String Phenomenology
München, 16th June 2005

Effective Theories and Potentials from Twisted Tori Compactifications with Fluxes

Gianguido Dall'Agata
(CERN - Geneva)

G.D. and S. Ferrara hep-th/0502066 (NPB 717)
G.D, R. D'Auria and S. Ferrara, hep-th/0503122 (PLB in press)
G.D. and N. Prezas, *in progress*

Outline

- Why twisted tori?
- Gauged supergravity algebras (for M-theory)
- Tensor fields and **Free Differential Algebras**
- M-theory on *twisted* G2-manifolds
  - IIA reduction
  - Hořava-Witten and Heterotic potentials
- Summary and Outlook

Why twisted tori?

- Standard approach: IIB on Calabi-Yau w/ fluxes fixes complex-structure moduli. Non-perturbative effects stabilize Kæhler ones.

- *Fluxes* generate *Backreaction* and hence *more general geometries*. Twisted tori provide **examples** of such backgrounds and can stabilize all moduli

- *Twisted Tori compactification = Scherk-Schwarz reduction* (We can use gauged supergravity techniques)

- "Experimental" way to derive general properties of effective theory and flux backgrounds

- Fluxes and Scherk-Schwarz determine non abelian gauge couplings:

$$\langle H_{IJK}(x,y)\rangle = h_{IJK}$$

$$\int H \wedge *H = \int d^4x\sqrt{-g}(\partial_\mu B_{\nu I} g^{\mu J} g^{\nu K} h_{JK}{}^{I}) + \cdots$$

- *Gauged SUGRA* (couplings and potential) fixed by the gauge group (and symplectic embedding)
- The gauge algebra follows from reduction of the 10/11d symmetries
- Vector fields from the metric $g_{\mu I}$ and tensors $B_{\mu I}$
- **Jacobi** identities = 10/11d **Bianchi** identities

Scherk-Schwarz reduction as Twisted Tori compactification

- Scherk-Schwarz reduction gives y-dependence to scalar fields

$$\Phi(x,y) = e^{imy}\,\Phi(x)$$

$$\Phi_\alpha(x,y) = U_\alpha{}^\beta(y)\,\Phi_\beta(x)$$

if $\Phi \rightarrow e^{i\alpha}\Phi$ is a global symmetry

- General reductions can use 2 types of symmetries
  - External: do not involve space-time. BERGSHOEFF ET AL. Example: type IIB has SL(2,R) symmetry and one can gauge a subgroup in S^1 compactifications
  - Internal: the matrix U(y) acts on the metric scalars

Scherk-Schwarz reduction as Twisted Tori compactification

- Metric ansatz:

$$g_{IJ}(x,y) = U_I{}^K(y) U_J{}^L(y) h_{KL}(x)$$

- In order for the y-dependence to disappear from the 4d Lagrangian one has to satisfy

Constant

$$U_I^{-1\,L}(y) U_J^{-1\,M}(y) \partial_{[L} U_{M]}{}^K = \tau_{IJ}^K$$

- This is the connection for a parallelizable group manifold (also twisted tori)

KALOPER-MYERS

$$e^I = dy^J\, U_J{}^I \qquad de^K + \frac{1}{2}\tau_{IJ}^K\, e^I \wedge e^J = 0$$

Example: M-theory on Twisted Tori with fluxes

G.D.-FERRARA

M-theory on Twisted T^7 with fluxes

- Fields $\quad G_{\mu\nu} \qquad G_{\mu I},\ A_{\mu IJ} \qquad G_{IJ}, A_{IJK}, A^I_{\mu\nu I}$

METRIC 28 VECTORS 70 SCALARS

$$\mathcal{L}_\omega = i_\omega d + d i_\omega$$

- Gauge transformations in 4d are relic of 10d translation and tensor gauge symmetries

$$de^I = \frac{1}{2}\tau^I_{JK} e^J \wedge e^K$$

$$e^I \longrightarrow e^I + i_\omega de^I + d i_\omega e^I$$

$$A \longrightarrow A + d\Sigma + i_\omega dA + d i_\omega A$$

- Resulting 4d gauge transformations

TENSOR

SCHERK-SCHWARZ

FLUX

$$\delta G^I_\mu = \partial_\mu \omega^I - \tau^I_{JK}\omega^J G^K_\mu$$

$$\delta A_{\mu IJ} = \partial_\mu \Sigma_{IJ} - 2\Sigma_{L[I}\tau^L_{J]K} G^K_\mu - \tau^S_{IJ}\Sigma_{\mu S}$$

$$-2\omega^K \tau^L_{K[I} A_{\mu J]K} - \omega^K g_{IJKL} G^L_\mu$$

M-theory on Twisted T^7 with fluxes

The gauge algebra

$$
\begin{aligned}
[Z_I, W^{JK}] &= 2\tau_{IL}^{[J} W^{|L|K]}, \\
[Z_I, Z_J] &= g_{IJKL} W^{KL} + \tau_{IJ}^{K} Z_K .
\end{aligned}
$$

Generators Z_I, W^{IJ} correspond to KK ω^I and

tensor gauge transformations Σ_{IJ}

The Jacobi identities give constraints : $d^2 e^I = 0$ Almost the B.I. $dG = 0$

$$\tau_{[IJ}^{L}\tau_{K]L}^{M} = 0 \qquad \tau_{[I[K}^{N} g_{LM]J]N} + \tau_{[KL}^{N} g_{M]IJN} = 0$$

General gauge algebras

- Generators: - Z_A : Kaluza-Klein generators
  - X_α : tensor gauge generators
- Algebra:

KALOPER-MYERS
G.D.-FERRARA
HULL-REID EDWARDS

$$
\begin{aligned}
[X_\alpha, X_\beta] &= 0, \\
[Z_A, X_\alpha] &= \left(f_{A\alpha}{}^\beta + \tau_{A\alpha}{}^\beta\right) X_\beta, \\
[Z_A, Z_B] &= f_{AB}{}^\alpha X_\alpha + \tau_{AB}{}^C Z_C.
\end{aligned}
$$

- This is an *ordinary Lie algebra* only when the structure constants satisfy the Jacobi identities

Tensor fields and Free Differential Algebras

- The M-theory Jacobi identities together with the 4-form Bianchi identities imply the **strong** condition:

$$\tau_{IJ}^{N}\, g_{KLMN} = 0$$

- This can be *relaxed* if one does ***not*** dualize the tensor fields
- *Massless tensors - Scalar fields*

$$e_{\Lambda}^{[I} m^{J]\Lambda} = 0$$

D'AURIA
SOMMOVIGO
VAULA'

- Couplings make them *Massive tensors - Massive vectors*

$$F^{\Lambda} + m^{\Lambda I} B_I \qquad dq^I + e_{\Lambda}^{I} A^{\Lambda}$$

- q appears only under derivatives (hence it does ***not*** appear in the potential)
- For *N=1* we have a holomorphic superpotential so the full complex modulus does not appear

Tensor fields and Free Differential Algebras

G.D.-D'AURIA-FERRARA

- The gauge symmetries of vector and tensor fields together realize a Free Differential Algebra

$$\begin{aligned} \mathcal{F}^\Lambda &= dA^\Lambda + \frac{1}{2} f^\Lambda{}_{\Sigma\Gamma} A^\Sigma \wedge A^\Gamma + m^{\Lambda i} B_i , \\ \mathcal{H}_i &= dB_i + (T_\Lambda)_i{}^j A^\Lambda \wedge B_j \\ &+ k_{i\,\Lambda\Sigma\Gamma} A^\Lambda \wedge A^\Sigma \wedge A^\Gamma . \end{aligned}$$

- The Jacobi identities get modified and the full BI is restored:

$$f^\Lambda{}_{\Sigma[\Gamma} f^\Sigma{}_{\Pi\Delta]} + 2 m^{\Lambda i} k_{i\,\Gamma\Pi\Delta} = 0$$

M-theory on Twisted G2-manifolds

M-theory on Twisted G2-manifolds

- We start with a *G2-holonomy* manifold which is a 7-torus:

$$X_7 = \mathbb{T}^7/(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2)$$

JOYCE

- $\mathbb{Z}_2$ action:

$$\begin{aligned}
\mathbb{Z}_2(x_i) &= \{-x_5, -x_6, -x_7, -x_8, x_9, x_{10}, x_{11}\} \\
\mathbb{Z}_2'(x_i) &= \{a_1 - x_5, a_2 - x_6, x_7, x_8, -x_9, -x_{10}, x_{11}\} \\
\mathbb{Z}_2''(x_i) &= \{a_3 - x_5, x_6, a_4 - x_7, x_8, a_5 - x_9, x_{10}, -x_{11}\}
\end{aligned}$$

- The topology gives $b_1=b_2=0$, $b_3=7$, so the effective theory is described by 1 graviton, 7 chiral multiplets and (for $a_1=a_5=0$, $a_2=a_3=a_4=1/2$) one can turn on 7 4-form fluxes and 6 geometrical deformations

M-theory on Twisted G2-manifolds

- The blowup of the singularities introduces further 36 scalars and 12 vectors
- The N=1 superpotential (without the blowup modes) reads

G.D.-PREZAS

$$
\begin{aligned}
W \;=\; & g_{1234567} - T_1 T_6\, \tau^1_{45} + T_1 T_4\, \tau^2_{46} \\
- \;& T_4 T_6\, \tau^3_{47} - T_4 T_7\, \tau^5_{67} - T_5 T_6\, \tau^6_{57} \\
- \;& T_1 T_2\, \tau^7_{56} + \mathrm{i}\,(-T_1\, g_{3456} + T_2\, g_{1256} \\
+ \;& T_3\, g_{1234} + T_4\, g_{1467} - T_5\, g_{1357} \\
- \;& T_6\, g_{2457} - T_7\, g_{2367})
\end{aligned}
$$

M-theory on Twisted G2-manifolds

The ***twist*** makes X_7 a *G2-structure* manifold

$$
\begin{aligned}
d\Phi &= W_1 \star \Phi + \Phi \wedge W_7 + W_{27}, \\
d \star \Phi &= \star\Phi \wedge W_7 + W_{14}.
\end{aligned}
$$

The allowed classes for *N=1* vacua have been classified by Kaste-Minasian-Petrini-Tomasiello, G.D.-Prezas and Behrndt-Jeschek.

Here we can see which types we can realize with twisted tori.

By explicit computation, W_7 and W_{14} vanish.

M-theory on Twisted G2-manifolds

The ***twist*** makes X_7 a *G2-structure* manifold

G.D.-PREZAS

$$d\Phi = W_1 \star \Phi + W_{27}, \quad d \star \Phi = 0$$

The ***Kæhler potential*** and ***Superpotential*** read

$$K = -3 \log \left(\frac{1}{7} \int_{X_7} \Phi \wedge \star \Phi \right)$$

Parts in
BEASLEY-WITTEN
ACHARYA
CURIO-KRAUSE
GRIMM-LOUIS
BEHRNDT-JESCHEK
HOUSE-MICU

$$W = \int_{X_7} \left[d\Phi + i \left(dC + 2g \right) \right] \wedge \left(\Phi + iC \right)$$

Torsion contribution *Defines the T-moduli*

M-theory on Twisted G2-manifolds

- The blowup of the singularities introduces further 36 scalars and 12 vectors
- The $N=1$ superpotential (without the blowup modes) reads

G.D.-PREZAS

$$
\begin{aligned}
W \;=\;& g_{1234567} - T_1 T_6\, \tau^1_{45} + T_1 T_4\, \tau^2_{46} \\
-\;& T_4 T_6\, \tau^3_{47} - T_4 T_7\, \tau^5_{67} - T_5 T_6\, \tau^6_{57} \\
-\;& T_1 T_2\, \tau^7_{56} + \mathrm{i}\,(- T_1\, g_{3456} + T_2\, g_{1256} \\
+\;& T_3\, g_{1234} + T_4\, g_{1467} - T_5\, g_{1357} \\
-\;& T_6\, g_{2457} - T_7\, g_{2367})
\end{aligned}
$$

M-theory on Twisted G2-manifolds

The ***twist*** makes X_7 a *G2-structure* manifold G.D.-PREZAS

$$d\Phi = W_1 \star \Phi + W_{27}, \quad d \star \Phi = 0$$

The ***Kæhler potential*** and ***Superpotential*** read

$$K = -3 \log \left(\frac{1}{7} \int_{X_7} \Phi \wedge \star \Phi \right)$$

Parts in
BEASLEY-WITTEN
ACHARYA
CURIO-KRAUSE
GRIMM-LOUIS
BEHRNDT-JESCHEK
HOUSE-MICU

$$W = \int_{X_7} \left[d\Phi + i \left(dC + 2g \right) \right] \wedge \left(\Phi + iC \right)$$

Torsion contribution *4-form flux*

M-theory on Twisted G2-manifolds

- The blowup of the singularities introduces further 36 scalars and 12 vectors
- The $N=1$ superpotential (without the blowup modes) reads

G.D.-PREZAS

$$
\begin{aligned}
W \;=\; & g_{1234567} - T_1T_6\,\tau^1_{45} + T_1T_4\,\tau^2_{46} \\
- \; & T_4T_6\,\tau^3_{47} - T_4T_7\,\tau^5_{67} - T_5T_6\,\tau^6_{57} \\
- \; & T_1T_2\,\tau^7_{56} + \mathrm{i}\,(-T_1\,g_{3456} + T_2\,g_{1256} \\
+ \; & T_3\,g_{1234} + T_4\,g_{1467} - T_5\,g_{1357} \\
- \; & T_6\,g_{2457} - T_7\,g_{2367})
\end{aligned}
$$

M-theory on Twisted G2-manifolds

The ***twist*** makes X_7 a *G2-structure* manifold

$$d\Phi = W_1 \star \Phi + W_{27}, \quad d \star \Phi = 0$$

The ***Kæhler potential*** and ***Superpotential*** read

$$K = -3 \log \left(\frac{1}{7} \int_{X_7} \Phi \wedge \star\Phi \right) W = \int_{X_7} \left[d\Phi + i \left(dC + 2g \right) \right] \wedge \left(\Phi + iC \right)$$

Vacua:

ALSO RELATED TO
DERENDINGER et AL.
VILLADORO-ZWIRNER

AdS with **complete stabilization** ***(X_7 is weak G2)***

Various **Minkowski** with no-scale potentials

Results change with blowup modes *(in progress...)*

IIA reduction and N=1 potentials

- Define the *fibered* G2-structure

JOY
CHI

SU(3) complex structure

unit form

$$\Phi = J \wedge \alpha + \rho$$

GRANA-LOUIS-MICU-WALDRAM
VILLADORO-ZWIRNER

- The 6d *Kæhler potential* and *superpotential* read

$$K = \int J^3 + \int \Omega \wedge \bar{\Omega} \quad W = \int e^{iJ_c} \wedge G + \int (H + idJ_c) \wedge \Omega_c$$

- When reducing also the blowup modes one gets consistent setups with D6-branes and O6-planes

KACHRU-McGREEVY

- Various vacua and in the leading sugra approximation also dS partly stabilized. *(D-brane D-terms)*

Hořava-Witten scenario and Heterotic Potentials

- Hořava-Witten setup is a simple G2-holonomy manifold

$$X_7 = CY \times S^1/\mathbb{Z}_2$$

- The naïve reduction to 6d does ***not*** reproduce the Heterotic potential
- X_7 has also a clear SU(3) structure
- *M-theory on SU(3) structure manifolds* gives an effective N=2 Lagrangian (bulk HW is an N=1 projection) G.D.-PREZAS
- Constructing the gravitino mass matrix and taking the appropriate projection reproduces precisely BECKER-BECKER-DASGUPTA-GREEN CARDOSO-CURIO-G.D.-LÜST-MANOUSSELIS-ZOUPANOS

$$W = \int_{Y_6} (H + idJ) \wedge \Omega$$

Summary

- Twisted tori are instances of non-Kæhler manifolds which lead to complete moduli stabilization
- Scherk-Schwarz = Twisted Tori
  Info on Υ_6 from low-energy gauged supergravity
- Effective theories are unusual gauged supergravities (with massive tensor multiplets)

THEIS-VANDOREN
LOUIS-MICU
G.D.-D'AURIA-SOMMOVIGO-VAULA'
LOUIS-SCHULGIN
D'AURIA-FERRARA-VAULA'...

- We can use them as a way to derive general properties of G-structure compactifications (potentials, vacuum structure...)
- To do: explicitly construct new theories, generalize to other SS reductions, complete vacua analysis ...