Compactifications on Generalized Geometries

Jan Louis

Universität Hamburg

work in collaboration with I. Benmachiche, M. Graña, D. Waldram

String Phenomenlogy 2005, München, June 2005

Introduction

String Theory: strings moving in 10-dimensional space-time background M_{10}

contact with particle physics: compactification

$$M_{10} = M_4 \times Y_6$$

M_4: 4 dim. space-time, Y_6: Calabi-Yau threefold

Problems:

- mechanism of supersymmetry breaking
- stabilization of moduli

Recently:

- Standard Model on D-branes
- supersymmetry breaking/moduli stabilization with background fluxes

  alternatively: compactifications on generalized class of manifolds

Purpose of this talk:

➪ discuss string compactifications on manifolds with $SU(3)$-structure

➪ compute Kähler potential and potential

Compactification: determine Y_6

Lorentz group on space-time background $M_{10} = M_4 \times Y_6$ decomposes

$$SO(1,9) \to SO(1,3) \times SO(6)$$

spinor decomposes accordingly: $\mathbf{16} \to (\mathbf{2}, \mathbf{4}) \oplus (\bar{\mathbf{2}}, \bar{\mathbf{4}})$

phenomenologically most interesting case: minimal amount of supersymmetry

$\Rightarrow$ impose two conditions:

1. demand that supercharge Q exist $\Rightarrow$ structure group of Y_6 has to be reduced

$$SO(6) \to SU(3) \quad \text{s.t.} \quad \mathbf{4} \to \mathbf{3} + \mathbf{1}$$

   $\Rightarrow$ invariant spinor η exists $\Rightarrow$ Y_6 has $SU(3)$-structure

2. background preserves supersymmetry

$$\delta\Psi_M = \nabla_M \eta + (\gamma \cdot F)_M = 0 \,, \qquad F = \text{background flux}$$

   - for $F = 0$: $\nabla_M \eta = 0 \Rightarrow Y_6$ is Calabi-Yau manifold Y
   - here: study $\nabla_M \eta \neq 0$

Manifolds with $SU(3)$ structure: characterized by two invariant tensors J, Ω

(follows from existence of invariant spinor η)

➪ two-form

$$J_{mn} = \eta^\dagger \gamma_{[m}\gamma_{n]}\eta \ , \qquad dJ \neq 0$$

$\Rightarrow$ almost complex structure

$$I_m{}^n = J_{mp} g^{pn} \ , \qquad I^2 = -1 \ , \qquad N(I) \neq 0$$

J is $(1,1)$-form with respect to I

➪ $(3,0)$-form

$$\Omega_{mnp} = \eta^\dagger \gamma_{[m}\gamma_n\gamma_{p]}\eta \ , \qquad d\Omega \neq 0$$

Remarks:

- $dJ, d\Omega \ \sim$ torsion of Y_6
- Calabi-Yau: $\nabla\eta = 0 \quad \Rightarrow \quad dJ = d\Omega = N(I) = 0$

Low energy effective action: [Strominger; Bodner,Cadavid,Ferrara; Candelas,de la Ossa]

$$
\begin{aligned}
S_{\mathsf{NS}} &= \int d^{10}x\sqrt{g}\,e^{-2\phi}\left[R + 4(\partial\phi)^2 - \tfrac{1}{12}H^2\right] \\
&= \int d^{10}x\sqrt{g^{(4)}}\left[R^{(4)} - 2(\partial\phi^{(4)})^2 - \tfrac{1}{12}e^{-4\phi^{(4)}}H^2_{(4)}\right. \\
&\qquad -\tfrac{1}{4}G^{mp}G^{nq}(\partial_\mu G_{mn}\partial^\mu G_{pq} + \partial_\mu B_{mn}\partial^\mu B_{pq}) + \ldots
\end{aligned}
$$

where $$G^{(4)}_{\mu\nu} = e^{-2\phi_4}G_{\mu\nu}\ , \qquad \phi^{(4)} = \phi - \tfrac{1}{4}\ln\det G_{mn}$$

interpret last term as metric on the space of metric/B-field-deformations and decompose

$$\delta G_{mn} = \left[\delta G_{mn}\right]_{\mathbf{8}} + \left[\delta G_{mn}\right]_{\mathbf{6}+\bar{\mathbf{6}}} = \delta J + \delta\Omega + \delta\bar{\Omega}$$

put S into σ-model form

$$S = -\int_{M_4} d^4x\sqrt{g^{(4)}}g_{ab}(z)\,\partial_\mu z^a\partial^\mu z^b + \ldots$$

Scalar fields of compactification: deformations of metric/B-field
$\simeq$ deformations of J, Ω

σ-model metric in 4d $\mathcal{L}_{\text{eff}}$: product of special geometries [Grana,Waldram,JL]

$$\mathcal{M} = \mathcal{M}_J \times \mathcal{M}_\Omega$$

with

$$g_{ab} = \partial_a \partial_b (K_J + K_\Omega) , \qquad e^{-K_J} = \int_{Y_6} J \wedge J \wedge J , \qquad e^{-K_\Omega} = \int_{Y_6} \Omega \wedge \bar{\Omega}$$

Remarks:

- same expression as in Calabi-Yau compactifications
  but with non-harmonic modes contributing $\Rightarrow$ Hitchin functional
- geometry can already be discovered in $D = 10$
  (i.e. including full tower of KK-states [de Wit, Nicolai])
- type II: add RR-sector [Cecotti,Ferrara,Girardello; Ferrara,Sabharwal]

$$\mathcal{M}_{J/\Omega} \subset \mathcal{M}_H^{QK} \qquad \text{(c-map)}$$

Compute potential from supersymmetry transformation of gravitino [Grana,Waldram,JL]

type II: $N = 2$ in $D = 4$

$$\delta\psi_{A\,\mu} = D_\mu \varepsilon_A + i\gamma_\mu S_{AB}\varepsilon^B + \dots\ , \qquad S_{AB} = \tfrac{i}{2}\, e^{\frac{1}{2}K_V}\,\vec{\sigma}_{AB}\vec{P}\ , \qquad A = 1,2$$

IIA: $\vec{P} = \text{Killing prepotentials}$

$$P^1 + iP^2 \;=\; e^{\frac{1}{2}K_\Omega + \phi^{(4)}} \int_{Y_6} e^{-(B+iJ)} \wedge d\Omega\ , \qquad P^3 \;=\; e^{2\phi^{(4)}} \int_{Y_6} e^{-(B+iJ)} \wedge F_{\mathsf{A}}$$

IIB $F \equiv \sum_{\text{RR-forms}} F^{\text{RR}}$

$$P^1 - iP^2 = e^{\frac{1}{2}K_J + \phi^{(4)}} \int_{Y_6} \Omega \wedge d e^{-(B+iJ)}\ , \qquad P^3 = -e^{2\phi^{(4)}} \int_{Y_6} \Omega \wedge F_{\mathsf{B}}$$

Remarks:

- potential: $V = V(\vec{P}, D\vec{P})$
- torsion $d\Omega$, dJ appear in $\vec{P}$ but not in K
- compute F, D-terms of $N = 1$ by further truncation

  [Andrianopoli,D'Auria,Ferrara,Trigiante,Vaula; Grimm,JL]

  or directly from the heterotic string

Heterotic string: $N = 1$ in $D = 4$

$$\delta\psi_\mu = D_\mu \varepsilon + i\gamma_\mu e^{K/2} W \varepsilon + \ldots \ , \qquad W = \text{superpotential}$$

$$W \ = \ \int_{Y_6} \Omega \wedge d e^{-(B+iJ)} \ ,$$

[Cardoso,Curio,Dall'Agata,Lüst,Manousselis,Zoupanos; Becker,Becker,Dasgupta,Green; Gurrieri,Lukas,Micu; Benmachiche,JL]

Expand in basis of forms [D'Auria,Ferrara,Trigiante,Vaula; Grana,Waldram,JL]

$$B + iJ = t^i \omega_2^i \ , \qquad \Omega = z^A \alpha_3^A - F_A(z) \beta_3^A$$

ω_2^i basis of 2-forms, (α_3^A, β_3^A) basis of 3-forms

with

$$d\omega_2^i = m_A^i \alpha^A - e_A^i \beta^A \ , \qquad H_3 = m_A^0 \alpha^A - e_A^0 \beta^A \ ,$$

torsion: m_A^i, e_A^i , H-flux: m_A^0, e_A^0

$$\Rightarrow \qquad W = z^A e_A^I t_I + t_I m_A^I F_A \ , \qquad t_I = (t_0, t_i)$$

[cf. Berglund, Mayr]

Summary

- ➪ compactifications on manifolds with $SU(3)$ structure
  - $\Rightarrow$ product of special geometries
  - $\Rightarrow$ $K = K_J + K_\Omega$ is independent on torsion
- ➪ potential depends on torsion and background fluxes

- ➪ phenomenological questions:
  - stabilization of moduli
  - inclusion of D-branes
  - supersymmetry breaking