

ANOMALIES, CHERN-SIMONS TERMS AND $Z, Z' \rightarrow \gamma\gamma$

background: [hep-th/0204153](#) [hep-th/0210263](#)

with P. Anastasopoulos, M. Bianchi, E. Dudas [to appear](#)

with P. Anastasopoulos, C. Coriano, N. Irges [to appear](#)

Plan of the presentation

- Introduction
- The content of this lecture
- Anomalies and anomalous U(1)s
- The presence of non-anomalous U(1)s
- Comments
- The general case
- The stringy origin
- Low string scale orientifold vacua
- Models $mLSO_A$ and $mLSO_{A'}$
- Models $mLSO_B$ and $mLSO_{B'}$
- The anomalous U(1)s
- EW Symmetry breaking
- Z-Z' mixing
- Signals at colliders
- Conclusions

Introduction

- The main plus of string theory is that it provides a perturbative theory of quantum gravity together with other interactions.
 - The theory also provides many classes of potentially realistic vacua , although some of the popular ones from the days of grand unification are so far conspicuously absent.
 - Classes of vacua appearing in perturbative string theory constructions provided novel effective field theories and new ideas on the extension of the SM.
 - Most radical departures appear in orientifold vacua. Some novel features of these vacua are:
- ♣ Product groups including $U(N)$, $Sp(N)$ and $O(N)$ groups but no exceptional groups.

♠ A very restricted set of representations, namely bi-fundamentals, symmetric, antisymmetric and adjoint representations. Spinor representations are conspicuously absent (perturbatively). This is due to the fact that gauge degrees of freedom arise from Chan-Paton factors.

◇ There are many $U(1)$ symmetries that are superficially anomalous. Their anomalies are cancelled by the Green-Schwarz-Sagnotti mechanism using couplings to various RR forms.

♠ Although there are no exact global symmetries, approximate global symmetries are possible (with an arbitrary degree of accuracy) emerging as broken anomalous $U(1)$ gauge symmetries. This is due to the fact that at the orientifold point of the moduli space, the associated Higgs potential is quartic and spontaneous breakdown of the associated global symmetry does not occur.

♡ Unlike the heterotic string, orientifold vacua allow the possibility (modulo hierarchy questions) that the string scale is far below the four-dimensional Planck scale. In this case many features of the vacua may be accessible to experiments.

The content of this lecture

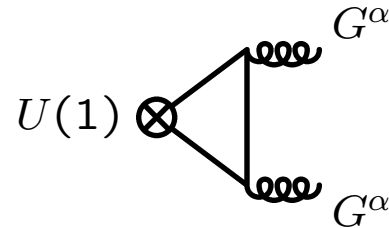
- In orientifold vacua, with several anomalous $U(1)$ s, 4-d anomalies are cancelled via appropriate couplings to axions.
- If some linear combinations of $U(1)$ s are non-anomalous (as we expect for the hypercharge in orientifold realizations of the SM) then, there are additional Chern-Simons couplings necessary for the cancellation of the anomalies.
- Such couplings can be computed by a stringy one-loop computation or inferred from anomaly cancellation.
- In the context of Low-Scale Orientifold Models (L.O.M.) the presence of such couplings has dramatic experimental consequences. It induces couplings $Z \rightarrow \gamma\gamma$ and $Z' \rightarrow \gamma\gamma$ that may be dominant at LHC masking the one-loop $Higgs \rightarrow \gamma\gamma$ signal.

Anomalies and anomalous U(1)s

We discuss the simplest example in 4d: A U(1) gauge symmetry that has a mixed triangle anomaly

(e.g. $\zeta = \text{Tr}[QT^a T^a] \neq 0$) with a non-abelian group.

The one-loop triangle diagram is non-zero



It induces a non-invariance to U(1) gauge transformations

$$A_\mu \rightarrow A_\mu + \partial_\mu \epsilon \quad , \quad \delta L_{1\text{-loop}} = \epsilon \zeta \text{Tr}[G \wedge G]$$

This is cancelled by a non-invariance of the classical (tree-level action).

$$\mathcal{L}_{\text{class}} \sim -\frac{1}{4g^2} F_{\mu\nu}^2 + \frac{1}{2} (\partial_\mu a + M A_\mu)^2 + \frac{\zeta}{M} a \text{Tr}[G \wedge G]$$

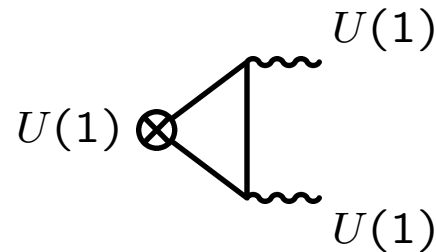
The axion now transforms as

$$a \rightarrow a - M \epsilon \quad , \quad \mathcal{L}_{\text{class}} \rightarrow \mathcal{L}_{\text{class}} - \zeta \epsilon \text{Tr}[G \wedge G]$$

The anomaly is cancelled.

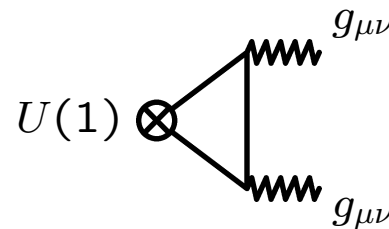
Comments

♣ The $U(1)^3$ anomaly associated to $\tilde{\zeta} = \text{Tr}[Q^3] \neq 0$ comes from the diagram



It is cancelled by an extra term $\delta L = \frac{\tilde{\zeta}}{M} a F \wedge F$

♠ The mixed gauge-gravitational anomaly associated to $\hat{\zeta} = \text{Tr}[Q] \neq 0$ is coming from the diagram



It is cancelled by the extra term $\delta L = \frac{\hat{\zeta}}{M} a R \wedge R$

◇ The axion that mixes with the gauge boson is typically a bulk axion emerging from the twisted RR sector. Rarely, it can be an untwisted RR axion as in the case of intersecting branes on T^6 .

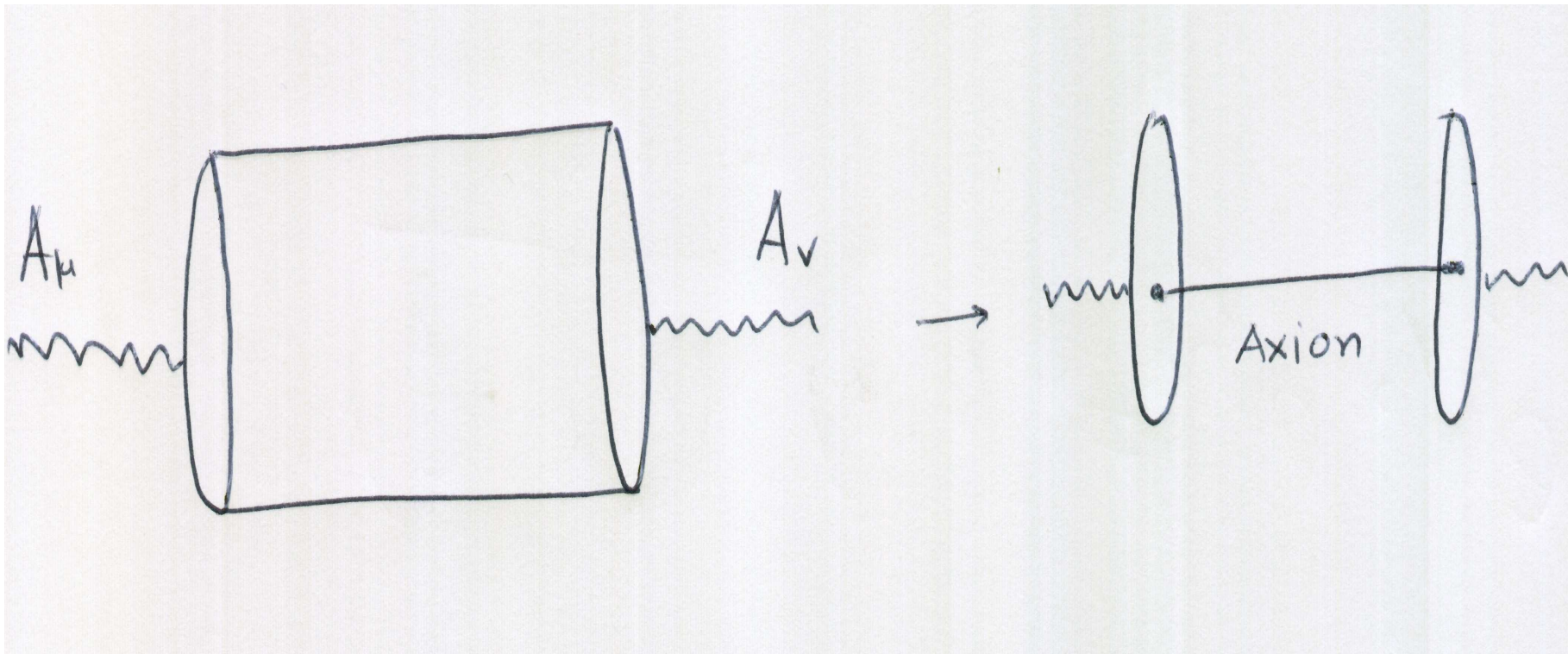
Comments

♥ The

$$\frac{1}{2}(\partial_\mu a + M A_\mu)^2$$

term in the action , mixing the U(1) gauge boson and the axion gives mass to the anomalous gauge-boson and breaks the U(1) gauge symmetry.

♣ The ultraviolet mass M can be computed from a string one-loop diagram and is given by an UV contact term.



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If the gauge boson belongs to a D-brane, D , and the associated axion is localized on a orbifold plane P , the mixing term scales as

$$M^2 \sim M_s^2 \frac{V_{D \cap P}}{V_{P-D \cap P}}$$

$$M_{\text{phys}}^2 = g^2 M^2 \sim \frac{M_s^2}{V_{D-D \cap P} V_{P-D \cap P}}$$

- The D-term-like potential is of the form

$$V \sim \left(s + \sum_i q_i |\phi_i|^2 \right)^2$$

where s is a bulk modulus. In SUSY Theories it is the chiral partner of the axion “eaten up” by the anomalous $U(1)$ gauge boson. If $\langle s \rangle = 0$, the global $U(1)$ symmetry remains intact. This happens at the orientifold point

Poppitz

The presence of non-anomalous U(1)s

Let us now consider another simple example that apart from the standard anomalous U(1) involves another "non-anomalous" U(1) Y_μ . By this we mean:

$$\text{Tr}[Y] = \text{Tr}[Y^3] = \text{Tr}[Y T^a T^a] = 0$$

However, the following mixed anomalies are non-zero (I ignore the gravitational ones $\sim \text{Tr}[Q]$)

$$\text{Tr}[Q^3] = c_3 \quad , \quad \text{Tr}[Q^2 Y] = c_2 \quad , \quad \text{Tr}[Q Y^2] = c_1 \quad , \quad \text{Tr}[Q T^a T^a] = \xi$$

Under a general gauge transformation

$$A_\mu \rightarrow A_\mu + \partial_\mu \epsilon \quad , \quad Y_\mu \rightarrow Y_\mu + \partial_\mu \zeta$$

$$\begin{aligned} \delta L_{1\text{-loop}} = & \epsilon \left[\frac{c_3}{3} F^A \wedge F^A + c_2 F^A \wedge F^Y + c_1 F^Y \wedge F^Y + \xi \text{Tr}[G \wedge G] \right] + \\ & + \zeta \left[c_2 F^A \wedge F^A + c_1 F^A \wedge F^Y \right] \end{aligned}$$

To cancel it we write the general anomaly cancelling terms as before

$$\mathcal{L}_{\text{class}} \sim -\frac{1}{4g^2}(F^A)^2 - \frac{1}{4g_Y^2}(F^Y)^2 + \frac{1}{2}(\partial_\mu a + M A_\mu)^2 +$$

$$+ D_0 a \text{Tr}[G \wedge G] + D_1 a F^A \wedge F^A + D_2 a F^A \wedge F^Y + D_3 a F^Y \wedge F^Y$$

- There is no mixing of Y_μ with an axion (unbroken Y-symmetry)
- The action above is Y-gauge invariant and therefore not enough! **We must have Y-non-invariant terms.**

$$L_{CS} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} [D_4 Y_\mu CS(A)_{\nu\rho\sigma} - D_5 A_\mu CS(Y)_{\nu\rho\sigma}]$$

$$= [D_4 Y \wedge A \wedge F^A - D_5 A \wedge Y \wedge F^Y]$$

Now we may cancel the anomalies to obtain

$$D_0 = \xi \quad , \quad D_1 = \frac{c_3}{3} \quad , \quad D_2 = 2c_2 \quad , \quad D_3 = 2c_1 \quad , \quad D_4 = c_2 \quad , \quad D_5 = c_1$$

Comments

- There are no ambiguities. The anomalies uniquely fix the CS terms in the effective action.

Can this situation arise in string theory?

- Anomalous $U(1)$ symmetries are generic.
- Non-anomalous linear combinations are also abundant.
- Some non-anomalous linear combinations are massive

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- Many non-anomalous linear combinations are massless and therefore realize the previous setup. This is quite generic

- An explicit example is the Z_6 orientifold

§ This orbifold has D9 and D5 branes

§ The gauge group is $U(6) \times U(6) \times U(4)$ from D9 branes and a similar copy from D5 branes.

There are therefore 6 U(1)s.

§ Out of these three are free of four-dimensional anomalies (mixed abelian-non-abelian, abelian, mixed gravitational).

§ Out of the three non-anomalous U(1)s, only one is massless. It is like the Y_μ gauge symmetry. The other two have non-zero masses because of uncanceled 6d anomalies. Their mass is $\sim V$ as $V \rightarrow 0$

P. Anastasopoulos

§ Therefore this situation realizes the simple situation presented previously.

The general case

We have N U(1)s A_μ^i .

$$L = - \sum_i \frac{1}{4g_i^2} (F^i)^2 - \frac{1}{2} \sum_I (\partial_\mu a^I + \sum_i B_i^I A_\mu^i)^2 + \\ + \sum_{I,j,k} C_{Ijk} a^I F^j \wedge F^k + \sum_{Ia} D_{Ia} a^I \text{Tr}[G^a \wedge G^a] + E_{ijk} S^{ijk}$$

$$S^{ijk} \equiv \int \epsilon^{\mu\nu\rho\sigma} A_\mu^i A_\nu^j F_{\rho\sigma}^k, \quad S^{ijk} = -S^{jik}$$

Under U(1) gauge transformations

$$\delta S^{ijk} = \int \epsilon^j F^i \wedge F^k - \epsilon^i F^j \wedge F^k$$

This implies that $E_{ijk} S^{ijk}$ with E_{ijk} completely antisymmetric is gauge invariant. By integrating by parts we obtain that the cyclic sum vanishes $S^{ijk} + S^{kij} + S^{jki} = 0$. Therefore if E_{ijk} is completely antisymmetric, $E_{ijk} S^{ijk}$ vanishes since it is a boundary term.

$$E \sim \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} \oplus \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array}$$

We may therefore take

$$E \sim \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}$$

Under gauge transformations

$$\delta L_{1-loop} = \sum_i \epsilon^i \left[A t_{ijk} F^j \wedge F^k + B t_{ia} \text{Tr}[G^a \wedge G^a] \right]$$

$$t_{ijk} = \text{Tr}[Q_i Q_j Q_k] \quad , \quad t_{ia} = \text{Tr}[Q_i (T^A T^A)_a]$$

where $(T^A T^A)_a$ is the quadratic Casimir of the non-abelian group G_a . The axions transform as

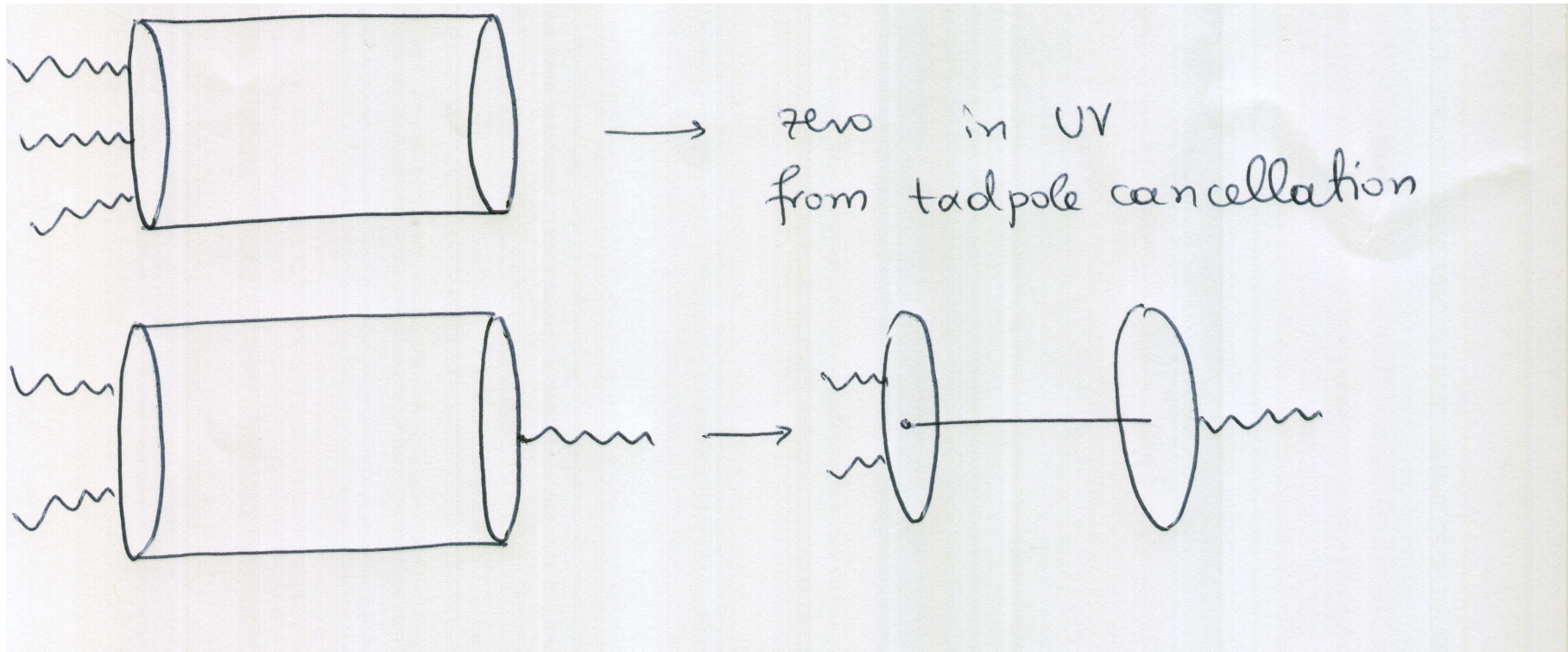
$$a^I \rightarrow a^I - B_i^I \epsilon^i$$

t_{ijk} is completely symmetric, C_{Ijk} is symmetric in jk . Cancelling the tree-level variation against the one-loop anomaly we finally obtain

$$\sum_I B_i^I C_{Ijk} + 2E_{ijk} = A t_{ijk} \quad , \quad \sum_I B_i^I D_{Ia} = B t_{ia}$$

The stringy origin

The CS terms can be alternatively calculated from an appropriate one-loop open amplitude



Low string scale orientifold vacua (LSO)

The previous discussion becomes interesting when the massive anomalous $U(1)$ gauge bosons are sufficiently close to experiment. This happens when the string scale is low ($\sim \text{TeV}$)

We take two of the six compact directions to be large. We wrap only the $U(1)'$ brane around them.

The charge assignments for the SM particles are parameterized as:

SM particle	$U(1)_3$	$U(1)_2$	$U(1)$	$U(1)'$
$Q(\mathbf{3}, \mathbf{2}, +\frac{1}{6})$	$+1$	w	0	0
$u^c(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$	-1	0	a_1	a_2
$d^c(\bar{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})$	-1	0	b_1	b_2
$L(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	0	$+1$	c_1	c_2
$e^c(\mathbf{1}, \mathbf{1}, +1)$	0	0	d_1	d_2
$H_u(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$	0	$-w$	c_3	c_4
$H_d(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	0	$-w$	c_5	c_6
$\nu^c(\mathbf{1}, \mathbf{1}, 0)$	0	0	d_1	d_2

♣ The charges are assigned using the principle that each end-point has charges ± 1

♣ Baryon number is a gauged symmetry namely, $U(1)_3$

♣ We must require that Lepton number is also a good symmetry

♣ The hypercharge must be a linear combination of the 4 $U(1)$ factors:

$$Y = k_3 Q_3 + k_2 Q_2 + k_1 Q_1 + k'_1 Q'_1$$

Since $U(1)'$ wraps large dimensions, to avoid a tiny α_Y we must take $k'_1 = 0$.

After taking into account also the matching of the gauge coupling constants there four possible configurations

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Anomalies and $Z \rightarrow \gamma\gamma$, E. Kiritsis

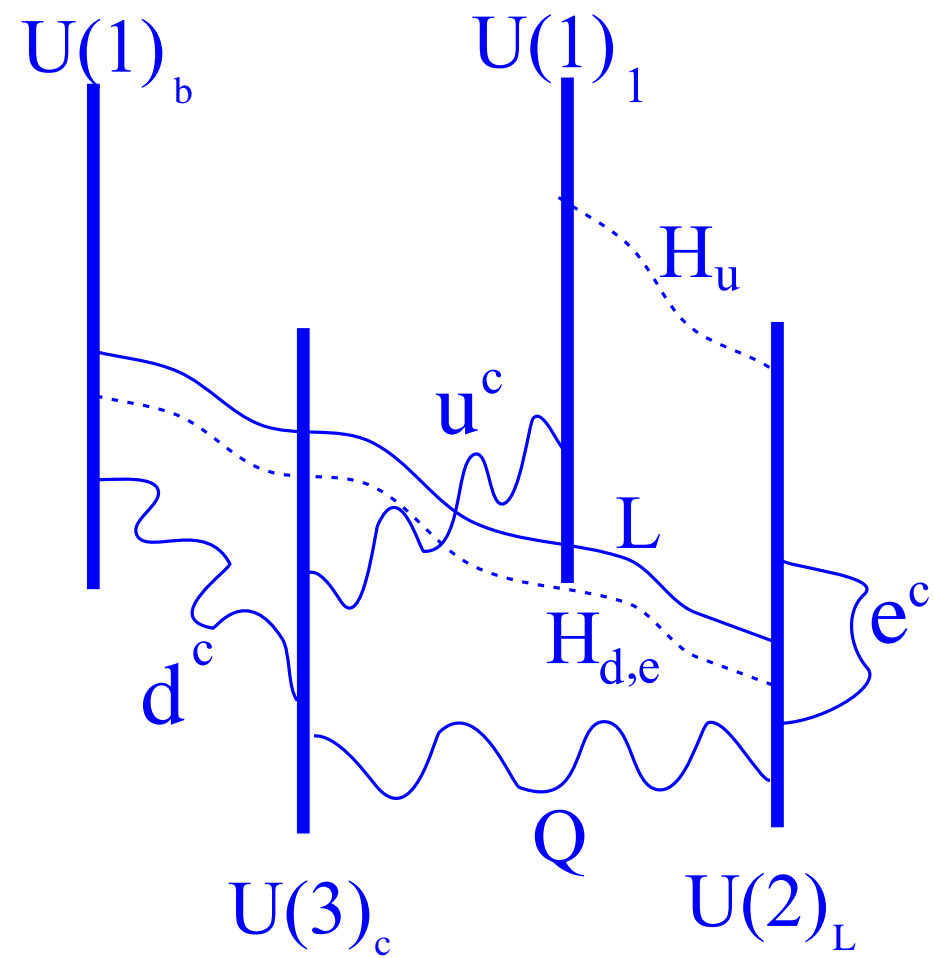
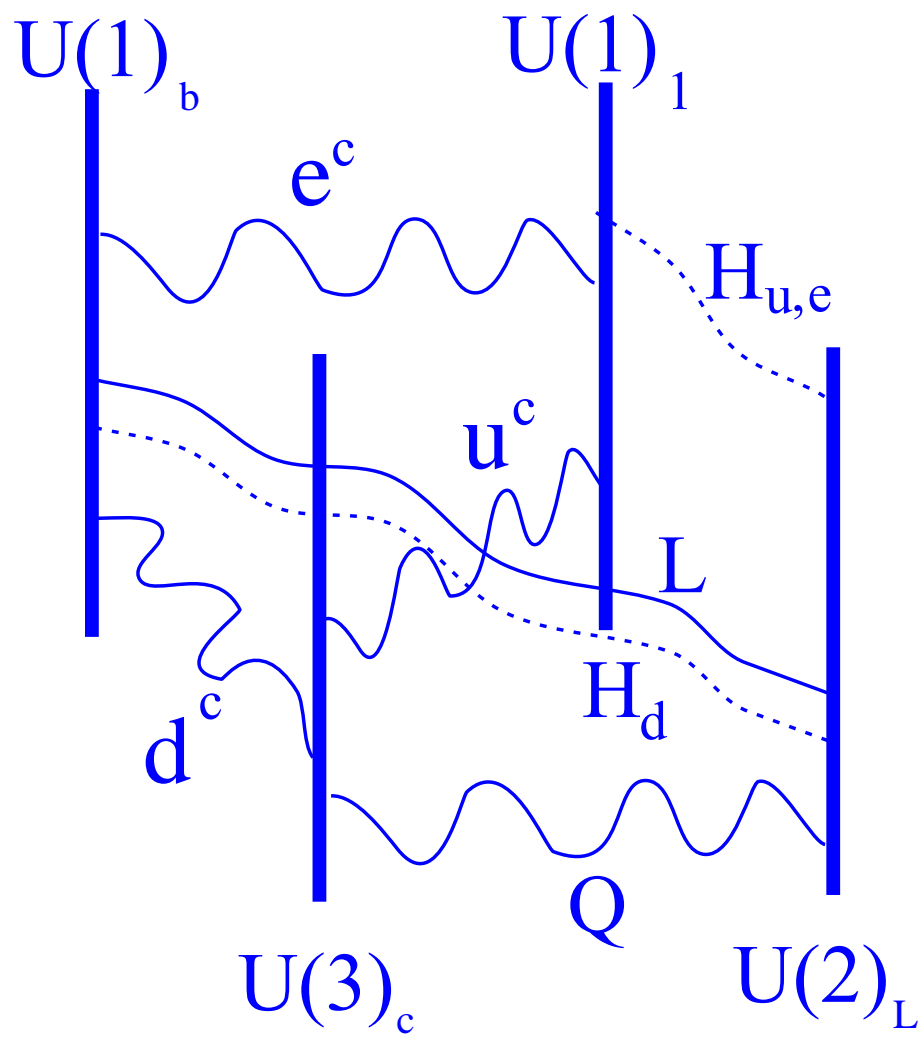
Models $m\text{LSO}_A$ and $m\text{LSO}_{A'}$

SM particle	$U(1)_3$	$U(1)_2$	$U(1)$	$U(1)'$
$Q(3, 2, +\frac{1}{6})$	+1	-1	0	0
$u^c(\bar{3}, 1, -\frac{2}{3})$	-1	0	-1	0
$d^c(\bar{3}, 1, +\frac{1}{3})$	-1	0	0	-1
$L(1, 2, -\frac{1}{2})$	0	+1	0	-1
$e^c(1, 1, +1)$	0	0(2)	1(0)	1(0)
$H_u(1, 2, +\frac{1}{2})$	0	1	1	0
$H_d(1, 2, +\frac{1}{2})$	0	-1	0	-1
$\nu^c(1, 1, 0)$	0	0	0	± 2

$$Y = -\frac{1}{3}Q_3 - \frac{1}{2}Q_2 + Q_1$$

Lepton Number $L = \frac{1}{2}(Q_3 + Q_2 - Q_1 - Q'_1)$

Peccei – Quinn $PQ = -\frac{1}{2}(Q_3 - Q_2 - 3Q_1 - 3Q'_1)$



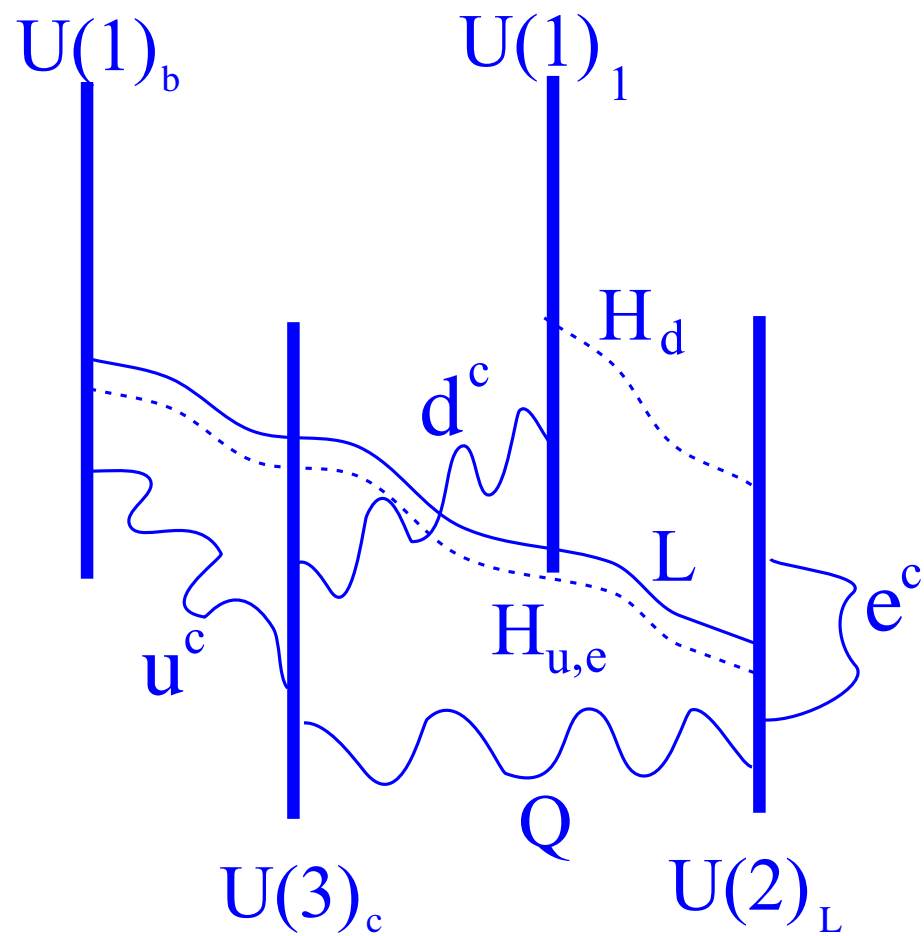
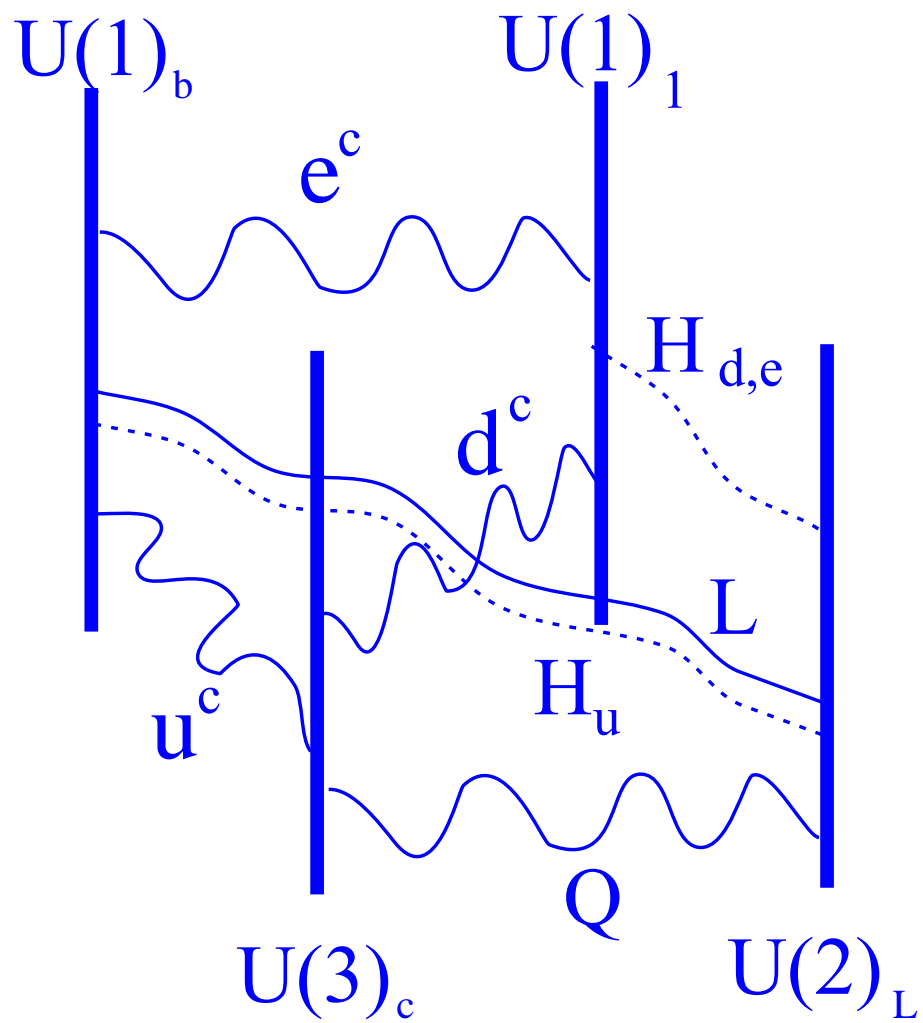
Models $m\text{LSO}_B$ and $m\text{LSO}_{B'}$

SM particle	$U(1)_3$	$U(1)_2$	$U(1)$	$U(1)'$
$Q(\mathbf{3}, \mathbf{2}, +\frac{1}{6})$	+1	-1	0	0
$u^c(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$	-1	0	0	1
$d^c(\bar{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})$	-1	0	1	0
$L(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	0	+1	0	-1
$e^c(\mathbf{1}, \mathbf{1}, +1)$	0	0(2)	1(0)	1(0)
$H_u(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$	0	-1	0	-1
$H_d(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$	0	1	1	0
$\nu^c(\mathbf{1}, \mathbf{1}, 0)$	0	0	0	± 2

$$Y = \frac{2}{3}Q_3 - \frac{1}{2}Q_2 + Q_1$$

$$\text{Lepton Number } L = -\frac{1}{2}(Q_3 - Q_2 + Q_1 + Q'_1)$$

$$\text{Peccei - Quinn } PQ = \frac{1}{2}(-Q_3 + 3Q_2 + Q_1 + Q'_1)$$



The anomalous U(1)s

In all four models above, we can label the four U(1)s as:

Y= Hypercharge

B= Baryon Number

L= Lepton Number

PQ= Peccei-Quinn-like symmetry

They have a non-trivial anomaly structure as in the cases we described earlier. We will therefore have CS terms of the structure described

$$L_{CS} = E_{ijk} A^i \wedge A^j \wedge F^k, \quad i, j, k \in (Y, B, L, PQ)$$

EW Symmetry breaking

The two EW Higgses are charged under Y and PQ but not B and L.

When EW breaking happens, both Y and PQ are spontaneously broken.

- There must be PQ violating terms in the potential otherwise a massless Goldstone boson (axion) remains with couplings that cannot be made small. This can be achieved by moving off the orientifold point.

There are in general two origins for the mass of the various gauge bosons:

♣ The UV mass-matrix of the anomalous U(1)s coming from

$$\sum_I (\partial_\mu a^I + B_i^I A_\mu^i)^2 \quad , \quad M_{ij}^2 = \sum_I B_i^I B_j^I$$

It can be obtained from a string calculation. Its eigenvalues are typically a half– a tenth of the string scale.

♠ The Higgs expectation value $v \simeq 100 - 200$ GeV

Z-Z' mixing

After the Higgs mechanism, the three mass eigenstates, the photon A , the Z^0 , and the PQ-related Z' -boson, are specific linear combinations of W^3 , Y and PQ gauge bosons. Inversely

$$\begin{pmatrix} W^3 \\ Y \\ PQ \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \begin{pmatrix} A \\ Z^0 \\ Z' \end{pmatrix}$$

We have

$$c_{11}, c_{12}, c_{21}, c_{22}, c_{33} \sim \mathcal{O}(1) \quad , \quad c_{13}, c_{23}, c_{31}, c_{32} \sim \mathcal{O}\left(\frac{M_Z}{M_s}\right) < 10^{-4}$$

The ρ -parameter, $\rho = \frac{M_W^2}{M_Z^2 \sin^2 \theta_W}$, is no more equal to the standard model value

$$\frac{\Delta\rho}{\rho_0} \sim \frac{M_Z}{M_s} < 6 \times 10^{-4}$$

and there are small modifications of the Z^0 couplings to the fermions.

Ghilencea, Ibanez, Irges, Quevedo

On the other hand the B and L gauge bosons are not affected by the Higgs mechanism. They give two extra massive Z' gauge bosons with masses $\sim M_s$.

Consider now the various anomaly cancelling Chern-Simons-like terms.

$$PQ \wedge Y \wedge dY \longrightarrow \left\{ \begin{array}{llll} Z^0 \wedge A \wedge dA & \Rightarrow & Z^0 \rightarrow \gamma\gamma & \sim \mathcal{O}\left(\frac{M_Z}{M_s}\right), \\ A \wedge Z^0 \wedge dZ^0 & \Rightarrow & Z^0 \rightarrow Z^0\gamma & \sim \mathcal{O}\left(\frac{M_Z}{M_s}\right), \\ Z' \wedge A \wedge dA & \Rightarrow & Z' \rightarrow \gamma\gamma & \sim \mathcal{O}(1), \\ Z' \wedge Z^0 \wedge dZ^0 & \Rightarrow & Z' \rightarrow Z^0 Z^0 & \sim \mathcal{O}(1), \\ Z' \wedge Z^0 \wedge dA & \Rightarrow & Z' \rightarrow Z^0\gamma & \sim \mathcal{O}(1) \end{array} \right.$$

Similarly for the other relevant CS term $Y \wedge PQ \wedge dPQ$.

Signals at colliders

- The three massive Z' associated to PQ, B, L have the standard Z' related couplings to the fermions.
- They can be seen in LHC if their masses are lower than 5 TeV in $pp \rightarrow Z' \rightarrow \ell^+ \ell^-$. More detailed info can be obtained from the Forward-Backward asymmetry for masses up to 2 TeV.

Dittmar, Nicollrat, Djouadi

- The current experimental limit $\Gamma(Z^0 \rightarrow \gamma\gamma)/M_Z^0 \leq 5 \times 10^{-7}$ puts a (mild) lower bound on M_s from the anomaly-induced $Z^0 \rightarrow \gamma\gamma$ vertex. It will be interesting if this signal can be seen directly.
- There is also a new vertex that will give two Z^0 s in the DY channel $pp \rightarrow \gamma \rightarrow Z^0 Z^0$. For LHC energies this is of the same order of magnitude as the $pp \rightarrow Z^0 \rightarrow \gamma\gamma$ process.

The Z' gauge bosons have also non-standard anomaly related couplings that distinguish them from other Z' models.

- There are $O(1)$ couplings that provide new production channels apart from DY, namely

$$pp \rightarrow Z' \rightarrow \gamma\gamma \quad , \quad pp \rightarrow Z' \rightarrow \gamma Z^0 \quad , \quad pp \rightarrow Z' \rightarrow Z^0 Z^0$$

- Moreover, the first signal is expected to be stronger than the $Higgs \rightarrow \gamma\gamma$ signal, that is one of the main channels for the discovery of the Higgs

Conclusions

- ♣ Anomalous U(1) gauge bosons are a generic prediction of orientifold vacua.
- ♣ If the string scale is low (few TeV region) such gauge bosons become the tell-tale signals of such vacua.
- ♣ The charge structure of such vacua is essentially fixed. This fixes all the minimal couplings of Z'_s
- ♣ Anomaly related CS-like couplings produce new signals that distinguish such models from any other Z' -model.
- ♣ such signals may be visible in LHC.