Towards Precision SUSY Studies at Colliders

W. Hollik, MPI München

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- Introduction
- Precision studies with standard particles
- Precision studies with Higgs bosons
- Precision studies with SUSY particles

Precision analysis required for

- Indirect tests of the MSSM
  $\rightarrow$ virtual SUSY effects in precision observables

- Precision studies for SUSY particles
  $\rightarrow$ determination of masses & couplings
  $\rightarrow$ reconstruction of model parameters

- Direct versus indirect tests
  $\rightarrow$ precision observables for precisely measured SUSY parameters
  $\rightarrow$ consistency check

Processes with external

(i) standard particles

(ii) Higgs bosons, especially light Higgs h^0

(iii) SUSY particles
– the chargino and neutralino sector
– the sfermion sector

recent review:

Heinemeyer, WH, Weiglein, hep-ph/0412214

(expected) experimental precision

error for	LEP/Tev	Tev/LHC	LC	GigaZ
M_W [MeV]	33	15	15	7
$\sin^2\theta_{\rm eff}$	0.00017	0.00021		0.000013
$m_{\rm top}$ [GeV]	4.3	2	0.2	0.13
$M_{\rm Higgs}$ [GeV]	–	0.1	0.05	0.05

together with

$\delta M_Z = 2.1$ MeV (LEP)

$\delta G_F/G_F = 1 \cdot 10^{-5}$ (μ lifetime)

Detailed analysis for SPS1a benchmark scenario: potential of LHC (300 fb^{-1}) alone and LHC $+$ LC

	LHC	LHC+LC
$\Delta m_{\tilde{\chi}_1^0}$	4.8	0.05 (input)
$\Delta m_{\tilde{l}_R}$	4.8	0.05 (input)
$\Delta m_{\tilde{\chi}_2^0}$	4.7	0.08
$\Delta m_{\tilde{q}_L}$	8.7	4.9
$\Delta m_{\tilde{q}_R}$	11.8	10.9
$\Delta m_{\tilde{g}}$	8.0	6.4
$\Delta m_{\tilde{b}_1}$	7.5	5.7
$\Delta m_{\tilde{b}_2}$	7.9	6.2
$\Delta m_{\tilde{l}_L}$	5.0	0.2 (input)
$\Delta m_{\tilde{\chi}_4^0}$	5.1	2.23

LHC+LC accuracy limited by LHC jet energy scale resolution

SPS 1a benchmark scenario:

favorable scenario for both LHC and LC

$\Rightarrow$ LC input improves accuracy significantly

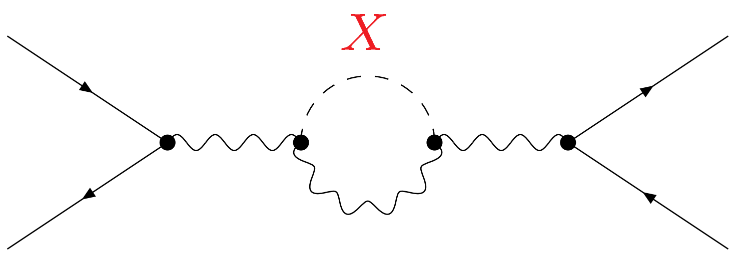
Comparison of electro-weak precision observables with theory:

EW Precision data: $\Delta\rho, \Delta r, M_W, \sin^2\theta_{\text{eff}}$	$\leftrightarrow$	Theory: SM, MSSM , . . .

$\Downarrow$

Test of theory at quantum level:

Sensitivity to loop corrections



$\Downarrow$

sensitivity to internal particles (X)

Precision observables: M_W, $\sin^2\theta_{\text{eff}}$, m_h, $(g-2)_\mu$, b physics, ...

1.) Theoretical prediction for M_W in terms of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\,\alpha}{\sqrt{2}\,G_\mu}\left(\frac{1}{1-\Delta r}\right)$$

$\Updownarrow$

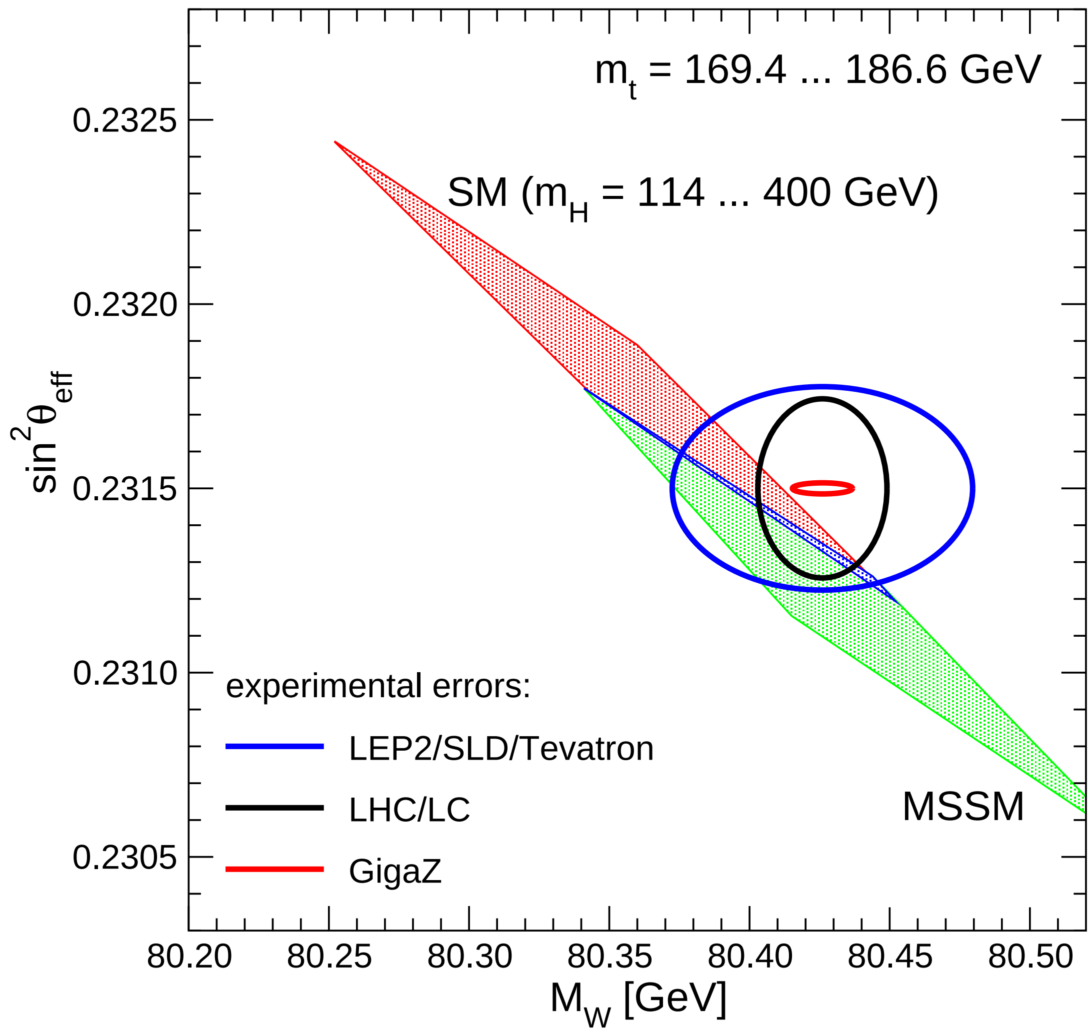
loop corrections

2.) Effective mixing angle:

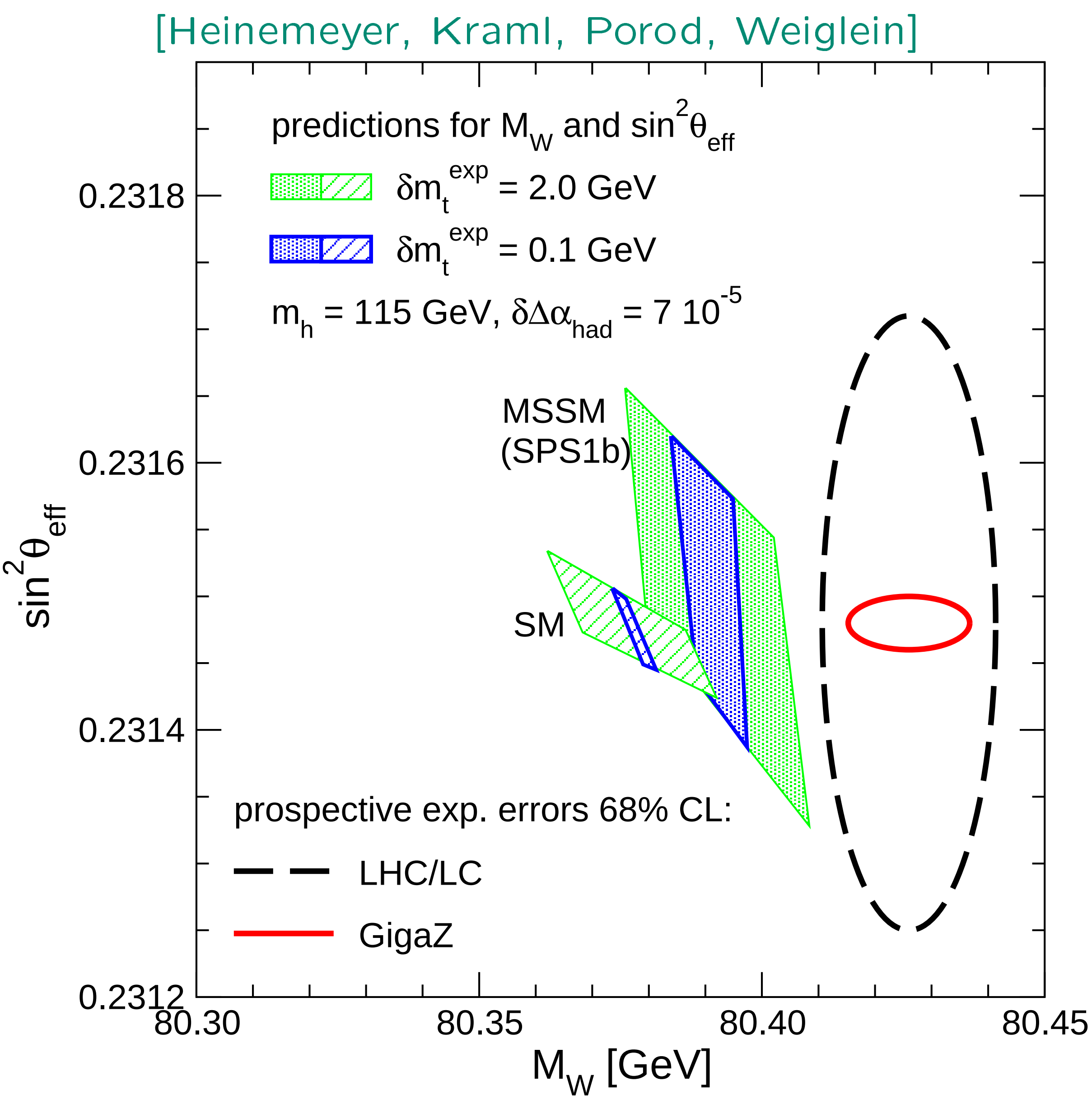
$$\sin^2\theta_{\text{eff}} = \frac{1}{4\,|Q_f|}\left(1 - \frac{\text{Re}\, g_V^f}{\text{Re}\, g_A^f}\right)$$

Higher order contributions:

$$g_V^f \to g_V^f + \Delta g_V^f, \quad g_A^f \to g_A^f + \Delta g_A^f$$



m_t = 169.4 ... 186.6 GeV
SM (m_H = 114 ... 400 GeV)
MSSM
experimental errors:
LEP2/SLD/Tevatron
LHC/LC
GigaZ
$\sin^2\theta_{eff}$
0.2325
0.2320
0.2315
0.2310
0.2305
80.20
80.25
80.30
80.35
80.40
80.45
80.50
M_W [GeV]


[Heinemeyer, Kraml, Porod, Weiglein]
predictions for M_W and sin^2θ_eff
δm_t^exp = 2.0 GeV
δm_t^exp = 0.1 GeV
m_h = 115 GeV, δΔα_had = 7 10^-5
MSSM
(SPS1b)
SM
prospective exp. errors 68% CL:
LHC/LC
GigaZ
0.2318
0.2316
0.2314
0.2312
80.30
80.35
80.40
80.45
M_W [GeV]
sin^2θ_eff

Models of SUSY breaking

generic MSSM: 105 parameters (masses, mixing angles, phases)

reduced to few parameters in specific models

mSUGRA: m_0, $m_{1/2}$, A_0, $\tan\beta$, $\mathrm{sign}(\mu)$

GMSB: M_{mess}, N_{mess}, $\tan\beta$, $\mathrm{sign}(\mu)$

AMSB: m_{aux}, m_0, $\tan\beta$, $\mathrm{sign}(\mu)$

$\rightarrow$ mass parameters at the electroweak scale
$(M_1, M_2, M_3, \mu, M_{\tilde{f}_{L,R}}, \ldots)$

Benchmark scenarios

"Snowmass points and slopes" (SPS),
hep-ph/0202233

examples (mSUGRA):

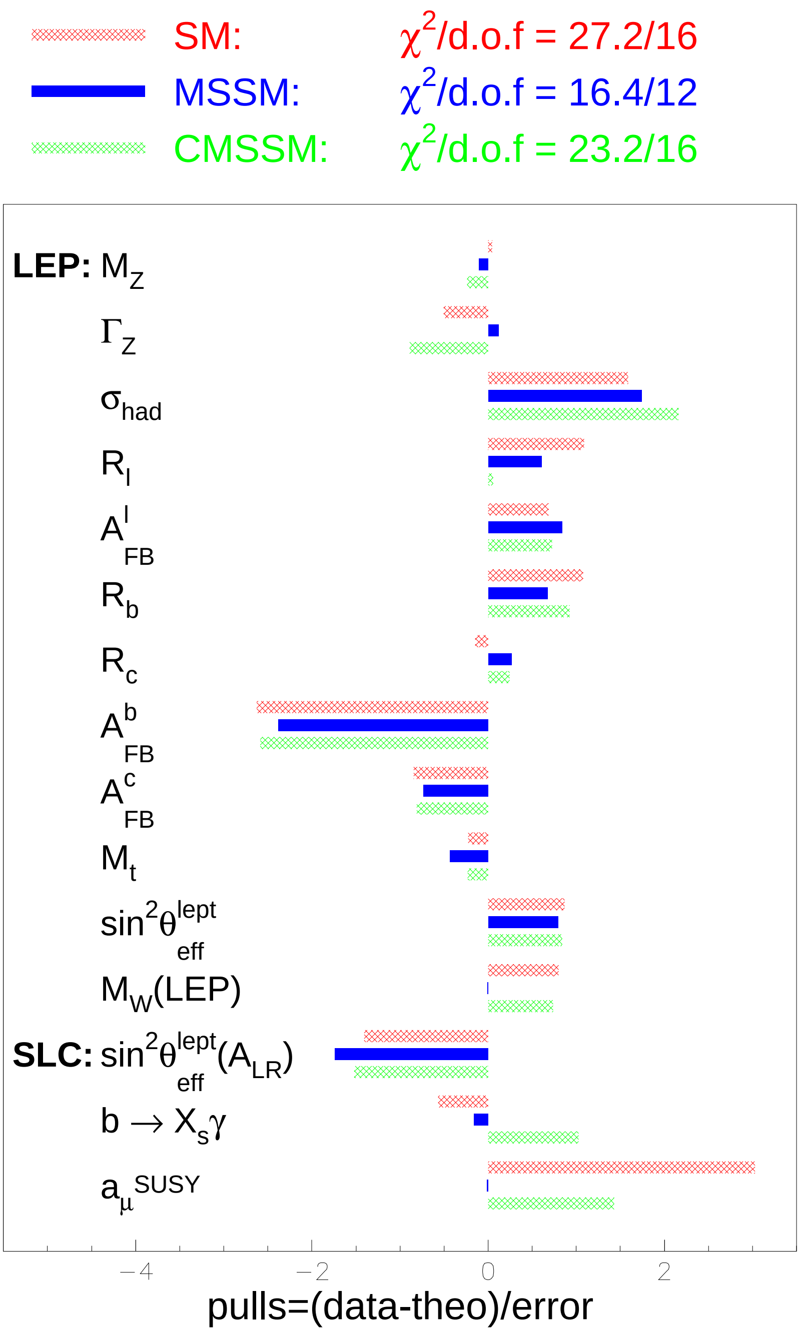
- SPS1a: $m_0 = 100$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -100$,
  $\tan\beta = 10$, $\mu > 0$.

- SPS1b: $m_0 = 200$ GeV, $m_{1/2} = 400$ GeV, $A_0 = 0$,
  $\tan\beta = 30$, $\mu > 0$.

Global fits in the MSSM

[de Boer, Dabelstein, WH, Mösle, Schwickerath]

[de Boer, Sander]



The Higgs sector of the MSSM

- Two $SU(2) \times U(1)$ doublets: $H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$

$$H_i^0 = \frac{v_i + S_i + i\,P_i}{\sqrt{2}} \qquad\qquad \tan\beta = \frac{v_2}{v_1}$$

- The soft SUSY-breaking mass terms for H_1^0 and H_2^0 are responsible for electroweak symmetry breaking (EWSB):

$$\begin{aligned} V_{\rm tree} &= (m_{H_1}^2 + \mu^2)\,|H_1^0|^2 + (m_{H_2}^2 + \mu^2)\,|H_2^0|^2 \\ &+ B\,\left(H_1^0 H_2^0 + \text{h.c.}\right) + \frac{1}{8}(g^2 + g'^2)\,\left(|H_1^0|^2 - |H_2^0|^2\right)^2 \end{aligned}$$

- Five physical states: $h\,,\ H\,,\ A^0\,,\ H^+\,,\ H^-$

- Tree–level mass matrix for the CP–even sector:

$$\left(\mathcal{M}_S^2\right)^{\rm tree} = \begin{pmatrix} m_Z^2\,c_\beta^2 + m_A^2\,s_\beta^2 & -\left(m_Z^2 + m_A^2\right) s_\beta\,c_\beta \\ -\left(m_Z^2 + m_A^2\right) s_\beta\,c_\beta & m_Z^2\,s_\beta^2 + m_A^2\,c_\beta^2 \end{pmatrix}$$

$\rightarrow$ m_h and m_H are predicted in terms of m_Z, m_A and $\tan\beta$

- Tree–level mass relation: $m_h^2 \leq \cos^2 2\beta\, m_Z^2$!!!

- Radiative corrections can push m_h well above the tree–level bound (e.g. $m_h \leq 135$ GeV for typical parameter choices) and introduce a dependence on many MSSM parameters.

dressed Higgs propagators

$$(\Delta_{\text{Higgs}})^{-1} = \begin{pmatrix} q^2 - m_H^2 + \hat{\Sigma}_H(q^2) & \hat{\Sigma}_{hH}(q^2) \\ \hat{\Sigma}_{Hh}(q^2) & q^2 - m_h^2 + \hat{\Sigma}_h(q^2) \end{pmatrix}$$

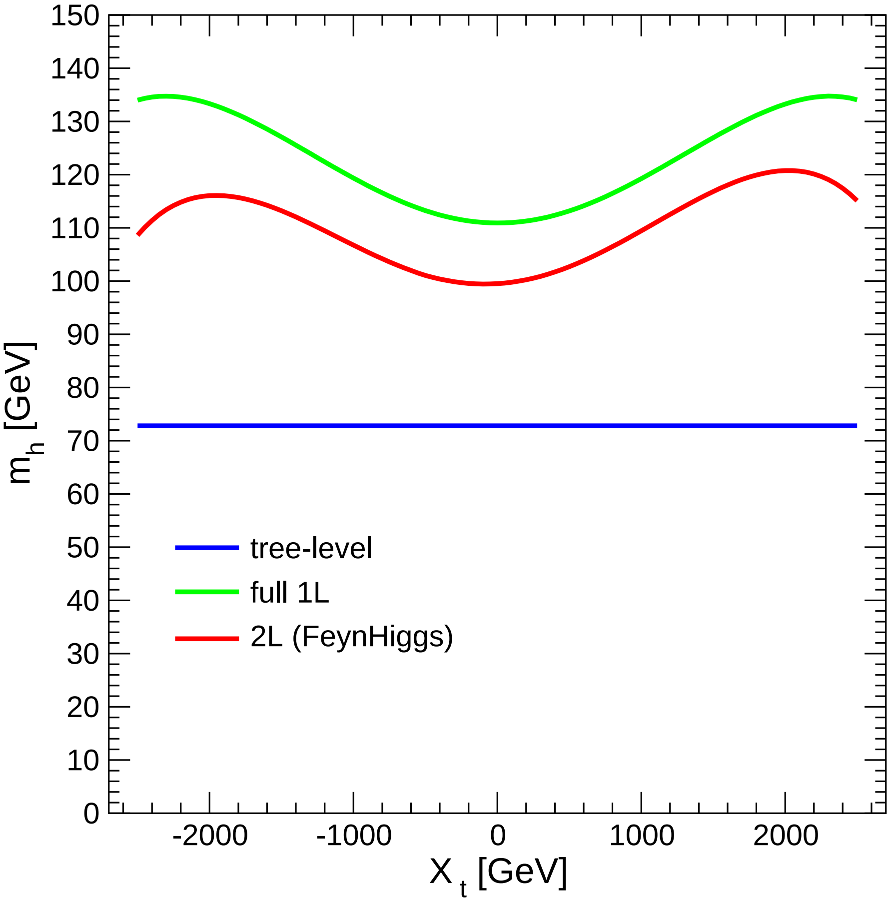
- det $= 0$ $\quad\rightarrow\quad$ $m_{h,H}^{\text{pole}}$
- diagonalization $\quad\rightarrow\quad$ effective couplings (α_{eff})

renormalized self-energies $\hat{\Sigma}$

1-loop: complete

2-loop: QCD corrections $\sim \alpha_s\alpha_t,\ \alpha_s\alpha_b$
Yukawa corrections $\sim \alpha_t^2$
[$\rightarrow$ FeynHiggs]

m_{h^0} prediction at different levels of accuracy:



$\tan\beta = 3$, $\quad M_{\tilde{Q}} = M_A = 1$ TeV, $\quad m_{\tilde{g}} = 800$ GeV

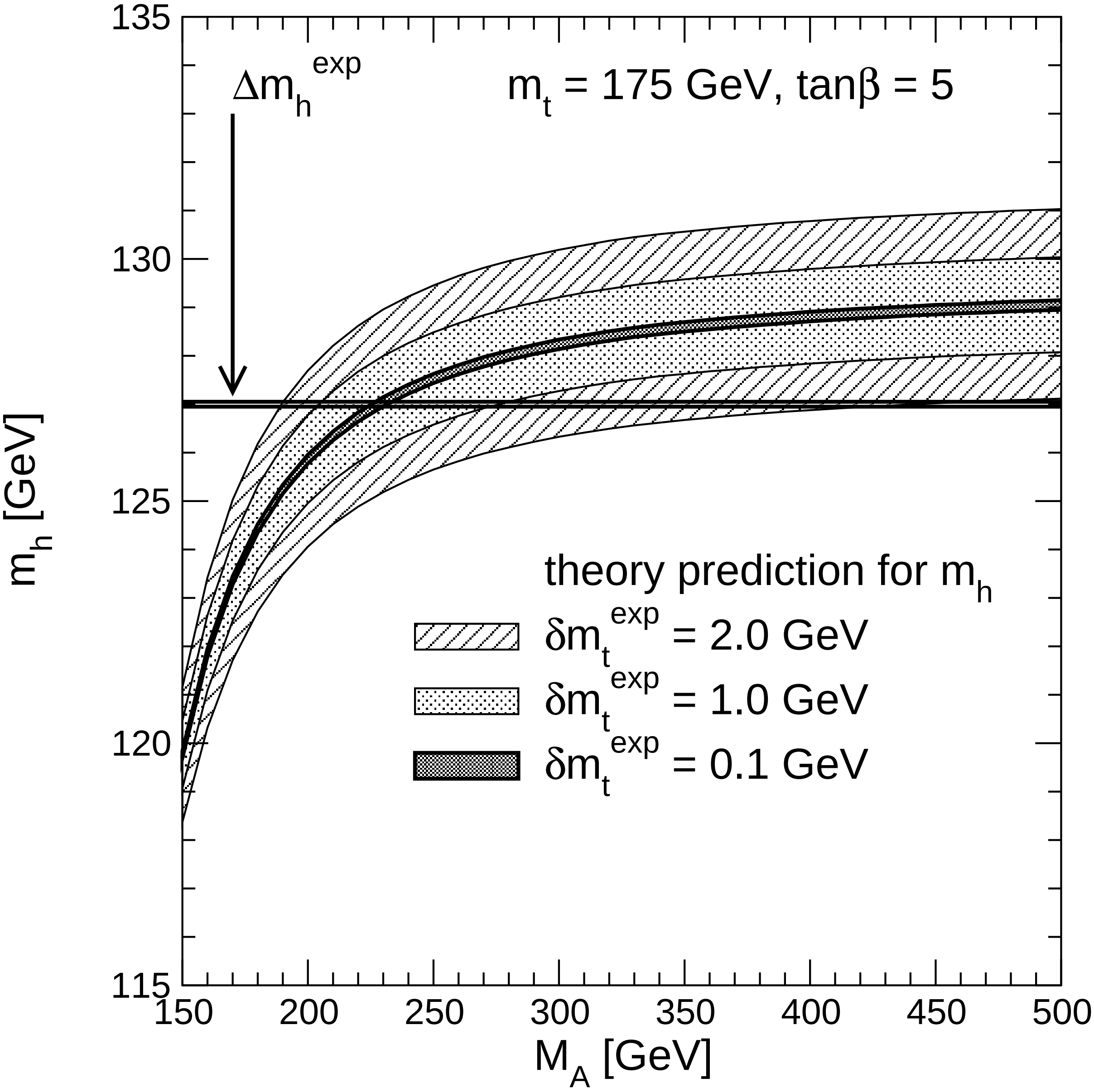
X_t : top-squark mixing parameter

$$X_t = A_t - \mu \cot\beta \,, \qquad \mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} m^2_{\tilde{t}_L} & m_t X_t \\ m_t X_t & m^2_{\tilde{t}_R} \end{pmatrix}$$

present theoretical uncertainty: $\quad \delta m_h \simeq 4$ GeV

[Degrassi, Heinemeyer, WH, Slavich, Weiglein]

[Heinemeyer et al.]



SUSY particles

- LHC will see SUSY if at low energy scale
- LC and LHC$\oplus$LC for precision studies
- Reconstruction of fundamental SUSY theory and breaking mechanism

from experiment:

$\rightarrow$ precision analyses of masses and couplings including higher orders

from theory:

$\rightarrow$ accurate theoretical predictions to match exp. data

$\rightarrow$ loop contributions Lagrangian param $\leftrightarrow$ observables

$\rightarrow$ RGEs for extrapolation to high scales

chargino/neutralino sector

complete at one loop [Fritzsche,WH/Eberl,Majerotto,...]
renormalization and mass spectrum
pair production in e^+e^- collisions

sfermion sector

renormalization and mass spectrum
[WH, Rzehak]

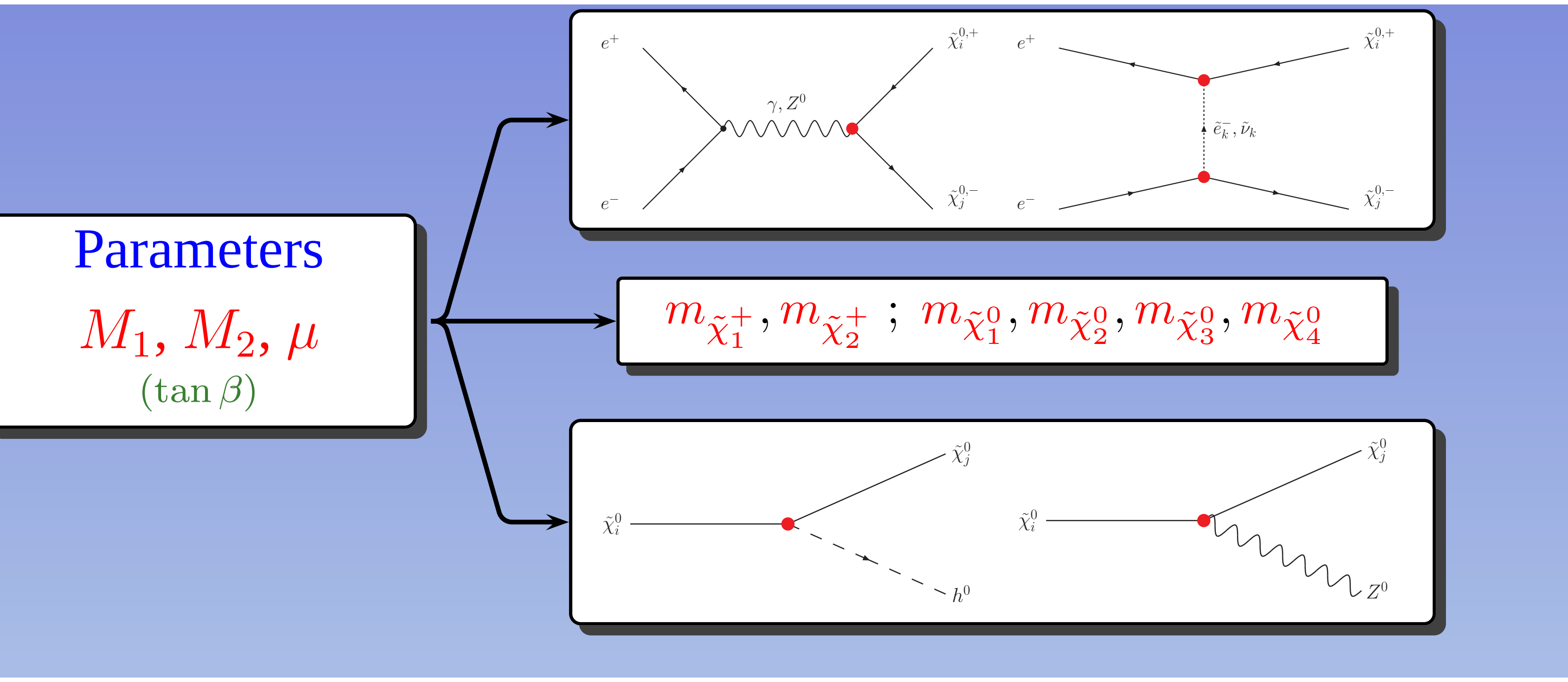
$$\begin{pmatrix} m_f^2 + M_L^2 + M_Z^2 c_{2\beta}(I_f^3 - Q_f s_W^2) & m_f(A_f - \mu\kappa) \\ m_f(A_f - \mu\kappa) & m_f^2 + M_{\tilde{f}_R}^2 + M_Z^2 c_{2\beta} Q_f s_W^2 \end{pmatrix}$$

sfermion pair production in e^+e^- collisions
complete at one-loop

[Arhrib, WH]	squarks, sleptons
[Kovarik, Weber, Eberl, Majerotto]	squarks
[Freitas, Miller, von Manteuffel, Zerwas]	sleptons

sfermion decays into fermions and -inos

complete at one-loop
[Guasch, WH, Solà]

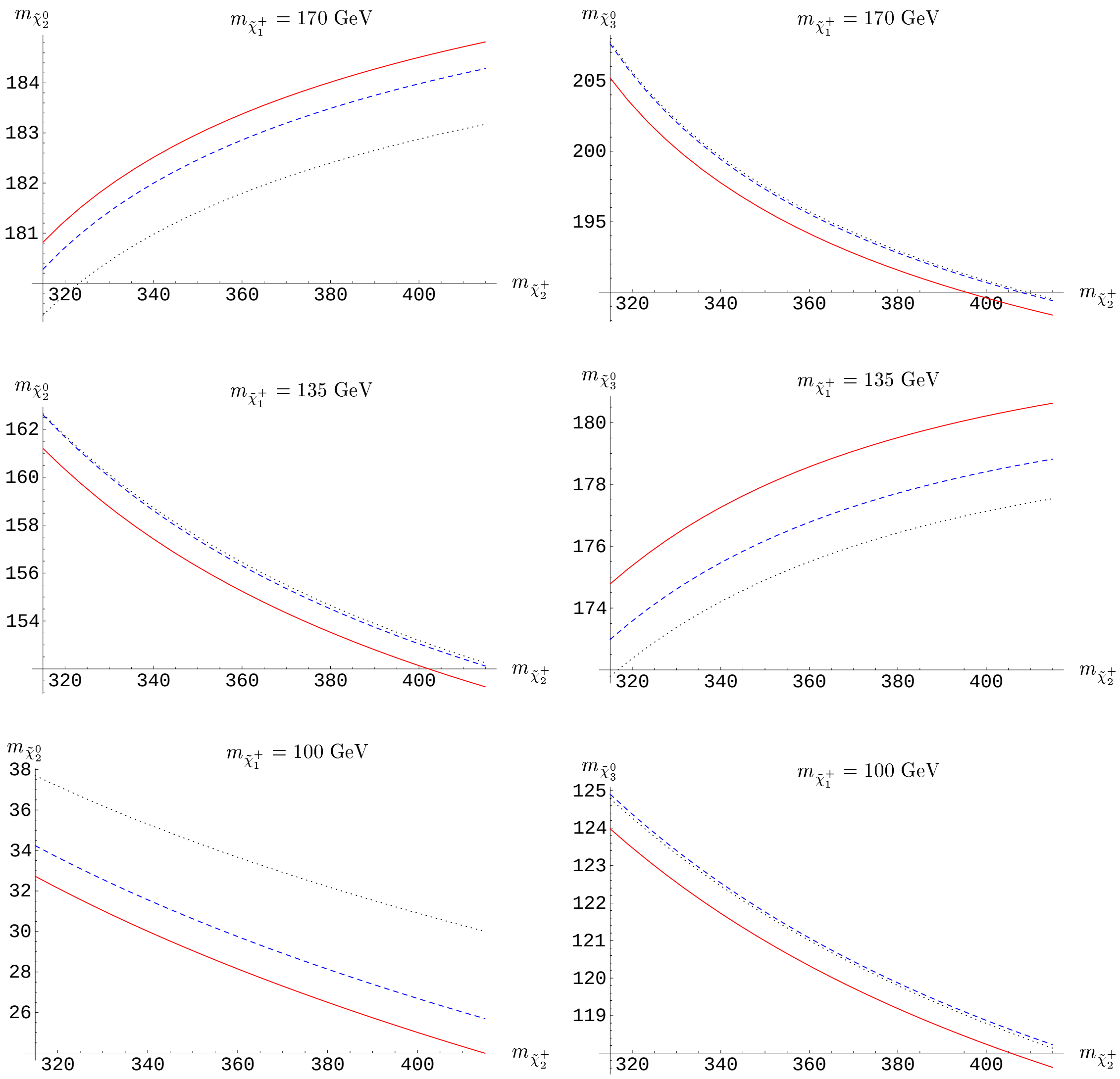


Parameters
M_1, M_2, μ
$(\tan\beta)$
e^+
e^-
γ, Z^0
$\tilde{\chi}_i^{0,+}$
$\tilde{\chi}_j^{0,-}$
$\tilde{e}_k^-, \tilde{\nu}_k$
$m_{\tilde{\chi}_1^+}, m_{\tilde{\chi}_2^+}$; $m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}$
$\tilde{\chi}_i^0$
$\tilde{\chi}_j^0$
h^0
Z^0

pole masses

$[M_{\tilde{f}} = 300\,\text{GeV},\ \tan\beta = 10]$

$M_{\chi_1^0} = 110\,\text{GeV}$



Born

complete 1-loop

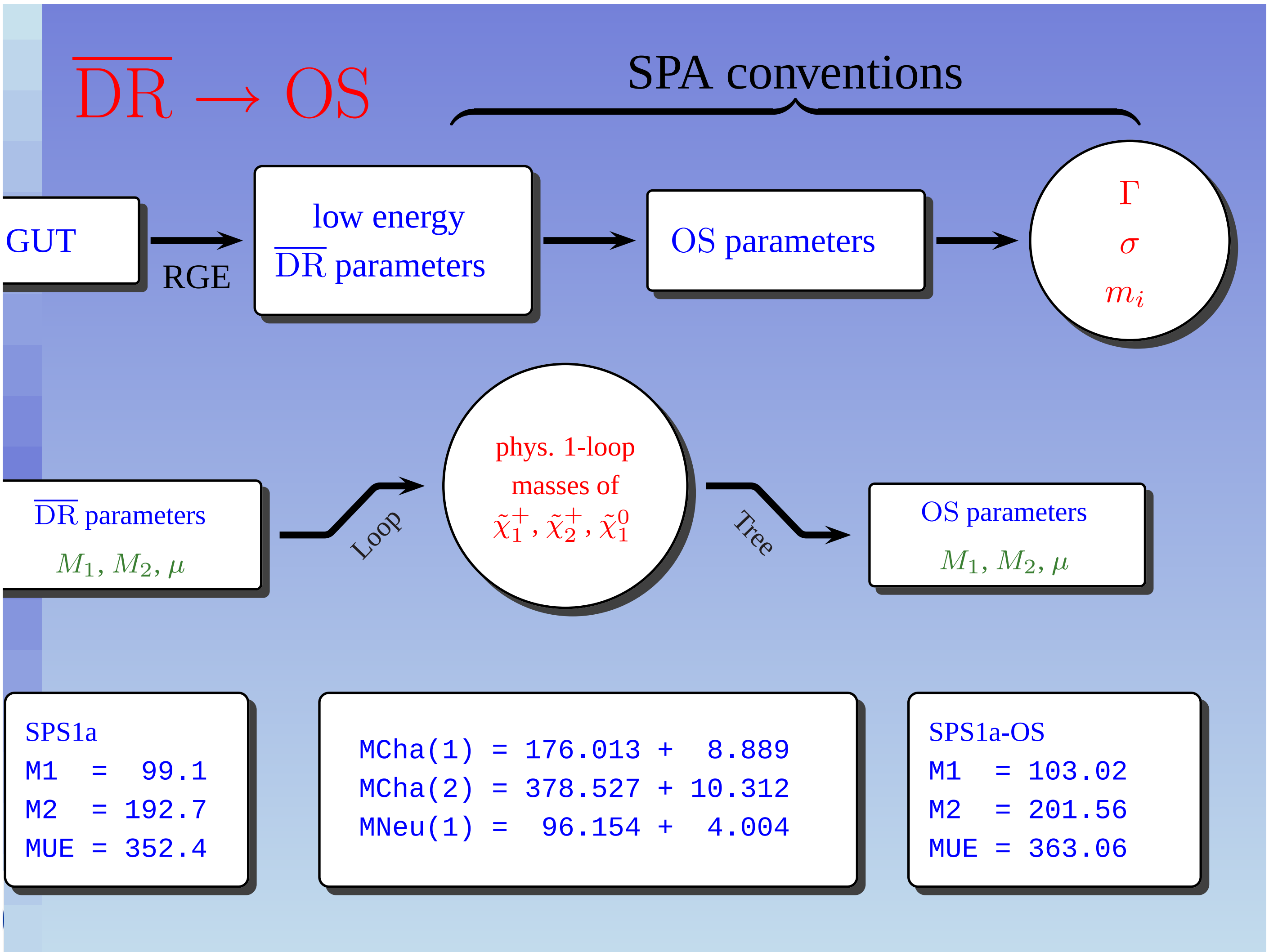
Renormalization schemes

$\overline{\mathrm{DR}}$ scheme:

- Loop integrals: $\frac{2}{\epsilon} - \gamma + \log 4\pi + \log \mu^2 \rightarrow \log \mu^2_{\overline{\mathrm{DR}}}$
- \+ easy to implement
- – observables are scale dependent in finite order perturbation theory
- \+ natural choice for GUT-inspired parameter sets (mSUGRA)

OS scheme:

- renormalization constants fixed by physical conditions
- – renormalization constants complicated
- \+ observables are scale independent
- \+ well suited for calculations of cross sections and decay rates (e.g. pole masses $\rightarrow$ correct kinematical thresholds)



$\overline{\mathrm{DR}} \to \mathrm{OS}$
SPA conventions
GUT
RGE
low energy
$\overline{\mathrm{DR}}$ parameters
OS parameters
Γ
σ
m_i
$\overline{\mathrm{DR}}$ parameters
M_1, M_2, μ
Loop
phys. 1-loop
masses of
$\tilde{\chi}_1^+$, $\tilde{\chi}_2^+$, $\tilde{\chi}_1^0$
Tree
OS parameters
M_1, M_2, μ
SPS1a
M1 = 99.1
M2 = 192.7
MUE = 352.4
MCha(1) = 176.013 + 8.889
MCha(2) = 378.527 + 10.312
MNeu(1) = 96.154 + 4.004
SPS1a-OS
M1 = 103.02
M2 = 201.56
MUE = 363.06

SPA

SUPERSYMMETRY PARAMETER ANALYSIS

The SPA project is a joint study of theorists and experimentalists working on LHC and Linear Collider phenomenology. The study focuses on the supersymmetric extension of the Standard Model. The main targets are

- High-precision determination of the supersymmetry Lagrange parameters at the electroweak scale
- Extrapolation to a high scale to reconstruct the fundamental parameters and the mechanism for supersymmetry breaking

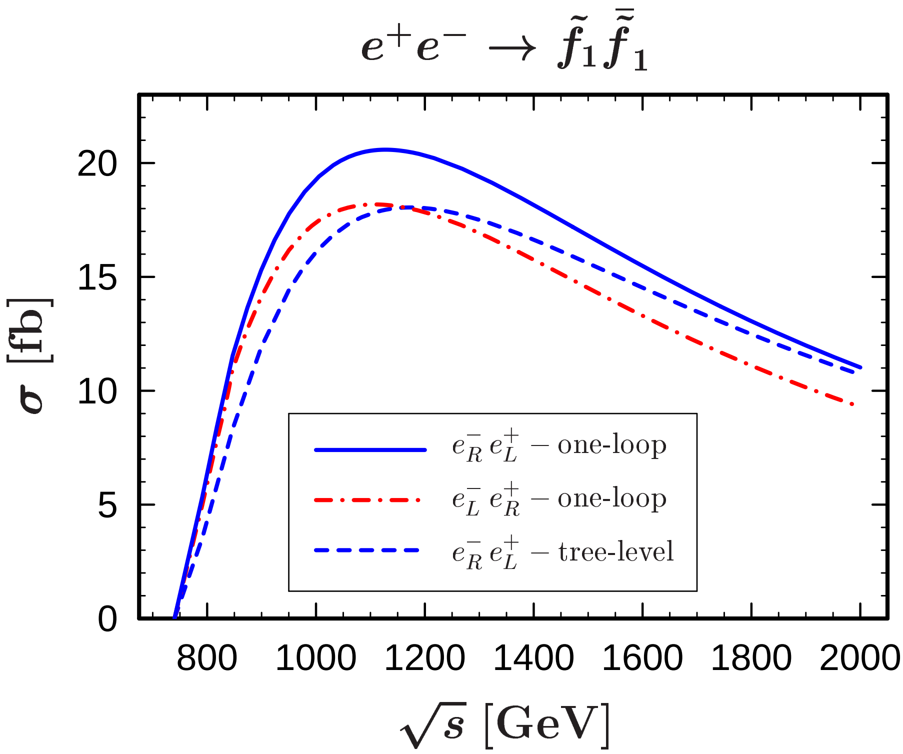
The SPA convention and the SPA Project are described in the report SPA.draft.ps.

`http://spa.desy.de/spa`

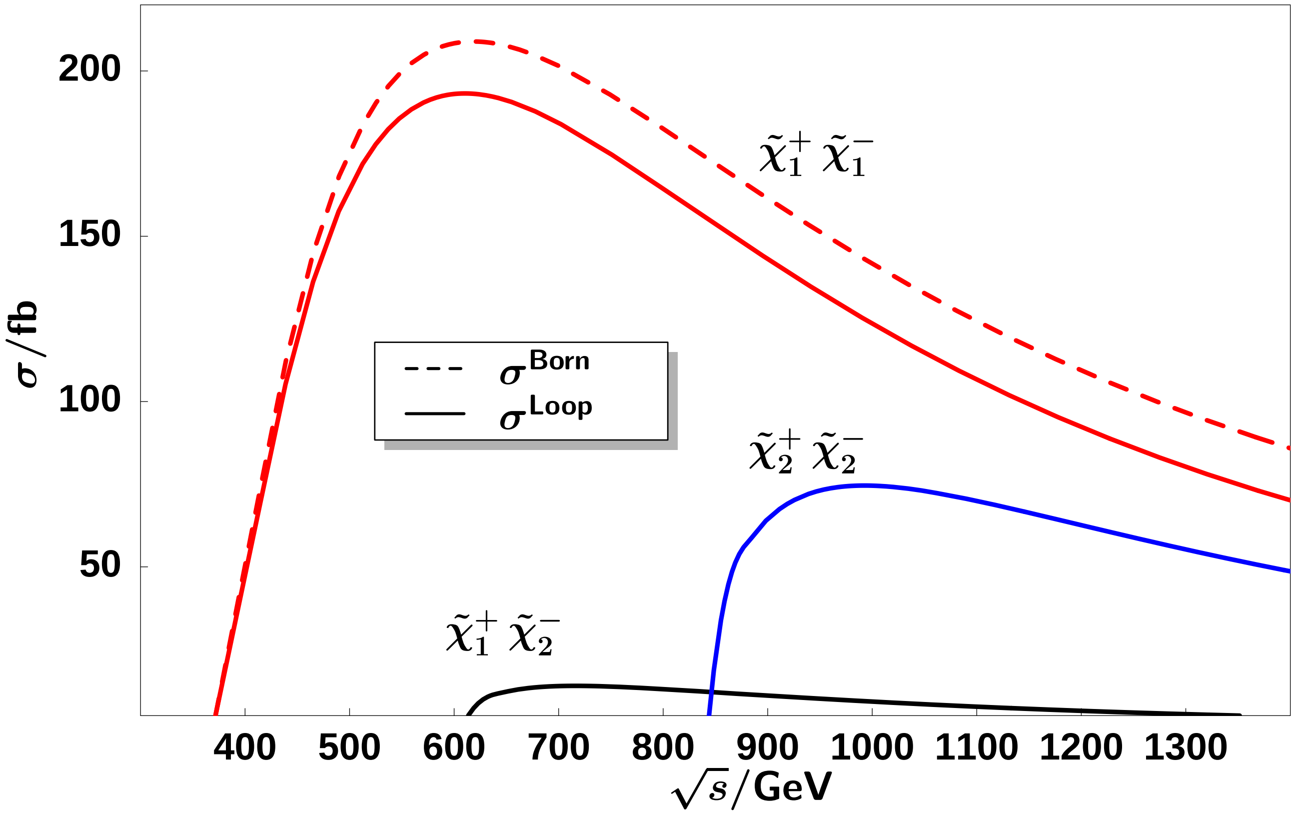
P. Zerwas, J. Kalinowski, H.U. Martyn,
W. Hollik, W. Kilian, W. Majerotto,
W. Porod, ...

SPA CONVENTION

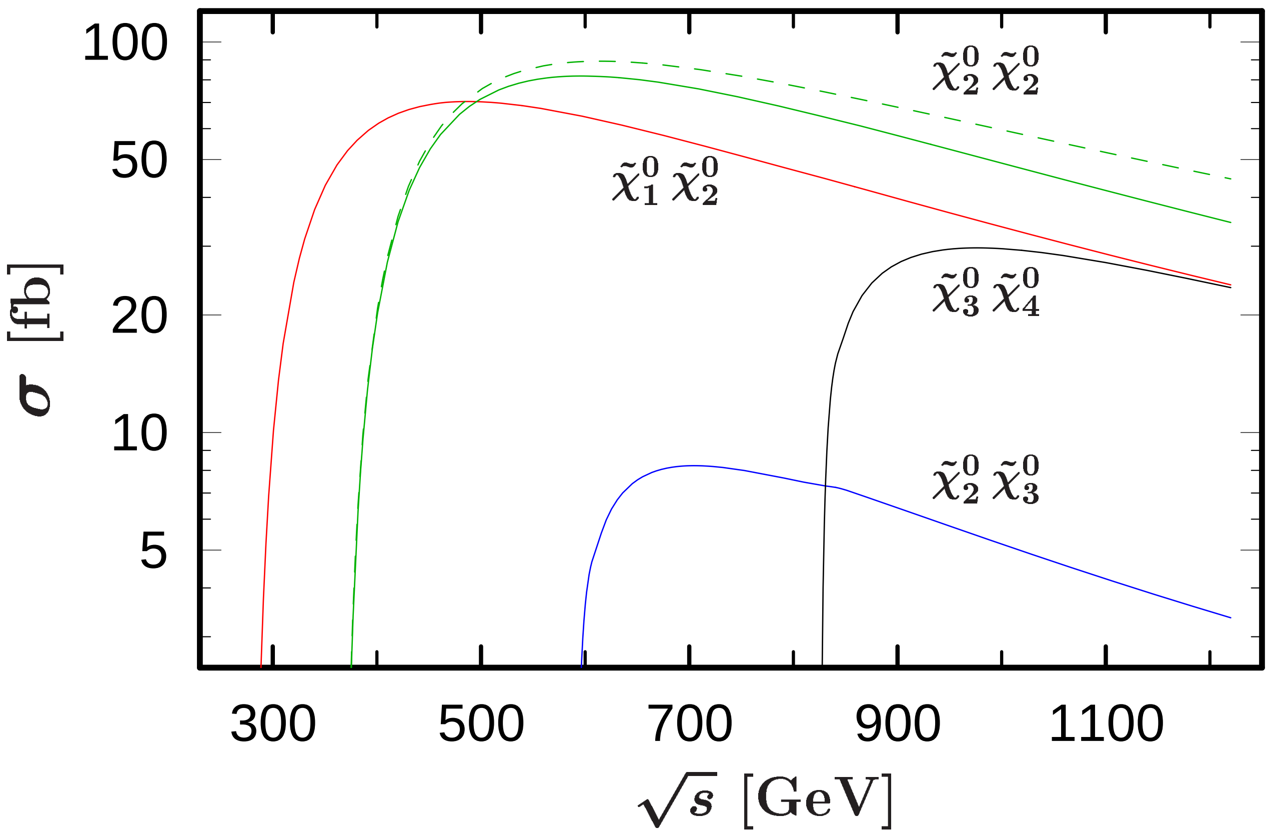
- The masses of the SUSY particles and Higgs bosons are defined as pole masses.
- All SUSY Lagrangian parameters, mass parameters and couplings, including $\tan\beta$, are given in the $\overline{DR}$ scheme and defined at the scale $\tilde{M} = 1$ TeV.
- Gaugino/higgsino and scalar mass matrices, rotation matrices and the corresponding angles are defined in the $\overline{DR}$ scheme at $\tilde{M}$, except for the Higgs system in which the mixing matrix is defined in the on-shell scheme, the scale parameter chosen as the light Higgs mass.
- The Standard Model input parameters of the gauge sector are chosen as G_F, α, M_Z and $\alpha_s^{\overline{MS}}(M_Z)$. All lepton masses are defined on-shell. The t quark mass is defined on-shell; the b, c quark masses are introduced in $\overline{MS}$ at the scale of the masses themselves while taken at a renormalization scale of 2 GeV for the light u, d, s quarks.
- Decay widths / branching ratios and production cross sections are calculated for the set of parameters specified above.



example: $\tilde{f}_1 = \tilde{t}_1$ [from SPA draft]



200
150
100
50
σ/fb
σ^Born
σ^Loop
$\tilde{\chi}_1^+ \tilde{\chi}_1^-$
$\tilde{\chi}_2^+ \tilde{\chi}_2^-$
$\tilde{\chi}_1^+ \tilde{\chi}_2^-$
400 500 600 700 800 900 1000 1100 1200 1300
$\sqrt{s}$/GeV



100
50
20
10
5
σ [fb]
$\tilde{\chi}_2^0 \tilde{\chi}_2^0$
$\tilde{\chi}_1^0 \tilde{\chi}_2^0$
$\tilde{\chi}_3^0 \tilde{\chi}_4^0$
$\tilde{\chi}_2^0 \tilde{\chi}_3^0$
300 500 700 900 1100
$\sqrt{s}$ [GeV]

REFERENCE POINT SPS1a′

SPS1a′ deriv. of Snowmass Point SPS1a: conform with Ω_{cdm}, LE data

mSUGRA values:

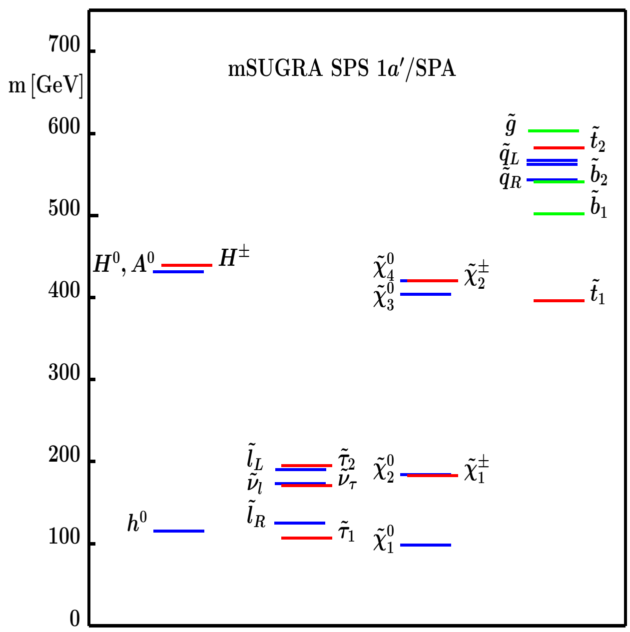
$M_{1/2}$	$=$	250 GeV	sign(μ)	$=$	$+1$
M_0	$=$	70 GeV	$\tan\beta$	$=$	10
A_0	$=$	-300 GeV			

LE/cosmic parameters:

	$BR(b \to s\gamma) = 3.0 \times 10^{-4}$	micrOMEGAs
	$\Delta[g_\mu - 2]/2 = 34 \times 10^{-10}$	FeynHiggs
	$\Omega_{cdm} h^2 = 0.10$	micrOMEGAs

POLE MASSES:

	m [GeV]		m [GeV]
h^0	115.4	$\tilde{e}_R$	125.2
H^0	431.1	$\tilde{e}_L$	190.1
A^0	431.0	$\tilde{\nu}_e$	172.8
H^+	438.6	$\tilde{\tau}_1$	107.4
$\tilde{\chi}_1^0$	97.75	$\tilde{\tau}_2$	195.3
$\tilde{\chi}_2^0$	184.4	$\tilde{\nu}_\tau$	170.7
$\tilde{\chi}_3^0$	406.8	$\tilde{u}_R$	547.7
$\tilde{\chi}_4^0$	419.6	$\tilde{u}_L$	565.7
$\tilde{\chi}_1^+$	184.2	$\tilde{t}_1$	368.9
$\tilde{\chi}_2^+$	421.1	$\tilde{t}_2$	584.9
$\tilde{g}$	607.6	$\tilde{b}_1$	506.3



$BR(\tilde{\nu} \to \nu \chi_1^0) = 100\%$ $\Rightarrow$ $\tilde{\nu}$ invis.

Measurements

– edge effects at LHC

– decay spectra at ILC

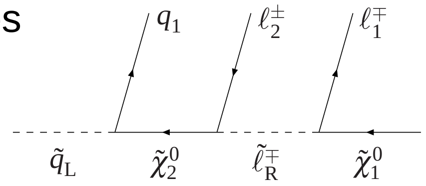
– cross sections/asymmetries at ILC

	Mass	"LHC"	"LC"	"LHC+LC"
h^0	115.4	0.25	0.05	0.05
H^0	431.1		1.5	1.5
$\tilde{\chi}_1^0$	97.75	4.8	0.05	0.05
$\tilde{\chi}_2^0$	184.4	4.7	1.2	0.08
$\tilde{\chi}_4^0$	419.6	5.1	3 – 5	2.5
$\tilde{\chi}_1^\pm$	184.2		0.55	0.55
$\tilde{e}_R$	125.2	4.8	0.05	0.05
$\tilde{e}_L$	190.1	5.0	0.18	0.18
$\tilde{\tau}_1$	107.4	5 – 8	0.24	0.24
$\tilde{q}_R$	547.7	7 – 12	–	5 – 11
$\tilde{q}_L$	565.7	8.7	–	4.9
$\tilde{t}_1$	368.9		1.9	1.9
$\tilde{b}_1$	506.3	7.5	–	5.7
$\tilde{g}$	607.6	8.0	–	6.5

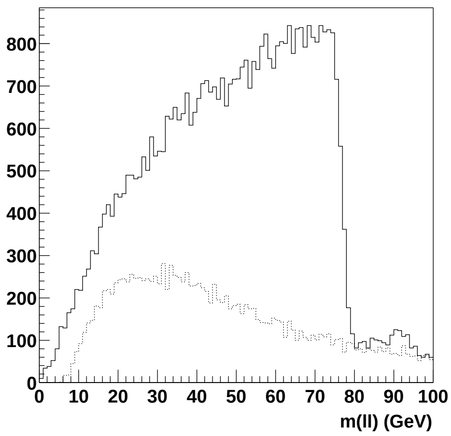
Example: Determination of SUSY parameters at LHC / LC

[*M. Chiorboli, B.K. Gjelsten, J. Hisano, K. Kawagoe, E. Lytken, U. Martyn, D. Miller, M. Nojiri, P. Osland, G. Polesello, A. Tricomi '03*]

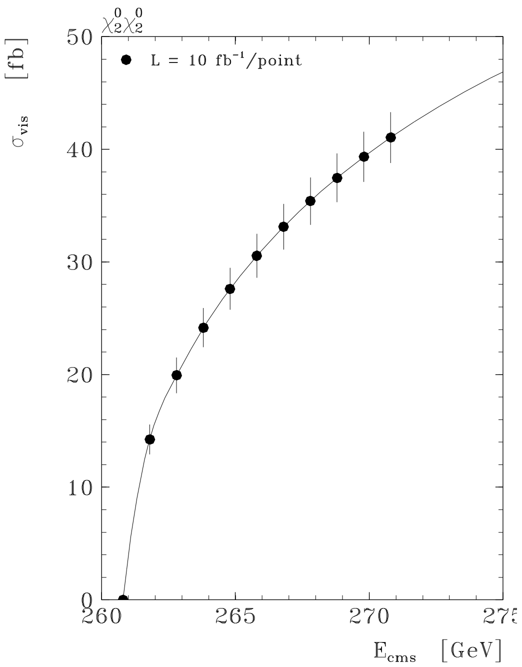
Cascade decays: complicated decay chains for squarks and gluinos



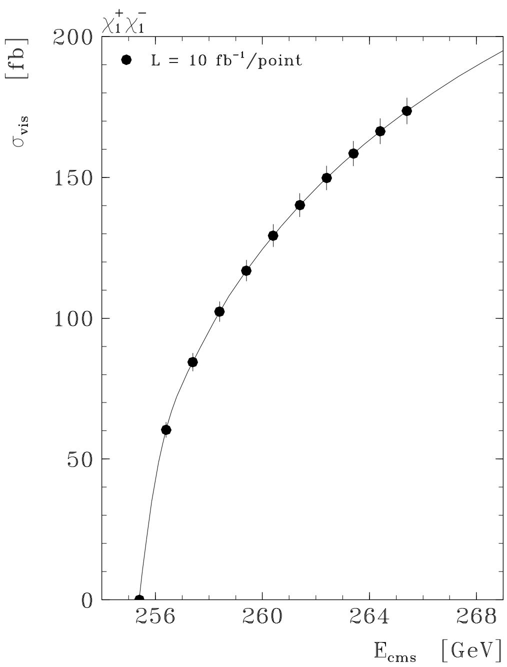
Main tool: dilepton "edge" from $\tilde{\chi}_2^0 \to \ell^+\ell^-\tilde{\chi}_1^0$



$$e^+e^- \to \chi_2^0\chi_2^0$$



$$e^+e^- \to \chi_1^+\chi_1^-$$



Reconstructing Lagrange param.

based on 82 simulated measurements at LHC and ILC

Parameter	SPS1a$'$value	Fit error [exp]
M_1	103.3	0.1
M_2	193.4	0.1
M_3	568.9	7.8
μ	400.4	1.1
$M_{\tilde{e}_L}$	181.3	0.2
$M_{\tilde{e}_R}$	115.6	0.4
$M_{\tilde{\tau}_L}$	179.5	1.2
$M_{\tilde{u}_L}$	523.2	5.2
$M_{\tilde{u}_R}$	503.9	17.3
$M_{\tilde{t}_L}$	467.7	4.9
m_{A}	374.9	0.8
A_{t}	-525.6	24.6
$\tan\beta$	10.0	0.3

High Scale Extrapolations

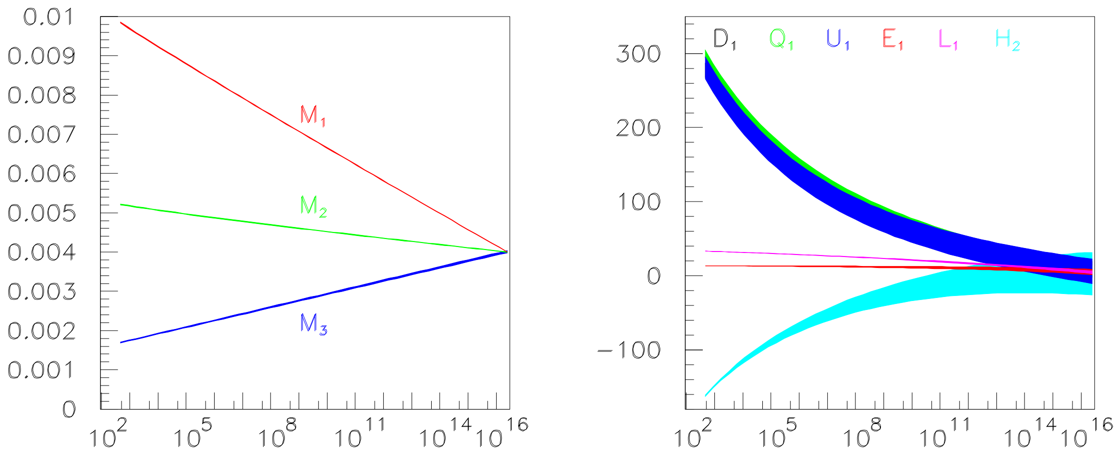


Fig. 1. *Running of the gaugino and scalar mass parameters in SPS1a′ [*`SPheno 2.2.2` *]. Only experimental errors are taken into account; theoretical errors are assumed to be reduced to the same size in the future.*

ERRORS `SPS1a′`:

mSUGRA	Parameter, ideal	"LHC+LC" errors
M_1	250. GeV	0.18 GeV
M_2	*ditto*	0.26 GeV
M_3		2.8 GeV
M_{L_1}	70. GeV	4.1 GeV
M_{E_1}	*ditto*	7.9 GeV
M_{Q_1}		11. GeV
M_{U_1}		31. GeV
M_{H_1}	*ditto*	7.5 GeV
M_{H_2}		72. GeV
A_t	−300. GeV	44. GeV

CONCLUSION:
- gauginos in excellent $\mathcal{O}$[per-mille] condition
- scalar leptons in good $\mathcal{O}$[per-cent] condition
- squarks in $\mathcal{O}$[1] condition

mSUGRA Fit:

	Param,ideal	Experimental error
M_U	$2.47 \cdot 10^{16}$ GeV	$0.02\ 10^{16}$ GeV
α_U^{-1}	24.17	0.06
$M_{\frac{1}{2}}$	250. GeV	0.2 GeV
M_0	70. GeV	0.2 GeV
A_0	-300. GeV	13. GeV
μ	402.9 GeV	0.3 GeV
$\tan\beta$	10.	0.3

General conclusion:

– universality can be tested in bottom-up approach in non-colored sector very well;

– colored sector needs improvement

– mSUGRA fit of high quality

Conclusions

- Era of electroweak precision physics:
  - quantum effects have been established
  - strong indication for a light Higgs boson

- The MSSM is competitive to the SM
  - global fits of similar quality (slightly better)
  - natural: light Higgs boson

- m_{h^0} is another precision observable
  - dependent on all SUSY sectors
  - accurate theoretical evaluation ($\delta m_{h^0} \simeq 4$ GeV), to be further improved

- one-loop studies for SUSY processes are underway, many results and tools already available