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Moduli stabilization

from

magnetic fluxes

with T. Maillard hep-th/0412008

A. Kumar and T. Maillard hep-th/0505260

Outline

- Motivations

- Framework

  Type I string theory with magnetized D9 branes

- T^6 example with O9 planes I.A-Maillard '04

  - SUSY conditions
  - tadpole cancellation
  - positivity requirements
  - explicit example
  - infinite discrete class with large volume(s)

- $T^6/\mathbb{Z}_2$ orientifold with O3 planes

  combined with 3-form fluxes

  I.A-Kumar-Maillard '05

  fix also the dilaton

Moduli stabilization with 3-form fluxes:
significant progress but

- no exact string description
  low energy SUGRA approximation

- fix only complex structure

Type I with internal magnetic fluxes:
alternative/complementary approach

- exact string description

- Kähler class stabilization
  T^6: all geometric moduli fixed

- natural implementation in intersecting
  D-brane models

General framework

- Type I string theory compactified in 4d on 6d Calabi-Yau

$\Rightarrow$ $N = 2$ SUSY in the bulk, $N = 1$ on branes

- Magnetic fluxes on 2-cycles

$\Rightarrow$ SUSY breaking

Dirac quantization: $H = \frac{m}{nA} \equiv \frac{p}{A}$

H: constant magnetic field

m: units of magnetic flux /1st Chern class

n: brane wrapping

A: area of the 2-cycle

Spin-dependent mass shifts for all charged states

$[p_i, p_j] = iqH\epsilon_{ij}$ $\qquad$ q: charge

$\Rightarrow$ Landau spectrum

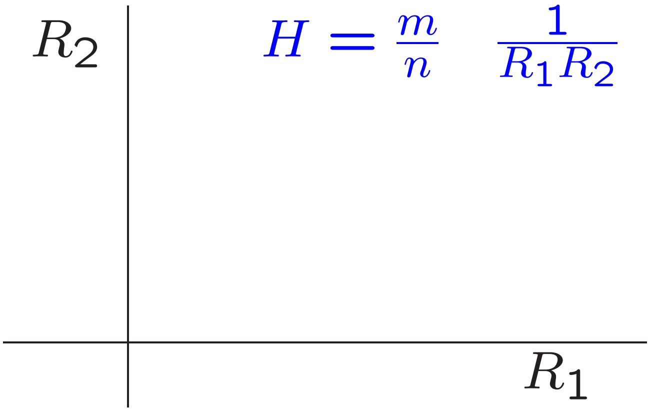
Exact open string description:

$q \to q_L + q_R$ endpoint charges

$qH \to \theta_L + \theta_R \quad ; \quad \theta_{L,R} = \arctan q_{L,R} H \alpha'$

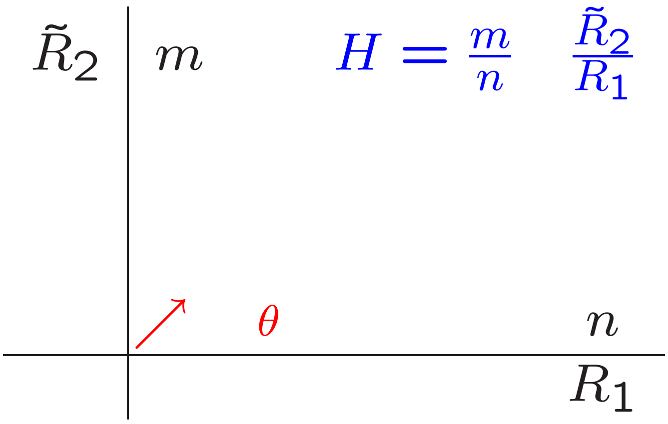
weak field limit $\Rightarrow$ field theory

T-dual representation:



$$H = \frac{m}{n} \quad \frac{1}{R_1 R_2}$$

$$R_2 \to \alpha'/R_2 \equiv \tilde{R}_2 \quad \Rightarrow$$



$$H = \frac{m}{n} \quad \frac{\tilde{R}_2}{R_1} = \tan\theta$$

m, n : wrapping numbers

Magnetic fluxes can be used to stabilize moduli

I.A.-Maillard '04

e.g. T^6: 36 moduli (geometric deformations)

internal metric: $6 \times 7/2 = 21 = 9 + 2 \times 6$

type IIB RR 2-form: $6 \times 5/2 = 15 = 9 + 2 \times 3$

$$\text{complexification} \Rightarrow \begin{cases} \text{Kähler class} & J \\ \text{complex structure} & \tau \end{cases}$$

9 complex moduli for each

magnetic flux: 6×6 antisymmetric matrix F

complexification $\Rightarrow$

$F_{(2,0)}$ on holomorphic 2-cycles: potential for τ

$F_{(1,1)}$ on mixed (1,1)-cycles: potential for J

T^6 parametrization/complexification

$x^i \equiv x^i + 1 \quad y_i \equiv y_i + 1 \quad i = 1, 2, 3$

orientation choice: $\int_{T^6} \prod_{i=1}^{3} \left(dx^i \wedge dy_i \right) = 1$

$z^i = x^i + \tau^{ij} y_i$ $\quad \tau$: complex structure matrix

$\delta g_{i\bar{j}}$: Kähler deformations $\rightarrow J = \delta g_{i\bar{j}} i dz_i \wedge d\bar{z}_j$

ω_r: real basis of 2-cycles

$H^2(T^6)$ cohomology

$\Rightarrow$ signs: $\mathcal{K}_{rst} = \int_{T^6} \omega_r \wedge \omega_s \wedge \omega_t$

$N = 1$ SUSY conditions:

(1) $F_{(2,0)} = 0 \quad \Rightarrow \tau \quad \tau^{\top} p_{xx} \tau - (\tau^{\top} p_{xy} + p_{yx} \tau) + p_{yy} = 0$

(2) $J \wedge J \wedge F_{(1,1)} = F_{(1,1)} \wedge F_{(1,1)} \wedge F_{(1,1)} \quad \Rightarrow J$

Appropriate choice of magnetic fluxes F^a

in several abelian directions $U(1)_a \Rightarrow$

all moduli vanish except the 6 radii of T^6

which are fixed in terms of the quantized fluxes

$T^6 = \prod_{I=1}^{3} T_I^2 \leftarrow$ orthogonal 2-torus

$\tau_I = \frac{R_I}{R'_I} \qquad J_I = R_I R'_I \qquad H_I^a = \frac{F_I^a}{J_I}$

(1) fixes the ratios τ_I

(2) fixes the sizes J_I

$H_1 + H_2 + H_3 = H_1 H_2 H_3 \Leftrightarrow \theta_1 + \theta_2 + \theta_3 = 0$

DBI action:

$$\{[J\wedge J\wedge J - J\wedge F\wedge F]^2 + [J\wedge J\wedge F - F\wedge F\wedge F]^2\}^{1/2}$$

becomes polynomial

$$\mathcal{S}_{\rm DBI} = -T_9 \int \sqrt{g}\,(1 - |H_1H_2| + |H_2H_3| + |H_1H_3|)$$

or permutation of the minus sign

I.A.-Maillard, Bianchi-Trevigne

Tadpole conditions

$Q_9 = \sum_a N_a \sum_{rst} \mathcal{K}_{rst} n_r^a n_s^a n_t^a = 16 \leftarrow$ O9 charge

sign due to orientation choice

$Q_5 = \sum_a N_a \sum_{st} \mathcal{K}_{rst} n_r^a m_s^a m_t^a = 0 \qquad \forall$ 2-cycle r

- volume positivity: $J\wedge J\wedge J - J\wedge F^a \wedge F^a > 0$
- no antibranes: $\mathcal{K}_{rst} n_r^a n_s^a n_t^a > 0$

Main ingredients for moduli stabilization

1. "oblique" magnetic fields $\Rightarrow$
   fix off-diagonal components of the metric

2. magnetized D9's $\Rightarrow$ -ve 5-brane tension

3. Non linear DBI action $\Rightarrow$ fix overall volume

(1)+(2) : necessary for Q_5 cancellation

(2)+(3) : not valid in six dimensions

Stabilization of RR moduli

- Kähler class: absorbed by massive $U(1)$'s

$dC_2 \wedge \star(A^a \wedge F^a) \quad \Rightarrow$

4d G-S kinetic mixing with magnetized $U(1)$'s

$\Rightarrow$ need at least 9 brane stacks

- Complex structure: get potential through mixing with NS moduli

Bianchi-Trevigne '05

Stack $\sharp$	Fluxes	Fixed complex structure	$5-$brane local.	Fixed Kähler class
$\sharp 1$	$(F^1_{x_1y_2}, F^1_{x_2y_1})$	$\tau_{31} = \tau_{32} = 0$ $p^1_{x_1y_2}\tau_{11} = \tau_{22}p^1_{x_2y_1}$	$[x_3, y_3]$	$V_{1\bar{2}} - V_{2\bar{1}} = 0$
$\sharp 2$	$(F^2_{x_1y_3}, F^2_{x_3y_1})$	$\tau_{21} = \tau_{23} = 0$ $p^2_{x_1y_3}\tau_{11} = \tau_{33}p^2_{x_3y_1}$	$[x_2, y_2]$	$V_{1\bar{3}} - V_{3\bar{1}} = 0$
$\sharp 3$	$(F^3_{x_1x_2}, F^3_{y_1y_2})$	$\tau_{13} = 0$ $\tau_{11}\tau_{22} = -\frac{p^3_{y_1y_2}}{p^3_{x_1x_2}}$	$[x_3, y_3]$	$V_{1\bar{2}} + V_{2\bar{1}} = 0$
$\sharp 4$	$(F^4_{x_2x_3}, F^4_{y_2y_3})$	$\tau_{12} = 0$	$[x_1, y_1]$	$V_{2\bar{3}} + V_{3\bar{2}} = 0$
$\sharp 5$	$(F^5_{x_1x_3}, F^5_{y_1y_3})$		$[x_2, y_2]$	$V_{1\bar{3}} + V_{3\bar{1}} = 0$
$\sharp 6$	$(F^6_{x_2y_3}, F^6_{x_3y_2})$		$[x_1, y_1]$	$V_{2\bar{3}} - V_{3\bar{2}} = 0$

$V_{i\bar{j}} = (J \wedge J)_{i\bar{j}} = 0 \Rightarrow J_{i\bar{j}} = 0 \;\; \forall\, i \neq j$

4th column: 5-brane charge localization on the 2-cycles $[x_i, y_i]$

Compatibility condition: p^4, p^5, p^6 constrained by p^1, p^2, p^3, so that τ_{ii} remain unchanged

Fix areas of the 3 T^2's $\Rightarrow$ add 3 more stacks:

Stack $\sharp$	Fluxes	$5-$brane localization
$\sharp 7$	$(F^7_{x_1y_1}, F^7_{x_2y_2}, F^7_{x_3y_3})$	$[x_1, y_1]$ $[x_2, y_2]$ $[x_3, y_3]$
$\sharp 8$	$(F^8_{x_1y_1}, F^7_{x_2y_2}, F^8_{x_3y_3})$	$[x_1, y_1]$ $[x_2, y_2]$ $[x_3, y_3]$
$\sharp 9$	$(F^9_{x_1y_1}, F^9_{x_2y_2}, F^9_{x_3y_3})$	$[x_1, y_1]$ $[x_2, y_2]$ $[x_3, y_3]$

$$\Rightarrow \begin{pmatrix} F^7_1 & F^7_2 & F^7_3 \\ F^8_1 & F^8_2 & F^8_3 \\ F^9_1 & F^9_2 & F^9_3 \end{pmatrix} \begin{pmatrix} J_2 J_3 \\ J_1 J_3 \\ J_1 J_2 \end{pmatrix} = \begin{pmatrix} F^7_1 F^7_2 F^7_3 \\ F^8_1 F^8_2 F^8_3 \\ F^9_1 F^9_2 F^9_3 \end{pmatrix}$$

Consistency conditions and tadpole cancellation are ok

- large volume:

- rescale $F^{7,8,9}_1 \to \Lambda F^{7,8,9}_1 \Rightarrow J_1 \to \Lambda J_1$: one large T^2

tadpoles are satisfied by appropriate rescaling of F^{1-6}

- rescale all fluxes and all J_I : three large T^2

Geometric moduli: fixed without breaking gauge symmetries

- more magnetic fields:

in general incompatible unless special fluxes or

turn on open string moduli

charged states modifying the D-terms

$\Rightarrow$ - fixed at non trivial values

- break gauge symmetries

- fixing the dilaton?

combine magnetic and 3-form fluxes

loose the exact string description

3-brane charge $\Rightarrow T^6/\mathbb{Z}_2$ with O3 planes

I.A.-Kumar-Maillard '05

T-duality to $T^6/\mathbb{Z}_2$

magnetic flux $m \leftrightarrow$ winding number n

- non-trivial because of 0-eignevalues $(m = 0)$
- weak string coupling $\Rightarrow$ minimize

  the 3-brane charge of magnetized branes
- new infinite discrete series

  with the same values for all moduli

Models

- use 3-form fluxes to fix only the dilaton

  compatible with the values of T^6 moduli
- part of moduli are fixed by magnetic

  and part by 3-form fluxes

$N = 1$ SUSY condition for Kähler moduli:

$J \wedge J \wedge J = J \wedge F_{(1,1)} \wedge F_{(1,1)} \Rightarrow$

$\mathcal{S}_{\rm DBI} = -T_9 \int \sqrt{g} \, (|H_1 H_2 H_3| - |H_1| + |H_2| + |H_3|)$

or permutation of the minus sign

Tadpole conditions

$Q_3 = \sum_a N_a \sum_{rst} \mathcal{K}_{rst} m_r^a m_s^a m_t^a = 16 - N_3$

3-form fluxes

$Q_7 = \sum_a N_a \sum_{st} \mathcal{K}_{rst} m_r^a n_s^a n_t^a = 0 \qquad \forall$ 2-cycle r

- volume positivity: $F^a \wedge F^a \wedge F^a - J \wedge J \wedge F^a > 0$
- no antibranes: $\mathcal{K}_{rst} m_r^a m_s^a m_t^a > 0$

0-eigenvalue $\Rightarrow \mathcal{K}_{rst} n_r^a m_s^a m_t^a > 0$

Example

- branes 1-6: - add 2 diagonal fluxes in each

  - min 3-brane charge = 6

$\Rightarrow$ fix the complex structure in a diagonal form

- branes 7-9: take out 1 diag flux from each

  $\rightarrow$ vanishing 3-brane charge

$\Rightarrow$ Kähler form diagonal and fixed

- particular rescaling of diag. fluxes in all branes

rescaled 7-brane charge of 1-6 cancel with 7-9

$\Rightarrow$ new ∞ series with the same moduli values

- 3-form flux compatible with above values $\Rightarrow$

$$N_3 = hf \qquad\qquad \frac{1}{g_s} = (\mathrm{Im}\tau_{33})\frac{f}{h}$$

h: NS-NS $\qquad$ f: R-R

g_s min $\Rightarrow$ h min $\qquad$ f max allowed by N_3

non-trivial $B \Rightarrow h_{\mathrm{min}} = 1 \qquad g_s = \frac{1}{4\mathrm{Im}\tau_{33}}$

- special rescalings $\Rightarrow$ large radii + small g_s