# Fixing All Moduli for M-Theory on K3 x K3

**Renata Kallosh**

**Stanford**

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Aspinwall, R. K. hep-th/0506014
R.K., Kashani-Poor, Tomasiello, hep-th/0503138
Bergshoeff, R. K., Kashani-Poor, Sorokin,Tomasiello, work in progress

# K3 surface

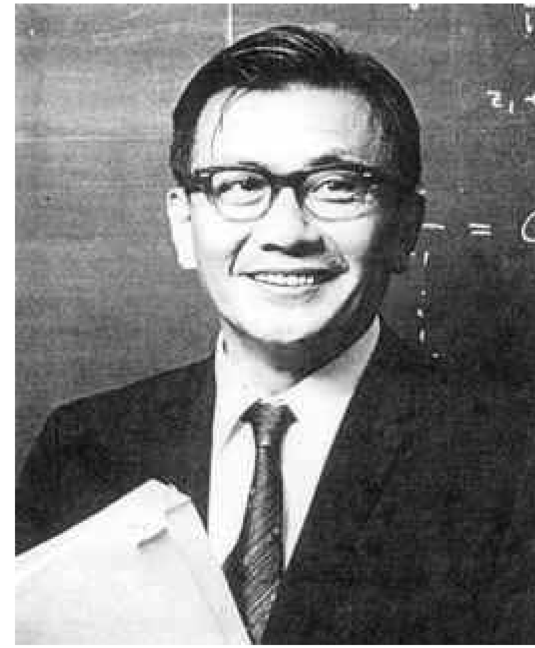
**Eduard Kummer**

**1810-1893**

**Erich Kähler**

**1906-2000**

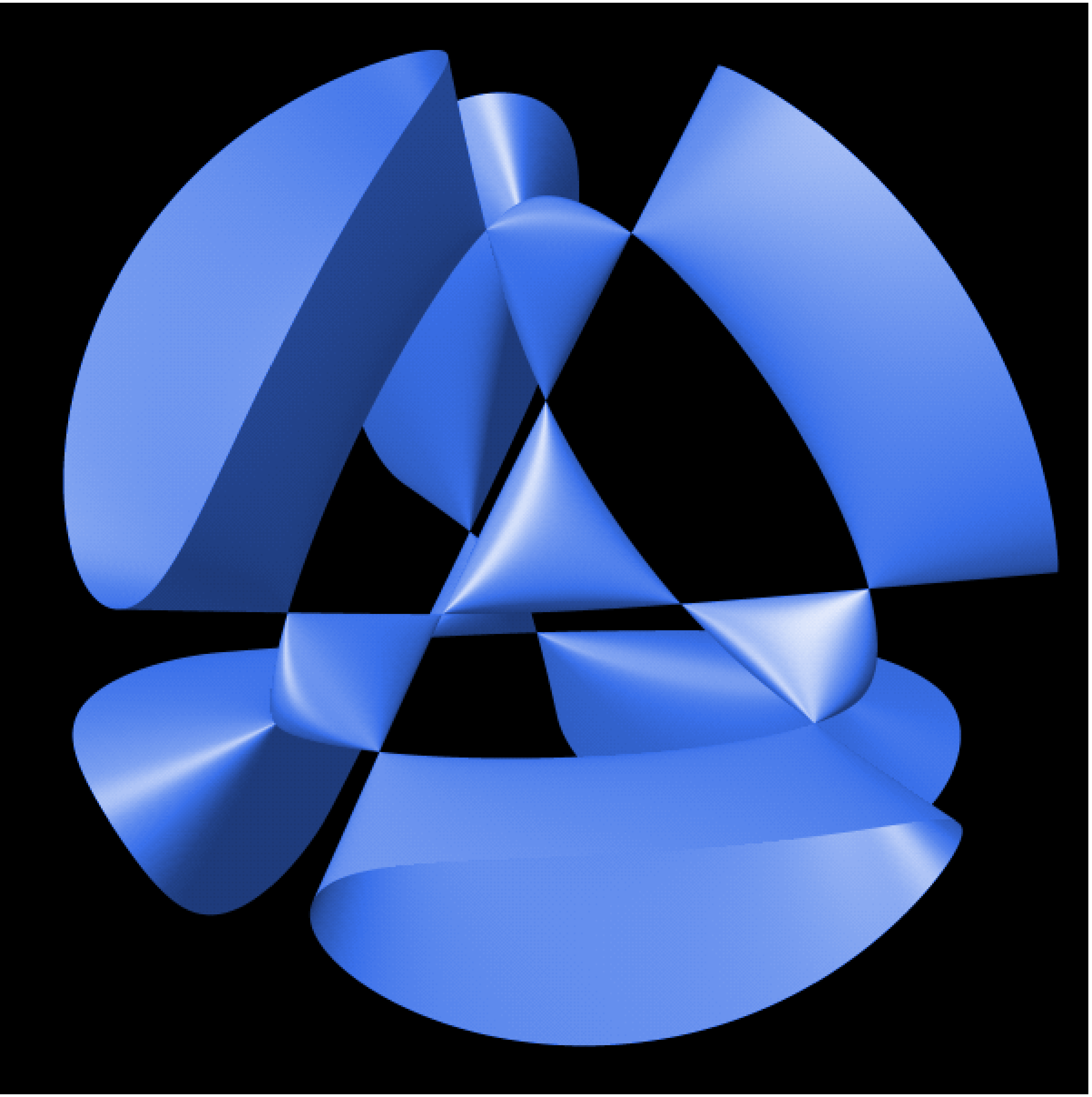
**Kunihiko Kodaira**



**1915-1997**

K2 (8,611 m) is the second highest mountain in the world and is regarded as one of the hardest to climb.

The name K3 surface was given in the 50’s when the mountain peak K2 was in the news



## Kummer surfaces

**In 1864**

Ernst Eduard Kummar gave the following real one-dimensional family of surfaces of degree four (quartics):

$$X_{4,\mu} = \left\{ \left(x^2 + y^2 + z^2 - \mu^2 w^2\right)^2 - \lambda\, pqrs = 0 \right\}$$

$$\lambda = \frac{3\mu^2 - 1}{3 - \mu^2}, \quad \mu \in \mathbb{R}$$

$$\begin{aligned} p &= w - z - \sqrt{2}\,x \\ q &= w - z + \sqrt{2}\,x \\ r &= w + z + \sqrt{2}\,y \\ s &= w + z - \sqrt{2}\,y \end{aligned}$$

A Kummer surface has **sixteen double points**, the maximum possible for a surface of degree four in three-dimensional space

# Paul Aspinwall's K3 movie

- K3 surface with the **variation of the deformation parameter**

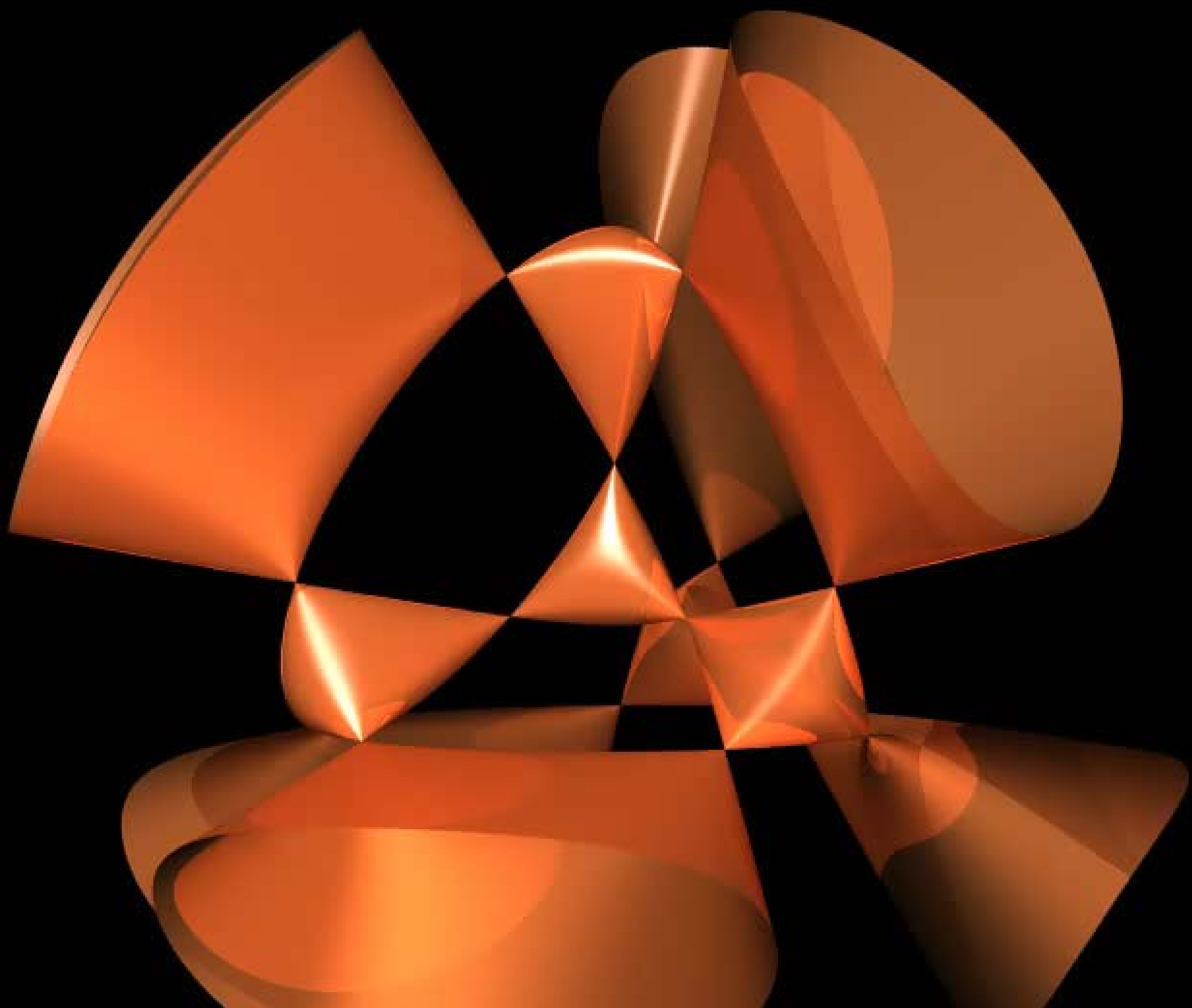
- A **non-singular** quartic surface in projective space of three dimensions is a K3 surface. This determines the Betti numbers:

  1, 0, 22, 0, 1.

# From Kummer to K3

- **Kummer surface** is the quotient of a two-dimensional abelian variety A by the action of $a \rightarrow -a$. This results in 16 singularities.

- Such a surface has a birational **desingularisation** as a quartic in *P*3, which provides the earliest construction of **K3 surfaces.**



# M-theory compactified on K3xK3:

## incredibly simple and elegant

- Without fluxes in the compactified 3d theory there are two 80-dimensional quaternionic **Kähler** spaces, one for each K3.

- With non-vanishing primitive (2,2) **flux**, (2,0) and (0,2), each K3 becomes an **attractive K3**: one-half of all moduli are fixed

- 40 in each K3 still remain moduli and need to be fixed by instantons. There are **20 proper 4-cycles in each K3 and they provide instanton corrections from M5-branes wrapped on these cycles**:

**moduli space is no more**

Aspinwall, R. K. hep-th/0506014

F-theory limit dual to an orientifold of

# **IIB string compactified on** K3x $\frac{T^2}{\mathbf{Z}_2}$

**A natural space for D3/D7 cosmological model**

- Flux vacua in this model were studied by Trivedi, Tripathy in string theory, Angelantonj, D'Auria, Ferrara, Trigiante in string theory and 4d gauged supergravity.
- The minimal remaining moduli space is

Vectors → | ← Hypers

$$\frac{U(1,1+n_3)}{U(1)\times U(1+n_3)} \times \frac{SO(2,18)}{SO(2)\times SO(18)}$$

KKLT stabilization possible!

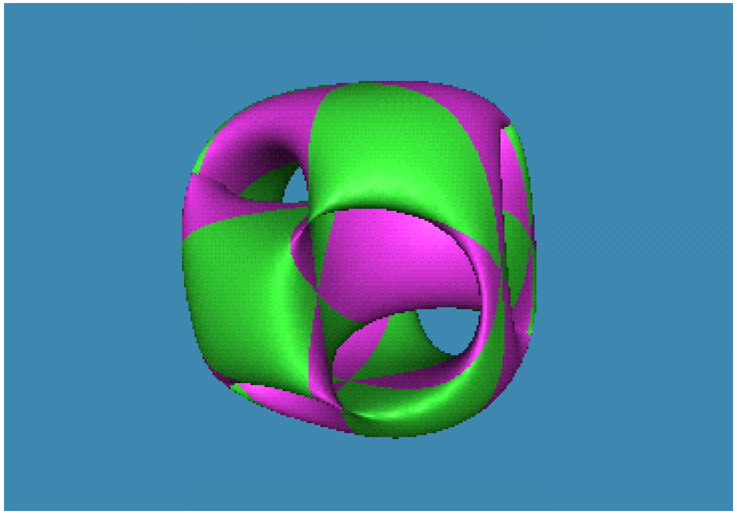
Includes the volume of K3

Includes the volume of $\frac{T^2}{\mathbf{Z}_2}$

**Is KKLT stabilization possible?**

# Major problem

The mechanism of volume stabilization in this (and many other models of string theory) does not seem to work.



KKLT 1: gaugino condensation: works only for vector multiplets, does not work for hypers

***KKLT 2: instantons from euclidean 3-branes wrapped on 4-cycles (may work for vector multiplets and hypers)***

- On K3x $\frac{T^2}{Z_2}$ there are 4-cycles which may or may not lead to non-vanishing instantons

## Status in 2004

According to the rules (**established before fluxes were introduced**), the relevant M-theory divisors in our model have an arithmetic genus 2 and therefore there are no instantons.

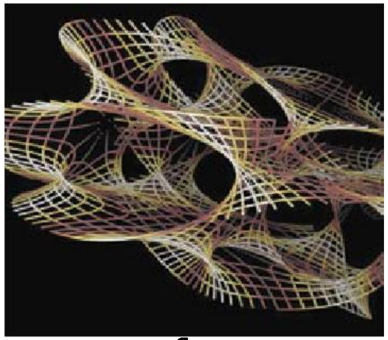
Witten 1996: in type IIB compactifications under certain conditions there can be corrections to the superpotential coming from Euclidean D3 branes. His argument was based on the M-theory counting of the fermion zero modes in the Dirac operator on the M5 brane wrapped on a 6-cycle of a Calabi-Yau four-fold. He found that

such corrections are possible

only in case that the four-fold admits divisors of arithmetic genus one,

$$\chi_D \equiv \sum(-1)^n h^{(0,n)} = 1$$

In the presence of such instantons, there is a correction to the superpotential which at large volume yields a new term



$$W_{\text{inst}} \sim \exp(-a\rho)$$

In type IIB string theory the leading exponential dependence comes from the action of an Euclidean D3 brane wrapping a 4-cycle.

Until recently, Witten's constraint $\chi_D = 1$ was a guide for constructing all models in which Kähler moduli are stabilized via superpotentials generated by Euclidean instantons.

- First indication that in presence of fluxes there are more possibilities were given by Gorlich, Kachru, Tripathy and Trivedi:

$$\chi_D \geq 1$$

- **We have performed a systematic study of a general case**
- Constructed the Dirac operator on M5 brane with background fluxes
- Performed the counting of fermionic zero modes and have found the generalized "index"
- Constructed the Dirac operator on D3 brane with background fluxes and defined its "index" (no need for F-theory limit)

# M5 brane

- Dirac action on M5 with background fluxes

$$\Gamma^a \hat{\nabla}_a \theta = 0$$

- Here $\hat{\nabla}_a$ is a covarinat derivative including **torsion when fluxes in the background M theory are present**

$$\Gamma^a (\nabla_a + T_a{}^{\underline{abcd}} F_{\underline{abcd}})\theta_- = 0$$

R. K. and Sorokin

# Counting fermionic zero modes M5 with fluxes

Saulina

R.K., Kashani-Poor, Tomasiello, hep-th/0503138

- New computation of the normal bundle U(1) anomaly

$$\chi_D(F) = \chi_D - (h^{(0,2)} - n)$$

- Here n is the dimension of solutions of the constraint equation which depends on fluxes. $0 \leq n \leq h^{(0,2)}$
- To have instantons we need $\chi_D(F) = 1$

$\chi_D \neq 1$

# Witten's condition is generalized

- New vacua with $\chi_D(F) = 1$ $\chi_D \neq 1$

  $\chi_D = 1$

- Examples include "friendly" parts of the landscape with flat directions

# Example of $K3 \times K3$ compactification

An M5 brane wrapps a 6-cycle $D = K3 \times \mathbb{P}^1$ of the 4-fold $X = K3 \times K3$. The Dirac equation in the fluxless case has only positive chirality solution,

$$\epsilon_+ = \phi|\Omega\rangle + \phi_{\overline{12}}\Gamma^{\overline{12}}|\Omega\rangle \, .$$

consider a flux which is a $(2,0)+(0,2)$ form on $K3$

$$\frac{F}{\pi} = \Omega_1 \wedge \bar{\Omega}_2 + \bar{\Omega}_1 \wedge \Omega_2$$

In this example, we lose a zero mode of the Dirac operator, namely $\phi_{\overline{a}\overline{b}}\Gamma^{\overline{a}\overline{b}}|\Omega\rangle$, upon turning on flux. Therefore, $\chi_D = 2$, while $\chi_D(F) = 1$.

**Stabilization of Kahler moduli is possible!**

# ATTRACTIVE K3 SURFACES

- G. Moore, in lectures on Attractors and Arithmetic
- **In M-theory on K3xK3** Aspinwall, R. K.
- **The complex structures are uniquely determined by a choice of flux G**

The K3 surface is attractive if the rank of the Picard lattice has the maximal value, 20 and the rank of the transcedental lattice (orthogonal complement of the Picard lattice) is 2.

- They are in one-to-one correspondence with $Sl(2,Z)$ equivalence classes of positive-definite even integral binary quadratic forms, which can be written in terms of a matrix

$$j = 1,2 \qquad Q_j = \begin{pmatrix} p_j^2 & p_j.q_j \\ p_j.q_j & q_j^2 \end{pmatrix} = \begin{pmatrix} 2a & b \\ b & 2c \end{pmatrix}$$

- **Both K3 surfaces whose complex structures are fixed by G are forced to to be attractive**

$$\Omega_j = p_j + \tau_j q_j \qquad \tau_j = \frac{-p_j.q_j + i\sqrt{\det Q_j}}{q_j^2}$$

# Flux vacua and attractors

- The attractor value of the area of the black hole horizon

$$\frac{A_j}{4\pi} = |Z|^2_j = \sqrt{detQ_j}$$

- The area of the unit cell $\sqrt{detQ_j}$ in the transcedental lattice of the attractive K3 is precisely the attractive value of the horizon area of the corresponding black hole.

- May be some deeper relation between flux vacua and attractors still to be discovered…

**We proved that the number of solutions is finite up to $Sl(2,Z)$ using some theorems from** **number theory**

**We have found finite number of solutions of the tadpole condition**

(for the simplest case without M2 branes)

$$\frac{1}{2}G^2 = 48$$

| $Q_1$ | $Q_2$ | $\gamma$ | O? | $Q_1$ | $Q_2$ | $\gamma$ | O? |
|---|---|---|---|---|---|---|---|
| $\begin{pmatrix}4 & 0\\0 & 2\end{pmatrix}$ | $\begin{pmatrix}4 & 0\\0 & 2\end{pmatrix}$ | $1+\frac{i}{\sqrt{2}}$ | ✗ | $\begin{pmatrix}4 & 2\\2 & 4\end{pmatrix}$ | $\begin{pmatrix}2 & 1\\1 & 2\end{pmatrix}$ | $2+\frac{2i}{\sqrt{3}}$ | ✓ |
| $\begin{pmatrix}4 & 2\\2 & 4\end{pmatrix}$ | $\begin{pmatrix}6 & 0\\0 & 2\end{pmatrix}$ | $1+\frac{i}{\sqrt{3}}$ | ✓ | $\begin{pmatrix}6 & 0\\0 & 4\end{pmatrix}$ | $\begin{pmatrix}6 & 0\\0 & 4\end{pmatrix}$ | $\frac{2i}{\sqrt{6}}$ | ✗ |
| $\begin{pmatrix}6 & 0\\0 & 6\end{pmatrix}$ | $\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$ | $1+i$ | ✗ | $\begin{pmatrix}6 & 0\\0 & 6\end{pmatrix}$ | $\begin{pmatrix}4 & 0\\0 & 4\end{pmatrix}$ | $1$ | ✓ |
| $\begin{pmatrix}8 & 4\\4 & 8\end{pmatrix}$ | $\begin{pmatrix}4 & 2\\2 & 4\end{pmatrix}$ | $1+\frac{i}{\sqrt{3}}$ | ✓ | $\begin{pmatrix}12 & 0\\0 & 2\end{pmatrix}$ | $\begin{pmatrix}12 & 0\\0 & 2\end{pmatrix}$ | $\frac{i}{\sqrt{6}}$ | ✗ |
| $\begin{pmatrix}12 & 0\\0 & 4\end{pmatrix}$ | $\begin{pmatrix}2 & 1\\1 & 2\end{pmatrix}$ | $1+\frac{i}{\sqrt{3}}$ | ✓ | $\begin{pmatrix}12 & 0\\0 & 4\end{pmatrix}$ | $\begin{pmatrix}6 & 0\\0 & 2\end{pmatrix}$ | $\frac{i}{\sqrt{3}}$ | ✓ |
| $\begin{pmatrix}12 & 0\\0 & 6\end{pmatrix}$ | $\begin{pmatrix}4 & 0\\0 & 2\end{pmatrix}$ | $\frac{i}{\sqrt{2}}$ | ✗ | $\begin{pmatrix}12 & 0\\0 & 12\end{pmatrix}$ | $\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$ | $1$ | ✓ |
| $\begin{pmatrix}16 & 8\\8 & 16\end{pmatrix}$ | $\begin{pmatrix}2 & 1\\1 & 2\end{pmatrix}$ | $1+\frac{i}{\sqrt{3}}$ | ✓ | | | | |

Table 1: The 13 pairs of matrices $Q_1, Q_2$ yielding the possible attractive K3 surfaces. The column headed "O?" shows 8 solutions when one of K3 surfaces is a "Kummer surface"

# Obstructed instantons

- For more general choice of fluxes we find conditions when the instantons can be generated.

- When these conditions are not satisfied fluxes will obstruct the existence of the stabilizing instanton corrections.

# The orientifold on K3x$\frac{T^2}{\mathbf{Z}_2}$

- ***Only F-theory compactifications on K3xK3 where one of the attractive K3 is a Kummer surface describes an orientifold in IIB***

$$Q = 2R$$

- Attractive K3 surface is a Kummer surface if, and only if, the associated even binary quadratic form is twice another even binary quadratic form.

# Instanton corrections

- With account of the new index theorem we find that instantons are generated for M5 branes wrapping

$$K3_1 \times P^1 \qquad P^1 \times K3_2$$

- For a flux of the form

$$G = Re(\gamma \Omega_1 \wedge \bar{\Omega}_2)$$

each K3 surface is attractive and, as such, has Picard number equal to 20. This leaves each K3 with 20 complexified Kahler moduli.

- **We proved that there are 40 independent functions on 40 variables. All moduli unfixed by fluxes are fixed by instantons.**

# Orientifold limit

- Take an F-theory limit of M-theory on K3xK3. We find an equivalent statement about instanton corrections for IIB on K3x $T^2/\mathbf{Z}_2$

- The M5 instanton must wrap either an elliptic fibre or a “bad fibre” (fibre which is not an elliptic curve), classified by **Kodaira**. With account of these two possibilities we find

- Instantons from D3 branes wrapping $P^1 \times \frac{T^2}{Z_2}$ from M5 on $P^1 \times K3_2$

- Instanton from D3 wrapped on K3xpt from M5 which was wrapped on $K3_1 \times P^1$ where $P^1$ is a “bad fibre”.

- We find the right number of cycles to fix all moduli which were not fixed by fluxes (one should be careful about obstructed instantons).

# The index of the Dirac operator on D3 brane with background fluxes

Bergshoeff, R. K. Kashani-Poor, Sorokin, Tomasiello

- We study the instanton generated superpotentials in Calabi--Yau orientifold compactifications **directly in IIB**.

- We derive the Dirac equation on a euclidean D3 brane in the presence of background flux, which governs the generation of a superpotential in the effective 4d theory by D3 brane instantons.

# EXAMPLES

- Applying the formalism to the K3x $\frac{T^2}{Z_2}$ orientifold we show that our results are consistent with conclusions attainable via duality from an M-theory analysis.

- In case of $T^6/Z_2$ we find that $\chi_D(F) = 1$ and instanton corrections are possible when the divisor hits the O-plane.

- We also find that $\chi_D(F) = 0$ and instanton corrections are not possible when the divisor is off the O-plane, in agreement with Trivedi,Tripathy

# IIB on K3x $\frac{T^2}{Z_2}$

- Our results shows that the goal of fixing all moduli (but the positions of D3 branes) in this model is now accomplished (either by duality from M-theory or directly in IIB).

- The model is consitent with 16 D7 branes stabilized at the fixed points of the thetrahedron.

- Fixing of some moduli by fluxes was achieved by Trivedi, Tripathy, and Angelantonj, D'Auria, Ferrara, Trigiante.
- All remaining moduli (but D3) can be fixed by instantons from Euclidean D3 branes wrapped on specific 4-cycles.

# Mobile D3 brane?

- In our direct approach only the positions of D3 branes are not fixed by either fluxes or known instantons from wrapped branes

**Berg, Haack, Kors ???**
**problem with calculations**
**1.Valid only in absence of flux**
**2. All 16 D7 on top of each other**
**3. Now we have no gaugino condensation**

**Assumption about the use of the worldsheet instantons and duality in presence RR fluxes with N=1 susy ???**

**D. Lust, P. Mayr, S. Reffert, S. Stieberger**

**Ori Ganor???**

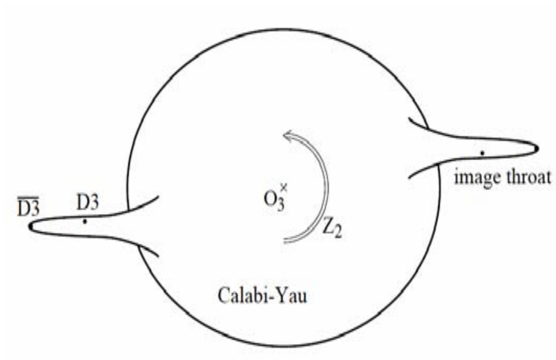
**Berglund, Myer ???**

# Stabilization of moduli via instantons

- When is this possible/impossible?

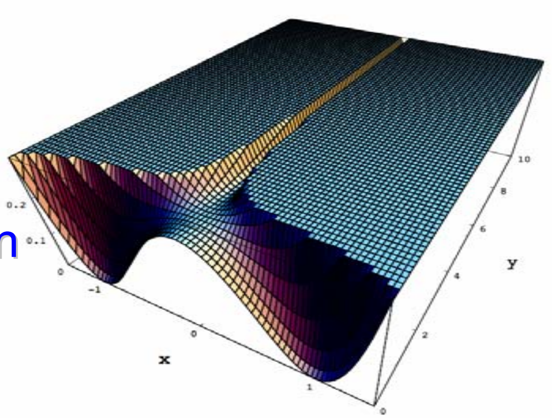
- Can we use fluxes and instantons (with account of other corrections) to fix all moduli but the inflaton?
- Would be most important for string cosmology.

# New class of inflationary models in string theory

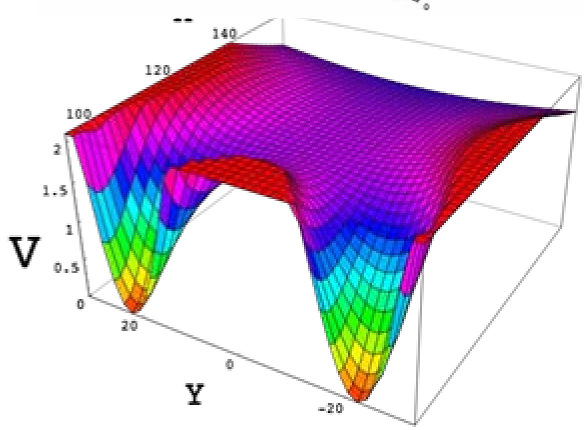



KKLMMT brane-anti-brane inflation

D-term inflation




**D3/D7 brane inflation**




**Racetrack modular inflation**

**DBI inflation**