

# Moduli Stabilization in Toroidal Type IIB Orientifolds

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in collaboration with

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- Introduction
- Toroidal Orientifold models
  - Orbifold/Orientifold action
  - The Closed String Moduli Space
  - Example: Finding cplx structure and Kähler moduli
- Turning on fluxes
- Vacuum Structure
- The non-perturbative superpotential
- Generalizations: Resolved Orbifolds
- Conclusions

Most string theory compactifications come with a large number of moduli which would lead to a fifth force if they were to stay.

Are they just an artifact of the leading approximation?

The complex structure moduli and the dilaton can be stabilized via 3-form fluxes.

Giddings, Kachru, Polchinski

Are the Kähler moduli stabilized via non-perturbative effects?

Kachru, Kallosh, Linde, Trivedi, etc.

Type IIB toroidal orientifold models exhibit many features of the standard model such as non-Abelian gauge groups, chiral fermions and family repetition.

$\mathbf{Z}_2 \times \mathbf{Z}_2$  is very well studied, but what about the other orbifolds?

Denef, Douglas, Florea, Grassi, Kachru

Here: No full-fledged models, only general properties of the orbifolds: geometry, allowed fluxes, existence of stable vacua...

We look at orientifolds of type IIB string theory compactified on  $X_6 = T^6/Z_N$  or  $X_6 = T^6/Z_N \times Z_M$  : Specify the lattice  $\Lambda$  and the point group  $\Gamma$ .

**Orbifold:** Abelian point groups, no discrete torsion, no vector structure.

N=2 SUSY  $\Rightarrow \Gamma \subset SU(3)$ , crystallographic action:

$$\begin{aligned}
 \mathbf{Z}_N: & \mathbf{Z}_3, \cancel{\mathbf{Z}_4}, \mathbf{Z}_{6-I}, \mathbf{Z}_{6-II}, \cancel{\mathbf{Z}_7}, \cancel{\mathbf{Z}_{8-I}}, \cancel{\mathbf{Z}_{8-II}}, \mathbf{Z}_{12-I}, \cancel{\mathbf{Z}_{12-II}}. \\
 \mathbf{Z}_N \times \mathbf{Z}_M: & \mathbf{Z}_2 \times \mathbf{Z}_2, \mathbf{Z}_3 \times \mathbf{Z}_3, \cancel{\mathbf{Z}_2 \times \mathbf{Z}_4}, \cancel{\mathbf{Z}_4 \times \mathbf{Z}_4}, \\
 & \mathbf{Z}_2 \times \mathbf{Z}_6, \mathbf{Z}_2 \times \mathbf{Z}_6'', \mathbf{Z}_3 \times \mathbf{Z}_6, \mathbf{Z}_6 \times \mathbf{Z}_6.
 \end{aligned}$$

Aldazabal, Font, Ibanez ,Violero;  
Zwart

**Orientifold action:**  $\Omega I_6$ ,  $\Omega$  : WS-orientation reversal,  
 $I_6 : z^i \rightarrow -z^i$ . O3/O7-planes.

Tadpole cancellation by adding D3/D7-branes

Twist $\Gamma$	Lattice	$h_{(1,1)}^{untw.}$	$h_{(2,1)}^{untw.}$	$h_{(1,1)}^{twist.}$	$h_{(2,1)}^{twist.}$
$Z_3$	$SU(3)^3$	9	0	27	0
$Z_{6-I}$	$SU(3) \times G_2^2$	5	0	24	5
$Z_{6-I}$	$G_2 \times SU(3)^2$	5	0	20	1
$Z_{6-II}$	$SU(2)^2 \times SU(3) \times G_2$	3	1	32	10
$Z_{6-II}$	$SU(3) \times SO(8)$	3	1	26	4
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$Z_{6-II}$	$SU(2) \times SU(6)$	3	1	22	0
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$Z_{12-I}$	$SU(3) \times F_4$	3	0	26	5
$Z_{12-I}$	$E_6$	3	0	22	1

Ibanez, Mas, Nilles, Quevedo; Eler, Klemm

Twist $\Gamma$	$h_{(1,1)}^{untw.}$	$h_{(2,1)}^{untw.}$	$h_{(1,1)}^{twist.}$	$h_{(2,1)}^{twist.}$
$Z_2 \times Z_2$	3	3	48	0
$Z_3 \times Z_3$	3	0	81	0
$Z_6 \times Z_6$	3	0	81	0
$Z_3 \times Z_6$	3	0	70	1
$Z_2 \times Z_6$	3	1	48	2
$Z_2 \times Z'_6$	3	0	33	0

Font, Ibanez, Quevedo

## Closed String Moduli Space:

- $h^{(1,1)}$  Kähler moduli  $\mathcal{T}^i$  (size),
- $h^{(2,1)}$  complex structure moduli  $\mathcal{U}^i$  (shape),
- dilaton  $S = i C_0 + e^{-\phi_{10}}$

The total moduli space is a direct product:

$$\mathcal{M} = \mathcal{M}_S \otimes \mathcal{M}_K \otimes \mathcal{M}_{CS}$$

$\mathcal{T}^i$  and  $\mathcal{U}^i$  follow directly from the geometry of the compactification manifold. For the effective field theory treatment, we must define new moduli:

$$T^i = e^{-\phi_{10}} \frac{\partial}{\partial \mathcal{T}^i} \text{Vol}(X_6(\mathcal{T}^j)) + i \int_{C_i} C_4 \quad U^i = \mathcal{U}^i$$

What do the **Kähler potentials** of the different moduli spaces look like?

$$\kappa_4^{-2} K_K = -\ln(S + \bar{S}) - \sum_{i=1}^3 \ln(T^i + \bar{T}^i) - \sum_{i=1}^3 \ln(U^i + \bar{U}^i)$$

$$\kappa_4^{-2} K_K = -\ln(S + \bar{S}) - \ln \left\{ (T^5 + \bar{T}^5) \left[ (T^1 + \bar{T}^1)(T^2 + \bar{T}^2) - (T^3 + \bar{T}^3)(T^4 + \bar{T}^4) \right] \right\}$$

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$$\kappa_4^{-2} K_K = -\ln(S + \bar{S}) - \ln \det(M + \bar{M})$$



All orbifolds in our list are Coxeter-orbifolds, i.e. the twist acting on the lattice is the Coxeter element of the respective Lie-algebra:  $Q = S_1 S_2 \dots S_{rank}$ . It consists of successive Weyl-reflections:

$$S_i(\mathbf{x}) = \mathbf{x} - 2 \frac{\langle \mathbf{x}, e_i \rangle}{\langle e_i, e_i \rangle} e_i$$

For the complex structure, we make the following ansatz:

$$dz^i = a_1^i dx^1 + a_2^i dx^2 + a_3^i dx^3 + a_4^i dx^4 + a_5^i dx^5 + a_6^i dx^6$$

We find the coefficients via

$$Q^t dz^i = e^{2\pi i v^i} dz^i$$

Twist must leave the scalar product invariant  $\rightarrow$  metric

$$Q^t g Q = g$$

Metric  $\rightarrow$  Kähler form  $J$

Find the  $h^{(1,1)}$  invariant  $(1,1)$ -forms of the orbifold, expand  $J$  in the real cohomology  $\rightarrow$  Kähler moduli.

Complexified Kähler moduli:

$$T^i = \int_{C_2^i} J + i \int_{C_4^i} C_4$$

Ramond 4-form  $\swarrow$

$\swarrow$   $\int_{C_2^i}$   $\searrow$   $\int_{C_4^i}$   $\swarrow$   
 2-cycle  $\longleftrightarrow$  4-cycle  
 Hodge-dual

# Example: $Z_{6-I}$ on $G_2^2 \times SU(3)$

$Z_{6-I}$ -twist:  $z^i \rightarrow e^{2\pi i v^i} z^i$ ,  $v^1 = \frac{1}{6}$ ,  $v^2 = \frac{1}{6}$ ,  $v^3 = -\frac{2}{6}$

Coxeter twist:  $G_2$   $Q = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 3 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}$   $SU(3)$

Metric:

$$g = \begin{pmatrix} R_1^2 & -\frac{1}{2}R_1^2 & R_1 R_3 \cos \theta_{13} & \frac{1}{\sqrt{3}} R_1 R_3 \cos \theta_{23} & R_1 R_3 \cos \theta_{13} & y & 0 & 0 \\ -\frac{1}{2}R_1^2 & \frac{1}{3}R_1^2 & \frac{1}{\sqrt{3}} R_1 R_3 \cos \theta_{23} & \frac{1}{3} R_1 R_3 \cos \theta_{13} & \frac{1}{\sqrt{3}} R_1 R_3 \cos \theta_{23} & \frac{1}{3} R_1 R_3 \cos \theta_{13} & 0 & 0 \\ R_1 R_3 \cos \theta_{13} & \frac{1}{\sqrt{3}} R_1 R_3 \cos \theta_{23} & R_3^2 & -\frac{1}{2}R_3^2 & 0 & 0 & 0 & 0 \\ y & \frac{1}{3} R_1 R_3 \cos \theta_{13} & -\frac{1}{2}R_3^2 & \frac{1}{3}R_3^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R_5^2 & -\frac{1}{2}R_5^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2}R_5^2 & R_5^2 \end{pmatrix}$$

$y = -\frac{1}{3}(3R_1 R_3 \cos \theta_{13} - \sqrt{3}R_1 R_3 \cos \theta_{23})$  **5 metric degrees of freedom!**

Complex structure:

$$dz^1 = dx^1 + \frac{1}{\sqrt{3}} e^{5\pi i/6} dx^2, \quad dz^2 = dx^3 + \frac{1}{\sqrt{3}} e^{5\pi i/6} dx^4, \quad dz^3 = dx^5 + e^{2\pi i/3} dx^6.$$

Five invariant (1,1)-forms:

$$\omega_1 = dx^1 \wedge dx^2, \quad \omega_2 = dx^3 \wedge dx^4, \quad \omega_3 = dx^5 \wedge dx^6,$$

$$\omega_4 = dx^2 \wedge dx^3 - dx^1 \wedge dx^4,$$

$$\omega_5 = 3 dx^1 \wedge dx^3 - 3 dx^1 \wedge dx^4 + dx^2 \wedge dx^4.$$

Kähler form (real cohomology):

$$\begin{aligned}
 J = & \overset{\text{Re}(\mathcal{T}^1)}{\boxed{\frac{1}{\sqrt{3}} R_1^2}} \omega_1 + \overset{\text{Re}(\mathcal{T}^2)}{\boxed{\frac{1}{\sqrt{3}} R_3^2}} \omega_2 \overset{\text{Re}(\mathcal{T}^3)}{\boxed{-\sqrt{3} R_5^2}} \omega_3 \overset{\text{Re}(\mathcal{T}^4)}{\boxed{-\frac{2}{\sqrt{3}} R_1 R_3 \cos \theta_{13}}} \omega_4 \\
 & + \overset{\text{Re}(\mathcal{T}^5)}{\boxed{\frac{2}{3} R_1 R_3 (13\sqrt{3} \cos \theta_{13} + 29 \cos \theta_{23})}} \omega_5
 \end{aligned}$$

# Turning on Fluxes



3-form flux:  $F_3 = dC_2$   $H_3 = dB_2$

$$G_3 = F_3 + i S H_3$$

Fluxes are quantized:  $\frac{1}{(2\pi)^2 \alpha'} \int_{C_3} F_3 \in n_0 \mathbf{Z}, \quad \frac{1}{(2\pi)^2 \alpha'} \int_{C_3} H_3 \in n_0 \mathbf{Z}$

Expressed in cplx basis:  $H^3 = H^{(3,0)} \oplus H^{(2,1)} \oplus H^{(1,2)} \oplus H^{(0,3)}$

$$\frac{1}{(2\pi)^2 \alpha'} G_3 = \sum_{i=0}^3 (A^i \omega_{A_i} + B^i \omega_{B_i}) + \sum_{j=1}^6 (C^j \omega_{C_j} + D^j \omega_{D_j})$$

$\nearrow dz^1 \wedge dz^2 \wedge dz^3$   $\nwarrow dz^1 \wedge d\bar{z}^1 \wedge dz^2$

Expressed in the real basis:

$$\frac{1}{(2\pi)^2 \alpha'} G_3 = \sum_{i=0}^3 [(a^i + iSc^i)\alpha_i + (b_i + iSd_i)\beta^i] + \sum_{j=1}^6 [(e^j + iSg^j)\gamma_j + (f_j + iSh_j)\delta^j]$$

On  $T^6$ , there are 20 3-forms. Which of them survive the orbifold-twist?

(3,0)                      (0,3)

always:  $dz^1 \wedge dz^2 \wedge dz^3, d\bar{z}^1 \wedge d\bar{z}^2 \wedge d\bar{z}^3$

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$d\bar{z}^1 \wedge dz^2 \wedge dz^3$   
 $dz^1 \wedge d\bar{z}^2 \wedge dz^3$   
 $dz^1 \wedge dz^2 \wedge d\bar{z}^3$

Twist $\Gamma$	$h_{(1,1)}^{untw.}$	$h_{(2,1)}^{untw.}$	$h_{(1,1)}^{twist.}$	$h_{(2,1)}^{twist.}$
$Z_2 \times Z_2$	3	3	48	0
$Z_3 \times Z_3$	3	0	81	0
$Z_6 \times Z_6$	3	0	81	0
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$dz^1 \wedge dz^2 \wedge d\bar{z}^3$

In this case,  $h^{(2,1)} = 0$ , therefore

$$\frac{1}{(2\pi)^2 \alpha'} G_3 = A_0 \omega_{A_0} + B_0 \omega_{B_0}$$

4 real parameters

Complex structure  $\rightarrow$

$$\begin{aligned} \omega_{A_0} = & dx^1 \wedge dx^2 \wedge dx^3 + \frac{1}{\sqrt{3}} e^{5\pi i/6} (dy^1 \wedge dx^2 \wedge dx^3 + dx^1 \wedge dy^2 \wedge dx^3) \\ & + e^{2\pi i/3} dx^1 \wedge dx^2 \wedge dy^3 + \frac{1}{3} e^{2\pi i/6} dy^1 \wedge dy^2 \wedge dy^3 \end{aligned}$$

Complex coefficients:

Integers

$$A_0 = \frac{1}{\sqrt{3}} (e^{2\pi i/12} a^0 - i\sqrt{3} b_0 + iS [e^{2\pi i/12} c^0 - i\sqrt{3} d_0]),$$

$$B_0 = \frac{1}{\sqrt{3}} (e^{-2\pi i/12} a^0 + i\sqrt{3} b_0 - iS [e^{-2\pi i/12} c^0 + i\sqrt{3} d_0])$$

Effective N=1 superpotential:

$$W = W_{\text{flux}}(S, U^j) + W_{\text{np}}(T^i) = \kappa_{10}^{-2} \int G_3 \wedge \Omega + \sum_{i=1}^3 g^i e^{-h^i T^i}$$

Scalar potential:

$$V = e^{\kappa_4^2 K} \left( |D_S W|^2 + \sum_i |D_{T^i} W|^2 + \sum_j |D_{U^j} W|^2 - 3|W|^2 \right)$$

Impose SUSY-conditions:

$$D_i W = \partial_i W + \kappa_4^2 W \partial_i K = 0, \quad i = S, U^i, T^i$$

Stability: The eigenvalues of the scalar mass matrix must be  $> 0$ .

$$\frac{\partial^2 V}{\partial M \partial \bar{N}} > 0, \quad (M, N = S, U^j, T^i)$$

$\leftrightarrow$  determinants of all upper-left sub-matrices must be positive



# Are there stable minima?



Dilaton and 1 Kähler modulus: **no!** Choi, Falkowski, Nilles, Olechowski, Pokorski

Dilaton and 3 Kähler moduli: **no!**

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no!

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# Are there stable minima?



- Dilaton and 1 Kähler modulus: **no!** Choi, Falkowski, Nilles, Olechowski, Pokorski
- Dilaton and 3 Kähler moduli: **no!**
- Dilaton and 5 Kähler moduli: **no!**
- Dilaton and 9 Kähler moduli: **no!**

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Dilaton and 3 Kähler moduli: **no!**

Dilaton and 5 Kähler moduli: **no!**

Dilaton and 9 Kähler moduli: **no!**

Dilaton and 3 Kähler moduli and 1 complex structure modulus: **YES!**

Dilaton and 3 Kähler moduli and 3 complex structure moduli: **YES!**

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YES!

# The non-perturbative superpotential



Two possible origins for the non-perturbative superpotential:

- (i) Euclidean D3/M5-brane instantons wrapping internal 4-cycles

$$W_{\text{np}} \sim g_i e^{-2\pi V_i}$$

- (ii) Gaugino condensation in the world-volume of D7-branes wrapped on internal 4-cycles

$$W_{\text{np}} \sim g_i e^{-\frac{2\pi V_i}{b}}$$

Condition for existence of non-vanishing non-pert. superpotential:

F/M-Theory:

Witten

$$\chi(\text{wrapped divisor}) = 1$$

Arithmetic genus changes in the presence of 3-form flux!

Tripathy, Trivedi; Kallosh, Kashani-Poor, Tomasiello; Saulina

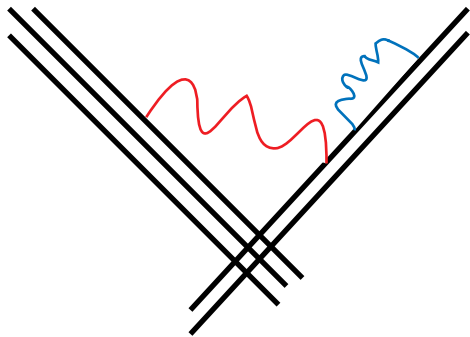
# The non-perturbative superpotential



D3/M5-instantons: F-theory lift seems inevitable

Gaugino condensates: field theoretic arguments

Existence of np-superpotential: **pure** super-Yang-Mills theory  
 $\leftrightarrow$  no massless bi-fundamental matter, no massless adjoint matter.  
 Depends again on the topology of the wrapped divisors.



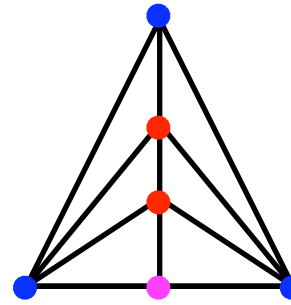
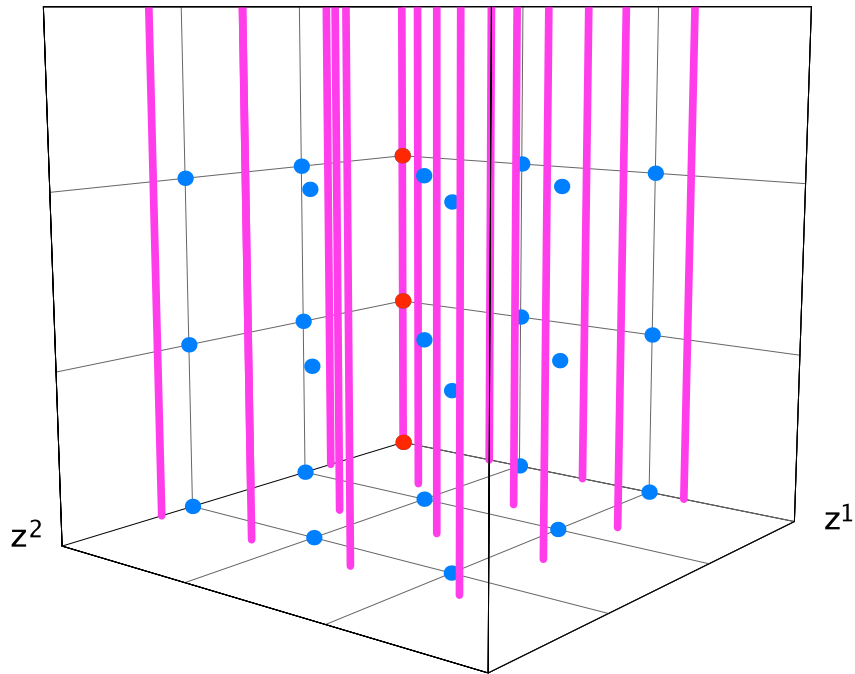
If the brane stacks don't intersect  $\rightarrow$   
**no bi-fundamental matter!**

If **massless adjoint** matter (D7-brane position fields and Wilson lines) exists, we can try to make them massive via 3-form fluxes!

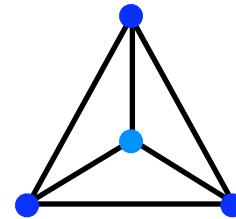


$\mathbf{Z}_{6-I}$  on  $G_2^2 \times SU(3)$

Fixed Sets:  $z^3$



Blowup of  $C^3/\mathbf{Z}_{6-I}$



Blowup of  $C^3/\mathbf{Z}_3$



Blowup of  $C^2/\mathbf{Z}_2$

Group El.	Twist	Fixed set	Conj. Cl.
$\theta$	$\mathbf{Z}_{6-I}$	3 fixed points	3
$\theta^2$	$\mathbf{Z}_3$	27 fixed points	15
$\theta^3$	$\mathbf{Z}_2$	16 fixed lines	6

Divisor count:  
 $3 \cdot 2 + 12 \cdot 1 + 6 \cdot 1 = 24$

Complex structures and geometric parameterization of Kähler moduli, allowed fluxes, existence of stable vacua at the orbifold point. “Phone book”

## Open Questions:

- Do we indeed get contributions to the non-perturbative superpotential for those models who allow stable vacua?
- What about **resolved** orbifolds?  
Exceptional divisors seem very likely to contribute to non-pert. superpotential.
- For which of these orbifold models, an F-theory lift exists?
- For which of these models can we get gaugino condensates?
- Consistent models with all moduli fixed other than  $\mathbf{Z}_2 \times \mathbf{Z}_2$  ?

work in progress

Thank you!