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EFFECTIVE N=1 SUPERGRAVITIES WITH GENERAL FLUXES & BRANES

Talk based on:

G. VILLADORO, F.Z.
JHEP 0506 (2005) XXX [hep-th/0503169]
GENERALIZED DIMENSIONAL REDUCTION

Complementary point of view to:
J.P. DERENDINGER, C. KONNAS, P.M. PETROPOULOS, F.Z.
NUCL. PHYS. B715 (2005) 211 [hep-th/0411276]
GAUGINGS OF N=4 SUPERGRAVITY
(see talk by J.-P. DERENDINGER)

... many other related talks ...

STRING PHENOMENOLOGY
MÜNCHEN 16 JUN 2005

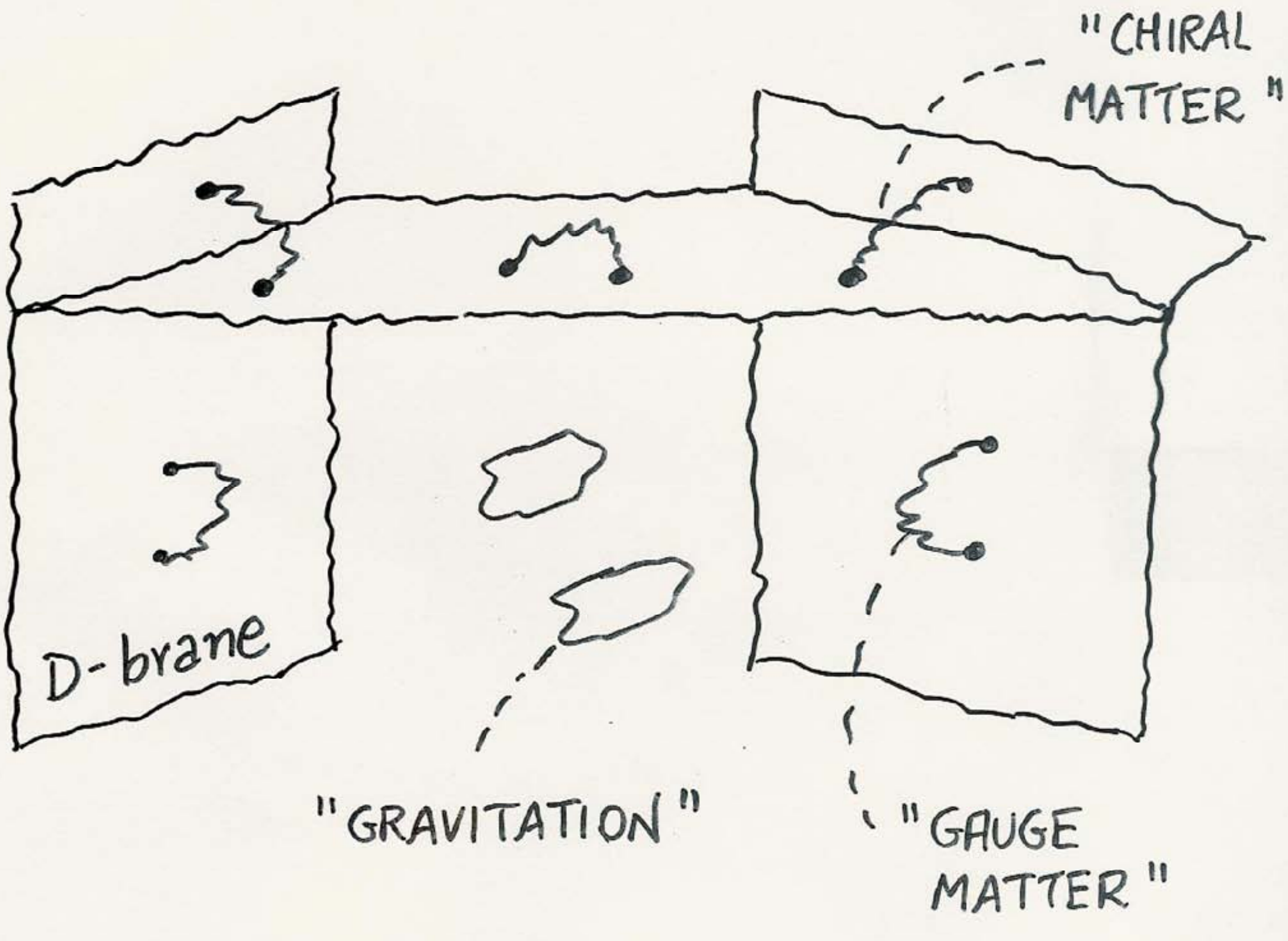
1. INTRODUCTION & OVERVIEW

SOME LONG-STANDING OPEN PROBLEMS
ON THE WAY FROM STRINGS TO THE OBSERVED WORLD:

- SUPERSYMMETRY BREAKING
- MODULI STABILIZATION
- VACUUM ENERGY (& SELECTION)

FIRST STUDIED IN HETEROTIC COMPACTIFICATIONS
CAN NOW BE STUDIED IN A NEW LIGHT
IN TYPE-II ORIENTIFOLD COMPACTIFICATIONS

THE BRANE-WORLD PICTURE:



THE ABOVE PROBLEMS ARE "INFRARED" ONES WITH SOME LIMITATIONS / CONSISTENCY-CONDITIONS CAN BE ADDRESSED IN THE EFFECTIVE SUPERGRAVITY

CONSIDER HERE A SPECIFIC $N=1$ EXAMPLE

- IIA ON $T^6/(Z_2 \times Z_2)$ PLUS ΩI_3 ORIENTIFOLD
- GENERAL SYSTEMS OF O6 / D6
- GENERAL BULK FLUXES (RR, NSNS, Sc-Sc)
- BRANE EXCITATIONS + FLUXES SET TO ZERO (EVENTUALLY TO BE INCLUDED)

AND DERIVE ITS EFFECTIVE $N=1$ SUGRA BY GENERALIZED DIMENSIONAL REDUCTION

THE METHOD

DUAL FORMULATION

... ...
BERGSHOEFF - KALLOSH - ORTIN - ROEST - VAN PROEYEN
... ...

suitably adapted to the present case

THE MAIN RESULTS:

- BIANCHI IDENTITIES WITH GENERAL FLUXES (AND MODIFIED TADPOLE CANCELLATION CONDITIONS)

ω 12 SCHERK-SCHWARZ GEOMETRICAL FLUXES

H 4 NS-NS THREE-FORM FLUXES

G 8 ALL R-R p-FORM FLUXES $\left(p = 0,2,4,6\right)$

- EFFECTIVE $N=1$ POTENTIAL & SUPERPOTENTIAL

  EXPLICITLY DERIVED BY DIMENSIONAL REDUCTION

  REPRODUCES DKPZ WHEN BOTH APPLICABLE

- GEOMETRICAL FORM OF SUPERPOTENTIAL

$$\underline{W} = \frac{1}{4}\int_{X_6} \underline{\bar{G}\, e^{iJ^c} - i\left(\bar{H} - i\omega J^c\right)\wedge \Omega^c}$$

GENERALIZES PREVIOUS RESULTS BY

IIB GUKOV-VAFA-WITTEN
TAYLOR-VAFA
...

IIA GUKOV GUKOV-HAACK
CARDOSO-CURIO-DALL'AGATA-LÜST
GURRIERI-LOUIS-MICU-WALDRAM
GRANA-MINASIAN-PETRINI-TOMASIELLO
GRIMM-LOUIS

- EXPLICIT EXAMPLES WITH PERTURBATIVE STABILIZATION OF ALL UNTWISTED BULK MODULI IN $N=1$ SUSY ADS_4 (≥ 1 AXION)

  INCLUDING SOME GENUINE $N=1$ ONES NOT CONTAINED IN $N=4$ GAUGINGS

2. DUAL FORMULATION OF IIA SUGRA

BOSONIC FIELDS :

G_{MN} METRIC $\quad \Phi$ DILATON $\quad B_{MN}$ NS 2-FORM

RR-FORMS : $C^{(1)} \quad C^{(3)} \quad C^{(5)} \quad C^{(7)} \quad C^{(9)}$

THE DUALITY TRICK

$$\boxed{C^{(p)}} \qquad \boxed{C^{D-p-2}}$$
$$\boxed{G^{(p+1)}} \qquad \boxed{G^{(D-p-1)}}$$

- Choose a pair, as independent variables
- Write the action equivalent to the standard one witout Fluxes and branes
- Vary w.r.t. to $C^{(p)} \Rightarrow$ BI
  For $G^{(D-p-1)}$: $dG^{(D-p-1)} + \ldots = 0$
- Solve the BI : $G^{(D-p-1)} = dC^{(D-p-2)} + \ldots$
  and identify the integrability conditions
- Plug the solution into the action

INITIAL CHOICE MUST BE ADAPTED TO THE ALLOWED FLUXES AND BRANES

STANDARD FORM OF MASSIVE IIA SUGRA (ROMANS)

NSNS : G_{MN}, ϕ, B_{MN} R-R : $C^{(1)}, C^{(3)}$

$$S_{bulk} = \int d^{10}x\, e_{10}\, \frac{1}{2} e^{-2\phi}\left[R_{10} + 4(\partial\phi)^2 - \frac{1}{2}(H)^2\right]$$

$$-\frac{1}{4} e_{10}\left[M^2 + (F'^{(2)})^2 + (\tilde{F}^{(4)})^2\right] - \frac{1}{4} B \wedge F'^{(4)} \wedge F'^{(4)}$$

DUAL FORMULATION (BERGSHOEFF-KALLOSH-ORTIN - ROEST- VAN PROEYEN)

$$S_{bulk} = \int d^{10}x\, e_{10}\, \frac{1}{2} e^{-2\phi}\left[R_{10} + 4(\partial\phi)^2 - \frac{1}{2}(H)^2\right]$$

$$-\frac{1}{4} e_{10}\left[(G^{(0)})^2 + (G^{(2)})^2 + (G^{(4)})^2 + (G^{(6)})^2\right]$$

$$+\frac{1}{2}\left[A^{(9)} - A^{(7)} + A^{(5)} - A^{(3)}\right] \wedge d\left[\mathbf{G} e^{B}\right]$$

RR - SECTOR : $G^{(0,2,4,6)}$ & $A^{(3,5,7,9)}$

$$S^{DBI}_{D6/O6} = -\sum_a^{D6/O6} T_a \int_{\pi_a} d^7x\, e_7\, e^{-\phi}$$

$$S^{WZ}_{D6/O6} = \sum_a^{D6/O6} \mu_a \int_{\pi_a} d^7x\, A^{(7)}$$

$T_a = \mu_a$ $\qquad \pi_a = \prod_{A=1}^{3} (n^a_A, m^a_A)$

NEGLECTING BRANE FLUCTUATIONS & LOCALIZED FLUXES

THE MODEL $\left(\dfrac{T^6}{Z_2 \times Z_2'} \,,\ \Omega I_3\right)$

	x^5	x^6	x^7	x^8	x^9	x^{10}
Z_2	$-$	$-$	$-$	$-$	$+$	$+$
Z_2'	$+$	$+$	$-$	$-$	$-$	$-$
I_3	$-$	$+$	$-$	$+$	$-$	$+$

Ω : $+1$ $G_{MN}, \phi, C^{(3)}, C^{(7)}$ $\quad -1$ $B_{MN}, C^{(1)}, C^{(5)}, C^{(9)}$

O6-PLANES : $(6,8,10)$; $(6,7,9), (5,8,9), (5,7,10)$

DUAL CHOICES : $[A^{(p)}, G^{(9-p)}]$ OR $[A^{(8-p)}, G^{(p+1)}]$

$$C^{(-1)} \leftrightarrow C^{(9)} : \quad \times \qquad G^{(0)} \qquad G^{(10)} \qquad C^{(9)}_{\mu\nu\rho 5678910}$$

$$C^{(1)} \leftrightarrow C^{(7)} : \begin{cases} \times & G^{(2)}_{\{56|\cdot|\cdot\}} & G^{(8)}_{\mu\nu\rho\sigma\{5678|\cdot|\cdot\}} & C^{(7)}_{\mu\nu\rho\{78910|\cdot|\cdot|\cdot\}} \\ \times & (\times) & \times & C^{(7)}_{\mu\nu\rho\sigma\{6810|\cdot|\cdot|\cdot\}} \end{cases}$$

$$C^{(3)} \leftrightarrow C^{(5)} : \begin{cases} \times & G^{(4)}_{\{5678|\cdot|\cdot\}} & G^{(6)}_{\mu\nu\rho\sigma\{56|\cdot|\cdot\}} & C^{(5)}_{\mu\nu\rho\{56|\cdot|\cdot\}} \\ C^{(3)}_{\{6810|\cdot|\cdot|\cdot\}} & G^{(4)}_{\mu\{6810|\cdot|\cdot|\cdot\}} & G^{(6)}_{\mu\nu\rho\{579|\cdot|\cdot|\cdot\}} & C^{(5)}_{\mu\nu\{579|\cdot|\cdot|\cdot\}} \\ C^{(3)}_{\mu\nu\rho} & G^{(4)}_{\mu\nu\rho\sigma} & G^{(6)}_{5678910} & \times \end{cases}$$

$$\mathbf{A} = e^{B}\mathbf{C} \qquad \mathbf{A} = \sum_{p=\text{odd}} A^{(p)} \qquad \mathbf{C} = \sum_{p=\text{odd}} C^{(p)}$$

3. BIANCHI IDENTITIES & THEIR SOLUTIONS

ALLOW FOR GEOMETRICAL FLUXES $\omega_{jk}{}^{i}$

(CONSTANT PARAMETERS OF SCHERK-SCHWARZ REDUCTIONS)

BI : $d^2 = 0 \quad \Rightarrow \quad \underline{\omega \cdot \omega = 0}$

Modified BI For the NS-NS 3-Form :

$$dH + \omega \cdot H = 0 \Rightarrow H = dB + \omega \cdot B + \bar{H}$$

$$\bar{H} = \text{CONSTANT} \qquad \underline{\omega \cdot \bar{H} = 0}$$

Modified BI For the RR Field-strengths :

obtained varying w.r.t. Lagrange multipliers $A^{(9-p)}$

(i) iF $A^{(9-p)}$ does not couple to localized sources

$$dG^{(p)} + \omega \cdot G^{(p)} + H \wedge G^{(p-2)} = 0 \quad \Rightarrow$$

$$G^{(p)} = dC^{(p-1)} + \omega \cdot C^{(p-1)} + H \wedge C^{(p-3)} + [\bar{G} e^{-B}]^{(p)}$$

$$\bar{G} = \text{CONSTANT} \quad \underline{\omega \cdot \bar{G}^{(p)} + \bar{H} \wedge \bar{G}^{(p-2)} = 0}$$

(ii) since $A^{(7)}$ couples to D6-branes & O6-planes

must consider "PHANTOM" components of $G^{(2)}$

$$\frac{1}{2}\left[dG^{(2)} + \omega \cdot G^{(2)} + H G^{(0)}\right] = \sum_a^{D6,O6} \mu_a \, \delta^{(3)}(\pi_a)$$

INTEGRABILITY CONDITIONS (RR-TADPOLES)

$$\frac{1}{2}\left(\omega\cdot G^{(2)}+\bar{H}\bar{G}^{(0)}\right)_{579} = \sum_a^{D6,O6} \mu_a\, m_1^a\, m_2^a\, m_3^a$$

$$\frac{1}{2}\left(\omega\cdot G^{(2)}+\bar{H}\bar{G}^{(0)}\right)_{5810} = \sum_a^{D6,O6} \mu_a\, m_1^a\, n_2^a\, n_3^a$$

$$\frac{1}{2}\left(\omega\cdot G^{(2)}+\bar{H}\bar{G}^{(0)}\right)_{6710} = \sum_a^{D6,O6} \mu_a\, n_1^a\, m_2^a\, n_3^a$$

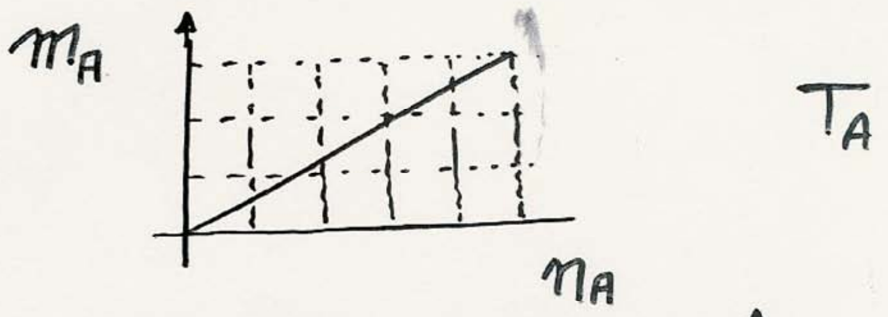
$$\frac{1}{2}\left(\omega\cdot G^{(2)}+\bar{H}\bar{G}^{0}\right)_{689} = \sum_a^{D6,O6} \mu_a\, n_1^a\, n_2^a\, m_3^a$$

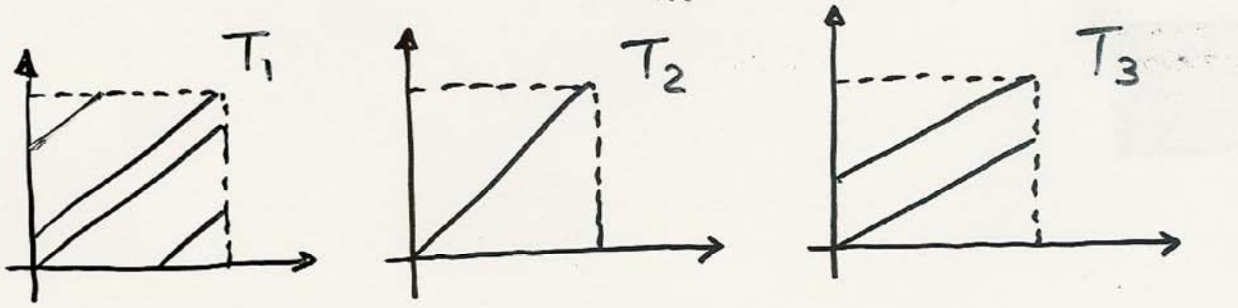
FLUXES

D6-BRANES
O6-PLANES

(n_A^a, m_A^a) WRAPPING NUMBERS ON A-TH 2-TORUS

MODIFIED RR-TADPOLE CANCELLATION CONDITIONS





4. N=1 EFFECTIVE POTENTIAL & SUPERPOTENTIAL

THE SEVEN 'MAIN' MODULI : $(A=1,2,3)$

$$S = s + i\sigma \quad U_A = u_A + i\nu_A \quad T_A = t_A + i\tau_A$$

$$G_{\mu\nu} = \hat{s}\,\tilde{g}_{\mu\nu} \quad G_{i_A j_A} = \begin{pmatrix} t_A \hat{u}_A & 0 \\ 0 & t_A/\hat{u}_A \end{pmatrix}$$

$$e^{-2\phi} = \frac{\hat{s}}{t_1 t_2 t_3} \quad B_{56|78|910} = \tau_{1|2|3}$$

$$C^{(3)}_{6810} = \sigma \quad C^{(3)}_{679|589|5710} = -\nu_{1|2|3}$$

$N=1$ COMPLEXIFICATION FOR

$$s = \sqrt{\frac{\hat{s}}{\hat{u}_1\,\hat{u}_2\,\hat{u}_3}} \qquad u_A = \sqrt{\frac{\hat{s}\,\hat{u}_B\,\hat{u}_C}{\hat{u}_A}}$$

4D EFFECTIVE ACTION OBTAINED BY SUBSTITUTING SOLUTIONS OF MODIFIED BI INTO THE DUAL 10D BULK + DBI + WZ ACTION

KINETIC TERMS CORRESPOND TO

$$K = -\log\left(s\,t_1 t_2 t_3\,u_1 u_2 u_3\right)$$

THE SCALAR POTENTIAL V

$$V = V_E + V_H + V_G + V_6$$

$$V_E = \frac{1}{8}\frac{e_{10}}{\tilde{e}_4} e^{-2\phi}\left(\omega_{jk}{}^{i}\omega_{j'k'}{}^{i'} g_{ii'} g^{jj'} g^{kk'} + 2\omega_{jk}{}^{i}\omega_{j'i}{}^{k} g^{jj'}\right) \quad \begin{matrix}\text{Sc.}\\ \text{Sc.}\end{matrix}$$

$$V_H = \frac{1}{4}\frac{e_{10}}{\tilde{e}_4} e^{-2\phi}\left(\bar{H} + \omega B\right)^2$$

$$V_G = \frac{1}{4}\frac{e_{10}}{\tilde{e}_4}\left[\left(G^{(0)}\right)^2 + \left(G^{(2)}\right)^2 + \left(G^{(4)}\right)^2 + \left(G^{(6)}\right)^2\right]$$

$$= \frac{1}{4}\frac{e_{10}}{\tilde{e}_4}\left[\left(\bar{H}\wedge C^{(3)} + \omega\cdot B\wedge C^{(3)} + \bar{G}^{(6)} - B\wedge\bar{G}^{(4)} + \frac{1}{2}B\wedge B\wedge\bar{G}^{(2)} - \frac{1}{6}B\wedge B\wedge B\wedge\bar{G}^{(0)}\right)^2 + \cdots\right.$$

$$V_6 = \sum_a T_a \frac{e_7^{(a)}}{\tilde{e}_4} e^{-\phi}$$

$N=1$

$\mathcal{J}(\Omega_a) \propto \theta_1 + \theta_2 + \theta_3 = 0$

BERKOOZ - DOUGLAS - LEIGH

$$= e^{K} t_1 t_2 t_3 \sum_a T_a \mathcal{R}(\Omega_a)$$

$$\mathcal{R}(\Omega_a) = m_1^a m_2^a m_3^a s - m_1^a n_2^a n_3^a u_1 - n_1^a m_2^a n_3^a u_2 - n_1^a n_2^a m_3^a u_3$$

BIANCHI IDENTITIES $\Rightarrow$ V_6

IN TERMS OF $\left[\omega\bar{G}^{(2)} + \bar{H}\bar{G}^{(0)}\right]$ 579158101 6710/689

AND (s, u_1, u_2, u_3)

THE SUPERPOTENTIAL W

$$V = V_E + V_H + V_G + V_6 \overset{BI}{=} e^K \Big[\sum_\alpha |W_\alpha + K_\alpha W|^2 - 3|W|^2 \Big]$$

$$\begin{aligned} 4W = & \left(\omega_{810}{}^6 T_1 U_1 + \dots\right) - S\left(\omega_{79}{}^6 T_1 + \dots\right) \\ & - \left(\omega_{89}{}^5 T_1 U_3 + \dots\right) + i\left(\bar{G}^{(4)}_{78910} T_1 + \dots\right) \\ & - \left(\bar{G}^{(2)}_{56} T_2 T_3 + \dots\right) + \bar{G}^{(6)}_{5678910} \\ & - i \bar{G}^{(0)} T_1 T_2 T_3 + i\left(\bar{H}_{579} S + \dots\right) \end{aligned}$$

AS DERIVED BY DERENDINGER, KOUNNAS, PETROPOULOS, FZ FROM CONSISTENT $N=1$ TRUNCATIONS OF $N=4$ GAUGINGS

GEOMETRICAL INTERPRETATION

$$J^c \equiv J + iB \qquad J = \frac{i}{2} \sum_A dz^A \wedge d\bar{z}^A \quad T_A$$

$$\Omega^c \equiv \mathcal{R}\left(i e^{-\phi} \Omega\right) + i C^{(3)} \quad \Omega = dz^1 \wedge dz^2 \wedge dz^3 \quad S, -U_A$$

$$W = \frac{1}{4} \int_{X_6} \bar{G} e^{iJ^c} + \left(\bar{H} - i\omega J^c\right) \wedge \Omega^c$$

$$\text{T-DUALITY}: \quad \bar{G} e^{iJ^c} \leftrightarrow \bar{G}^{(3)} \wedge \Omega_{IIB}$$

cfr. with $\int_{X_6} \left(\bar{G}^{(3)} - i S \bar{H}\right) \wedge \Omega_{IIB}$ GUKOV VAFA WITTEN

5. MODULI STABILIZATION ON N=1 AdS_4

$$V = e^K \Big[\sum_\alpha |W_\alpha + K_\alpha W|^2 - 3|W|^2 \Big]$$

$$\langle W_\alpha + K_\alpha W \rangle = 0 \qquad \langle W \rangle \neq 0$$

N=1 SUSY AdS_4: $\langle V \rangle = -3 \langle e^K |W|^2 \rangle = -3\, m_{3/2}^2$

PLANE-SYMMETRIC CASE ($T_1 \leftrightarrow T_2 \leftrightarrow T_3$)
AS IN DKPZ

IMPOSING ONLY N=1 & B.I. :

$$\frac{1}{9} \overline{G}^{(6)} = -t_0^2\, \overline{G}^{(2)} = \frac{t_0 u_0}{6} \omega_1 = \frac{s_0 t_0}{2} \omega_2 = \frac{t_0 u_0}{6} \omega_3$$

$$\frac{t_0}{3} \overline{G}^{(4)} = \frac{t_0^2}{5} \overline{G}^{(0)} = -\frac{s_0}{2} \overline{H}_0 = \frac{u_0}{2} \overline{H}_1$$

ALL MODULI STABILIZED A LA BREITENLOHNER-FREEDMAN

$$m^2 > -(9/4)\, m_{3/2}^2$$

TWO INDEPENDENT PARAMETERS ; 1 GOLDSTONE AXION
(LINEAR COMBINATION OF $\sigma, \nu_1, \nu_2, \nu_3$)

REQUIRING N=4 TRUNCATION AS IN DKPZ :

$$5 u_0^2\, \overline{H}_1 = 3 s_0^2 t_0^2 \omega_2^2 \quad \text{(1 EXTRA CONDITION)}$$

STILL ONE-PARAMETER FAMILY OF AdS_4 VACUA

6. CONCLUSIONS & OUTLOOK

SUMMARY OF RESULTS

IIA ON $T^6/(Z_2 \times Z_2)$ AND ΩI_3 ORIENTIFOLD WITH GENERIC FLUXES AND BRANES ($N=1$)
(BUT THE METHOD IS MORE GENERAL)

- DUAL FORMULATION $\Rightarrow$ MODIFIED BI
  VZ $\Rightarrow$ GENERAL TADPOLE CANCELLATION CONDITIONS

- EFFECTIVE POTENTIAL AND SUPERPOTENTIAL
  DKPZ VZ FOR THE SEVEN 'MAIN' MODULI

- VZ GEOMETRICAL EXPRESSION FOR W

- $N=1$ SUSY AdS_4 VACUA WITH ALL 7
  DKPZ VZ GEOMETRICAL MODULI PERTURBATIVELY FIXED

$N=1$ SUSY AdS_4 vacua also studied by
BEHRNDT-CVETIC LÜST-TSIMPIS
looking at Fermionic variations

EXTENSIONS / GOALS

- INCLUSION OF BRANE FLUCTUATIONS
- LOCALIZED MAGNETIC FLUXES (WITH THEIR GENERALIZED BI AND TADPOLE CANCELLATION CONDITIONS)
- CONSISTENT INCLUSION OF D-TERMS WITH THEIR FULL FIELD-DEPENDENCE
- SEARCH FOR STABLE NON-adS_4 VACUA (CONSISTENT SUPERGRAVITY 'UPLIFTINGS')
- CALCULATION OF SOFT TERMS AROUND SEMI-REALISTIC VACUA
- PERTURBATIVE AND NON-PERTURBATIVE CORRECTIONS TO THE CLASSICAL RESULTS
- ...

RECENT RELATED DEVELOPMENTS

- DE WOLFE - GIRYAVETS - KACHRU - TAYLOR hep-th/0505160

  $\mathrm{IIA}\quad T^6/(\mathbb{Z}_3 \times \underset{\mathbb{Z}_{FA}}{\mathbb{Z}_3})\quad \Omega I_3\quad D6/O6$

  only $\bar{G}^{(4)}, \bar{G}^{(0)}, \bar{H}\ [, \bar{G}^{(2)}]$ Fluxes

  [ω and $\bar{G}^{(6)}$ not considered]

  - explicit discussion of large volume/small coupling
  - discussion of blow-up modes of orbifold
  - some discussion of vacuum statistics

- CAMARA - FONT - IBÁÑEZ hep-th/0506066

  $\mathrm{IIA}\quad T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)\quad \Omega I_3\quad D6/O6$

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