gauged Supergravity**
(Supersymmetry broken to N · 2 by flux)

$$W_F = (W_{GKP} + W_{open}) + W_{n.p.}(D3)$$

F-theory:
GKP+open+D3

$$W_{sugra} = P_A X^A - P^A F_A$$

$$W_F = (W_{GVW} + W_{GCY}) + W_{n.p.}(F1)$$

Type II:
GVW+?+F1

= easy to compute

D3-instanton sum

Berglund, P.M.

- What one expects: $$\boxed{W^D_{inst} = \Delta(\phi) e^{-Vol(D)+i\varphi}}$$ $$W_{n.p.} = \sum_{''good''\ D} W^D_{inst}$$

- What we find: $$\boxed{W_{n.p} = \langle F_* \rangle \sum_{k=1}^{\infty} \Delta_k(\phi) e^{(-Vol(D)+i\varphi).k}}$$ **Computable**

- Divisors D with $\chi \neq 1$ contribute

- proportional to **a single U(1) flux F*** for given D

Instanton < -> magnetic monopole < -> U(1) charge< -> F*

- For given D -> infinite # of Multi-instanton contributions of charge k>1 with leading weight 1/k^2

-$W_{n.p}$ depends on **all** moduli, including

2 volume moduli and D3 brane moduli

All moduli fixed in F-theory/K3xK3.

(-> M-theory: Kallosh, Aspinwall, hep-th0506014)

Comparison to KKLT

Kachru, Kallosh, Linde, Trivedi

F-theory on K3 x K3
+ flux → Simple, explicitly computable realization of KKLT

1) Perturbative flux potential

$$\boxed{\int \Omega \wedge G_3} + \boxed{m(G_3) \sum C_A^2 + \sum_{A=1}^{16} C_A \int \omega \wedge F}$$

2) D3- Instanton corrections

$$W_{n.p} = \boxed{A e^{(-Vol(D)+i\varphi)}} + \boxed{\ldots} = \sum_{k=1}^{\infty} \langle F_* \rangle \Delta_k(\phi) e^{(-Vol(D)+i\varphi).k}$$

3) Positive energy contribution

$$\boxed{V_D \sim D_A^2} \qquad D_A \sim \int j \wedge F_A$$

(cf. Burgess, Kallosh, Quevedo)

The 1-loop determinant $\Delta(\phi)$

– $\Delta(\phi)$ is a non-trivial section of a line bundles *Witten; Ganor*

- One can explicitly compute this in dependence of moduli *Berglund, PM*

  - Zeros for special complex structure (7-brane moduli !)

$$\Delta(\phi_*) \equiv 0 \longrightarrow W = 0$$

  - Singularities at special moduli

$$W = (W_{GKP} + W_{open}) + \sum \cancelto{\infty}{\tilde{\Delta}}_k(\phi_*) e^{-k\, S_{D3}}$$

D3-instanton expansion breaks down

-> Physics: IR singularity, e.g. confining gauge theory

- Perturbative vacuum near ϕ_* is artifact of an unvalid approximation
- Strong coupling regime can still be computed using duality

The 1-loop determinant $\Delta(\phi)$

- Non-trivial H^3(D) -> $\Delta(\phi)$ is a sum over world-volume fluxes (Witten)

- "Expansion coefficents" of 4d D3 superpotential = partition function of (NC) string
  Details depend on global compactification (7-brane geometry and fluxes)

Compute -> $$\Delta_1 = \frac{E_4 E_6}{\eta^{24}}$$ 1-loop heterotic string

$$\Delta_1' = \frac{E_4}{\eta^{12}}$$ 1-loop NC E8 string

- Higher coefficients: Highly non-trival modular functions with zeros and poles

Important for vacuum equations

Perturbative Action for 7 and 3 branes

- Duality also computes perturbative action for the system

geometry + 7-branes + 3-branes

- Example: loop corrected Kaehler function:

$$K = \quad -\ln(i(T-\bar{T})(U-\bar{U})(S-\bar{S}))$$

$$+\frac{1}{(T-\bar{T})(U-\bar{U})}(\frac{1}{2}(B-\bar{B})^2 - \frac{7}{24}(S-\bar{S})^2 + \frac{105 i\zeta(3)}{\pi^3(S-\bar{S})}) + \ldots$$

- Large 1-loop corrections for D3-brane theory !

$$\frac{1}{g_{D3}^2} \sim S \;+\; (1-loop)$$

Bad approximation
for general 7-brane moduli

Link: Effective 4d field theory = **N= 2 gauged Supergravity**
(Supersymmetry broken to N · 2 by flux)

$$W_F = (W_{GKP} + W_{open}) + W_{n.p.}$$

F-theory:

GKP+open+D3

$$W_{sugra} = P_A X^A - P^A F_A$$

$$W_F = (W_{GVW} + W_{GCY}) + W_{n.p.}$$

Type II:

GVW+?+F1

Generalized type IIB superpotential

- Flux in F-theory:

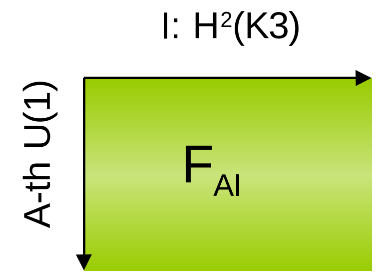
$$W_{open} = \Pi^A F_{AI} \tilde{\Pi}^I$$

Bilinear in K3 periods

$$\Pi = \int_\gamma \omega^{2,0}$$

U(1) ' vectormult.

H²(K3) ' hypermult.



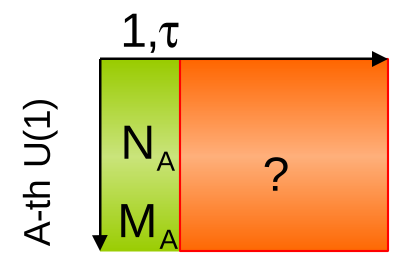
- "Flux" in type IIB:

$$W_{GVW} = (\int \Omega)_A \; (N_A + \tau M_A)$$

Linear in CY periods

U(1) ' vectormult.

?



- We have already matched the fields in the two dual theories in common effective action

-> can compute the type IIB potential dual to F-theory compactification

nice answer …

Generalized type II superpotential

$$W_{II} \sim \left(\int \Omega(Z)\right)_A \cdot F_{AI} \cdot \left(\int \Omega(Z')\right)_I$$

U(1) ' vectormult.

$H^3(Z')$ ' hypermult.

$$\rightarrow \quad W_{GVW} = \int \Omega \wedge G_3$$

$$\rightarrow \quad W_F = \Pi^A F_{AI} \tilde{\Pi}_I$$

$$\rightarrow \quad \ldots$$

- Bilinear in periods of a mirror pair (Z,Z') of CY 3-folds

- s Ω(Z) : Quantum periods on (Z,Z') defined by topological string

$$\int \Omega(Z') = \int_Z e^{B+iJ} + (\mathsf{WS\ instantons}) + (\mathsf{D\ instantons})$$

- Related to conjecture S-duality of topological strings *(Neitzke, Nekrasov, Ooguri, Vafa)*

- Depending on choice of “fluxes” -> GVW, F-theory, … much more

Generalized type II superpotential

W_{II} is the superpotential for type II compactification on a Generalized CY manifold (Hitchin)

F-theory on K3xK3
+ 2-form fluxes
$\Longleftrightarrow$
Type II strings on
GCY "Y"
(no branes)

Idea: Spontaneous supersymmetry breaking ->

Superpotential for type II on GCY Y can be computed in CY parents (Z,Z')

$$W_{II} \sim \left(\int \Omega(Z)\right)_A \cdot F_{AI} \cdot \left(\int \Omega(Z')\right)_I$$

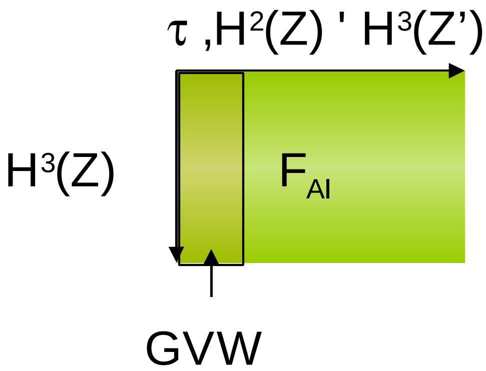
Flux: Vector bundles on $(M_Z, M_{Z'})$ $\xrightarrow{F_{AI}}$ Line bundle on M_Y
$W_{II}(Y)$

-The effective gauged supergravity is of the "gauged Heisenberg algebra" type described by D' Auria, Ferrara, Trigiante, Vaula, hep-th0410290

-W_{II} is in agreement with the general form for the potential on a GCY Y obtained by Louis, Grana, Waldram, hep-th 0505264

(duality -> can be computed on CY parents)

Generalized type II superpotential



$$W_{II} \sim \left(\int \Omega(Z)\right)_A \cdot \boxed{F_{AI}} \cdot \left(\int \Omega(Z')\right)_I$$

$$\sim \; W_{GVW} \; + \; \ldots$$

2) Describes BPS charges of a large class of domain walls with apparent T-duality group SL(b_2+1,Z)£ SL(b_3,Z); NS5 and D5 brane mix with new domain walls under T-duality

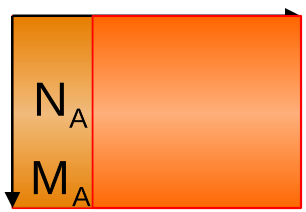
2) Depends on all moduli multiplets of the closed string type II compactification !

(Generically) all moduli fixed in the type II theory without branes …

VACUA I: Statistics

- GVW superpotential: $\longrightarrow$ $N_{vac}^{GVW} \sim \sqrt{M}^{2b_3}$ *Ashok, Douglas*

- Generalized superpotential W_{II}:



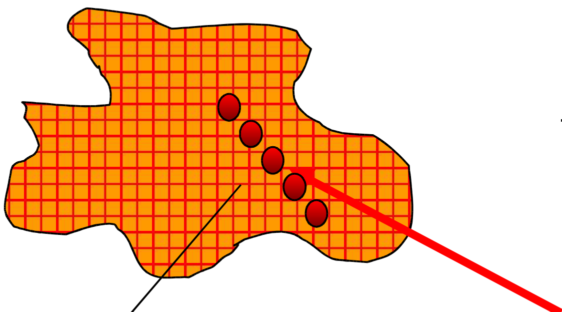
Integral lattice:

$$\begin{array}{ccc} b_2+1 & \times & b_3 \\ (H^2(Z),\tau) & & H^3(Z) \end{array} \longrightarrow N_{vac} \gg N_{vac}^{GVW}$$

SL(b_2+1,Z) x SL(b_3,Z)

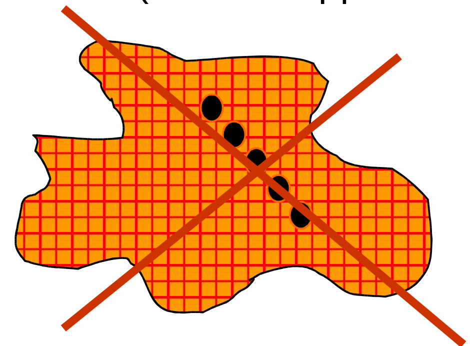
VACUA II: Poles and IR physics

- Perturbatively: oo # of N=1 Minkowski vacua for F-theory on K3xK3 (no scale approximation)

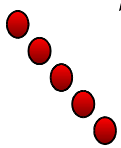


Delete strongly coupled regions

(perturbative approx. meaningless)



- Finite # of Poles of " instanton prefactor" $\Delta(\phi)$
< -> strongly coupled IR physics



Weakly coupled vacua
Perturbative approximation meaningful

Destabilized by instantons

Strongly coupled vacua (computable)
Perturbative approximation meaningless

Stabilized by strong IR effects

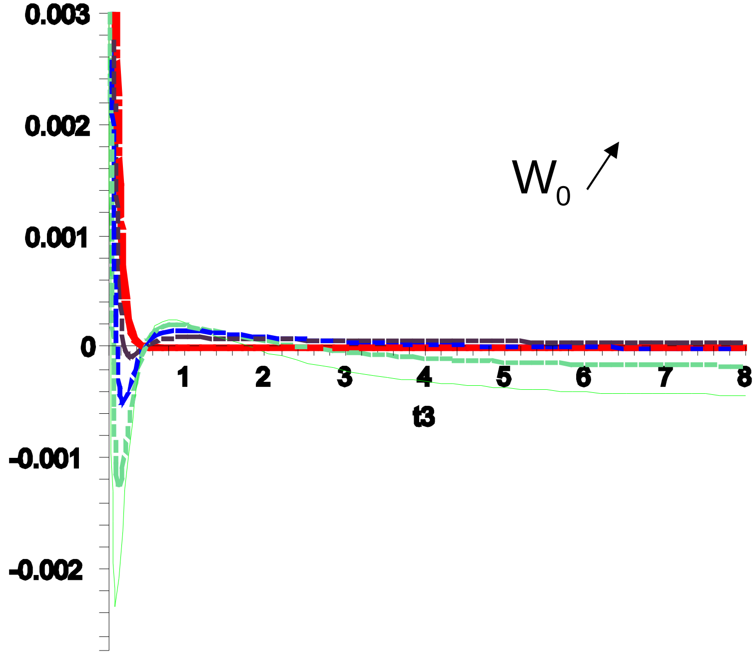
Taking into account instanton effects we get a very different structure:

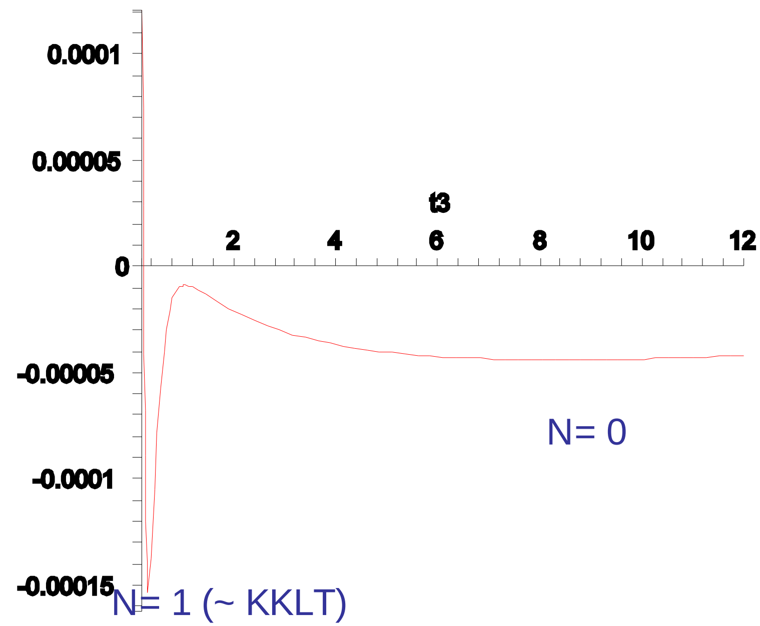
Perturbative approx.:	→	Non-perturbatively:
Infinite # of "random" vacua		Finite # of "semi-realistic" vacua

VACUA III: Quantitative results

- Can compute infinite instanton sums to high precision at all points in moduli (also small volume)

- Rich structure of supersymmetric and non-supersymmetric vacua, depending on flux





- Non-supersymmetric vacua similar to *Balasubramanian et. al.*
- Playground for study of “semi-realistic” string vacua at generic, small volumes

“Flux”
vacua

String vacua with spontaneous susy breaking

- Many more possibilties
- Often exactly computable
- All moduli can be fixed by “flux”
- Quantum effects are essential