On the Open String Landscape

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J. Gomis, F. M., D. Mateos, hep-th/0506179

Motivation

Moduli Problem

- Standard Calabi-Yau $\mathcal{N} = 1$ string compactifications suffer from a well-known problem in string phenomenology: plenty of massless scalars.

- **Insight:** **Flux** compactifications have **less moduli**

[Dasgupta, Rajesh, Sethi]
[Giddings, Kachru, Polchinski]

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- **Insight:** **Flux** compactifications have **less moduli** [DRS,GKP]

⋆ We have a good understanding of moduli in CY compactifications

⋆ What about $\mathcal{N}=1$ $SU(3)$-structure compactifications?

. . . the hope is that they have less moduli (or none).

Moduli and Fluxes

Simplest Example

- Type IIB on a conformal Calabi-Yau, with background 3-form fluxes

$$G_3 = F_3 - \tau H_3 \qquad \begin{cases} F_3 & \text{RR flux} \\ H_3 & \text{NSNS flux} \\ \tau & \text{complex dilaton} \end{cases}$$

$\longrightarrow$ New ingredient: GVW superpotential $W = \int G_3 \wedge \Omega$

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- The existence of W imposes new constraints on complex structure moduli: $DW = W = 0$ [Kachru, Schulz, Trivedi]

$\longrightarrow$ Reduced moduli space: $\mathcal{M}_{cpx}^{G_3} \subset \mathcal{M}_{cpx}^{\mathbf{CY}}$

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- Equivalently, $\mathcal{N} = 1$ conditions read G_3 harmonic (2,1)-form

  $\rightarrow$ Given an $\mathcal{N} = 1$ vacuum, only by moving in $\mathcal{M}_{cpx}^{G_3} \subset \mathcal{M}_{cpx}^{\mathbf{CY}}$ this condition will not be spoiled

Fluxes and D-branes

What about D-branes?

- We know by some partial results that the moduli space of D-branes can also be affected by G_3

- Examples:

  – D7-branes

  [Görlich, Kachru, Tripathy, Trivedi]
  [Cascales, Uranga]
  [Cámara, Ibáñez, Uranga]
  [Lüst, Mayr, Reffert, Stieberger]

  – Euclidean D3-branes

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... but we don't have a complete and general understanding of these facts

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- Examples:
  - D7-branes [GKTT, CU, CIU, LMRS]
  - Euclidean D3-branes [TT,Saulina,KKT]

IDEA

Implement the same approach
for Open Strings
as done for Closed Strings

D-branes and Moduli

- First approach: F/M-theory lift + GVW superp. $W = \int G_4 \wedge \Omega$

  But. . .

  - Needs global knowledge of $SU(4)$-holonomy geometries
  - D-brane intuition is lost

D-branes and Moduli

- First approach: F/M-theory lift + GVW superp. $W = \int G_4 \wedge \Omega$

- Second approach: Type IIB background + D-brane probe approx.
  - Write down SUSY conditions for D-branes
  - Find the D-brane deformations that do not spoil them

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In the conformal CY case, we hope to find a description in terms of

$$\left.\begin{array}{l}\text{Old moduli space}\\ \text{New restrictions}\end{array}\right\} \quad \Rightarrow \quad \text{Reduced moduli space}$$

D-branes & SUSY

- Which are the supersymmetric D-branes in flux compactifications?
  - Possible approach: generalized calibrations [Cascales, Uranga]

D-branes & SUSY

- Which are the supersymmetric D-branes in flux compactifications?
  - Possible approach: generalized calibrations [Cascales, Uranga]
  - We will use another technique: κ-symmetry

$$\Gamma\,\epsilon = \epsilon$$

ϵ : cov. constant spinor of the background

Γ : encodes the D-brane embedding

SUSY & κ-symmetry

- Previous analysis of Mariño, Minasian, Moore, and Strominger
  - Calabi-Yau background $\mathcal{M}_6$
  - Euclidean D$(2n-1)$-branes wrapping $\mathcal{S}_{2n} \subset \mathcal{M}_6$
  - $U(1)$ gauge bundle $\mathcal{F}$ in $\mathcal{S}_{2n}$

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  - $U(1)$ gauge bundle $\mathcal{F}$ in $\mathcal{S}_{2n}$

Result:

$$e^{iJ-\mathcal{F}}|^{\text{top}}_{\mathcal{S}_{2n}} = e^{i\theta}\sqrt{|g^{\mathcal{S}_{2n}}+\mathcal{F}|}\,\frac{\text{dvol}_{\mathcal{S}_{2n}}}{\sqrt{|g^{\mathcal{S}_{2n}}|}}$$

$$\iota_X\Omega \wedge e^{iJ-\mathcal{F}}|^{\text{top}}_{\mathcal{S}_{2n}} = 0, \qquad \forall X \in T\mathcal{M}_6$$

Ω : Holomorphic 3-form $\quad$ J : Kähler 2-form $\quad$ $\mathcal{F} = 2\pi\alpha' F + B_2|_{\mathcal{S}_{2n}}$

$\theta : \mathcal{N}=1 \subset \mathcal{N}=2$ parameter $\quad$ $\left(\iota_X C_{p+1}\right)_{a_1\ldots a_p} = X^\rho \left(C_{p+1}\right)_{\rho a_1\ldots a_p}$

κ-symmetry & Fluxes

• One can extend the MMMS computation to more general backgrounds, since the main ingredients needed are

– Hermitian metric

– p-forms Ω, J defined as spinor bilinears

– 2 SUSY generators $\eta_+^* = \eta_-$

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– Hermitian metric

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– 2 SUSY generators $\eta_+^* = \eta_-$

$\Rightarrow$ Conditions satisfied for any $SU(3)$-structure manifold

[Graña, Minasian, Petrini, Tomasiello]

$$\Downarrow$$

MMMS equations still apply

(Warning! Now $d\Omega$, dJ, $d\mathcal{F} \neq 0$)

Fluxes & D-branes

Simplest Example

- Warped Calabi-Yau with ISD G_3 fluxes

$$ds^2 = \Delta(y)^{-1}\,\eta_{\mu\nu}dx^\mu dx^\nu + \Delta(y)\,g^{\mathbf{CY}}_{mn}dy^m dy^n,$$

$$\Omega = \Delta^{3/2}\Omega^{\mathrm{CY}} \qquad J = \Delta J^{\mathrm{CY}}$$

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$$e^{iJ-\mathcal{F}}|^{\mathrm{top}}_{\mathcal{S}_{2n}} = -\sqrt{|g^{\mathcal{S}_{2n}} + \mathcal{F}|}\,\frac{d\mathrm{vol}_{\mathcal{S}_{2n}}}{\sqrt{|g^{\mathcal{S}_{2n}}|}}$$

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$\mathcal{S}_{2n}$ holomorphic $\qquad \mathcal{F}^{(2,0)} = 0$

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MMMS Eqs:

$$J|_{\mathcal{S}_2} + i\mathcal{F} = i\sqrt{|g^{\mathcal{S}_2} + \mathcal{F}|}\,\frac{d\mathrm{vol}_{\mathcal{S}_2}}{\sqrt{|g^{\mathcal{S}_2}|}}$$

$\mathcal{S}_2$ holomorphic

- $\mathbf{n = 1}$ $\rightarrow$ D5-branes or Euclidean D1's

$$J|_{\mathcal{S}_2} \rightarrow 0 \qquad \Rightarrow \qquad \text{fractional D3-brane}$$

Fluxes & D-branes

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$$J|_{\mathcal{S}_4} \wedge \mathcal{F} = 0$$

$$\mathcal{S}_4 \text{ holomorphic} \qquad \mathcal{F}^{(2,0)} = 0$$

- $\mathbf{n = 2}$ $\rightarrow$ D7-branes or Euclidean D3's

$$\mathcal{F} \text{ is (1,1) primitive} \quad \Rightarrow \quad *_4\mathcal{F} = -\mathcal{F}$$

D3-brane induced charge

Fluxes & D-branes

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MMMS Eqs:

$$\frac{1}{3!}J^3 = \frac{1}{2!}J \wedge \mathcal{F}$$
$$\mathcal{F}^{(2,0)} = 0$$

- $\mathbf{n = 3}$ $\rightarrow$ D9-branes or Euclidean D5's

$d\mathcal{F} = H_3$ is not defined globally $\quad \Rightarrow \quad$ Freed-Witten anomaly

We need extra sources of $\mathcal{F}$

D7-branes

- Recall that

$$\mathcal{F} = 2\pi\alpha' F + B_2|_{\mathcal{S}_4} \quad \left\{ \begin{array}{l} F = dA \in H^2(\mathcal{S}_4, \mathbb{Z}) \\ dB_2 = H_3 \end{array} \right\} \quad \Rightarrow \quad d\mathcal{F} = H_3|_{\mathcal{S}_4}$$

$\longrightarrow$ $\mathcal{F}$ is well-defined only if $[H_3|_{\mathcal{S}_4}] \in H_3(\mathcal{S}_4, \mathbb{Z})$ vanishes

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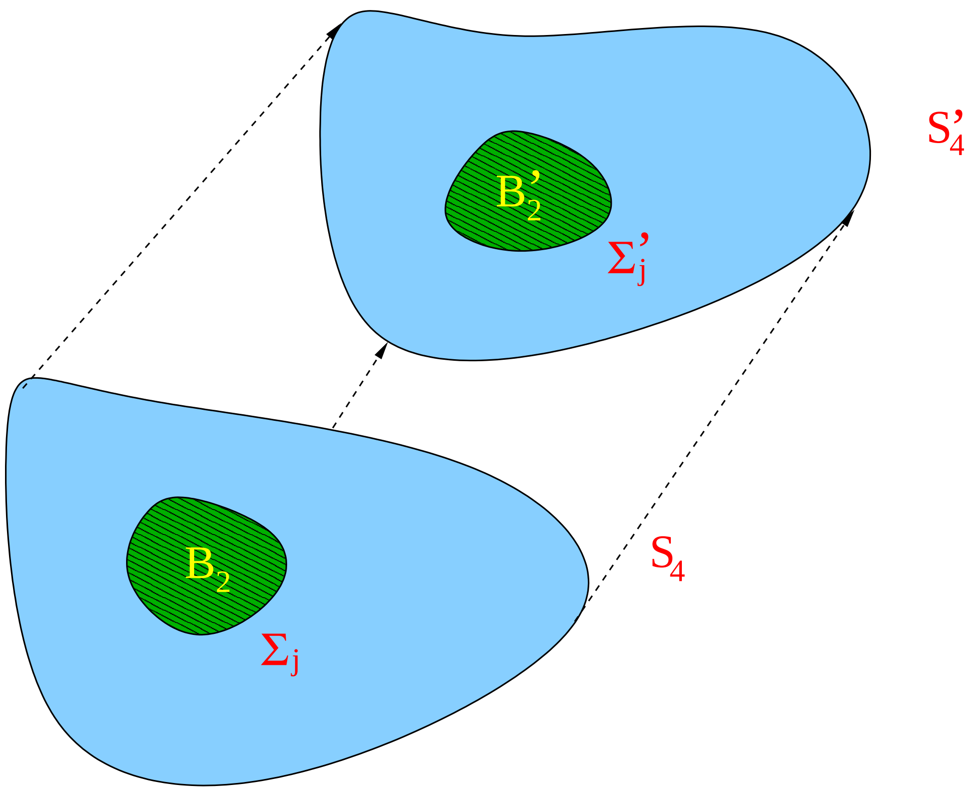
- Since B_2 is non-closed, its pull-back depends on the 4-cycle $\mathcal{S}_4$ that the D7-brane is wrapping



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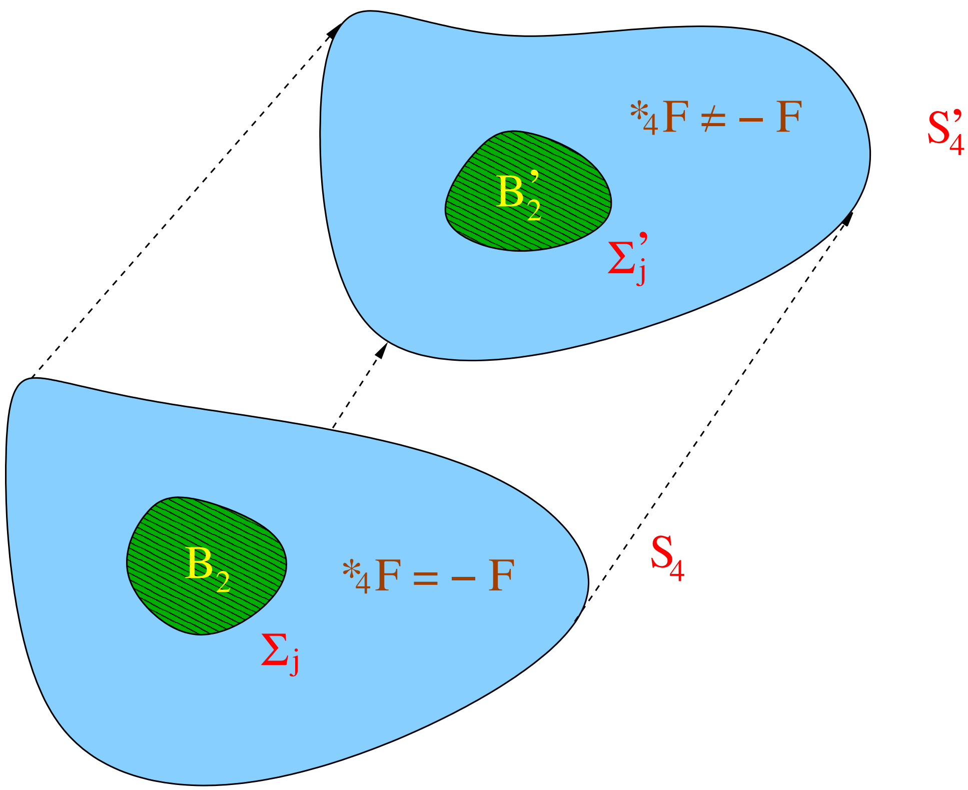
- A deformation $\mathcal{S}_4 \rightarrow \mathcal{S}'_4$ may not preserve the supersymmetry condition $*_4\mathcal{F} = -\mathcal{F}$



D7-branes II

- In absence of H_3 flux, the moduli space of a D7-brane is given by

$$\text{Geometric moduli}: \quad \zeta^a \quad a = 1, \ldots, H^{(0,2)}(\mathcal{S}_4)$$

$$\text{Wilson line moduli}: \quad \xi^b \quad b = 1, \ldots, H^{(0,1)}(\mathcal{S}_4)$$

[Jockers, Louis]

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- The harmonic part of $\mathcal{F}$ has the form

$$\mathcal{F}^{har} = \sum_{i=1}^{h^{2,0}} a_i(\{\zeta^a\})\,\alpha^i + \sum_{j=1}^{h_p^{1,1}} b_j(\{\zeta^a\})\,\beta^j + \sum_{k=1}^{h_{np}^{1,1}} c_k(\{\zeta^a\})\,\gamma^k + \text{c.c.}$$

$$\bar{\alpha}^i \in \mathcal{H}^{2,0} \quad \beta^j \in \mathcal{H}_p^{1,1} \quad \alpha^i \in \mathcal{H}^{0,2}$$
$$\gamma^k \in \mathcal{H}_{np}^{1,1}$$

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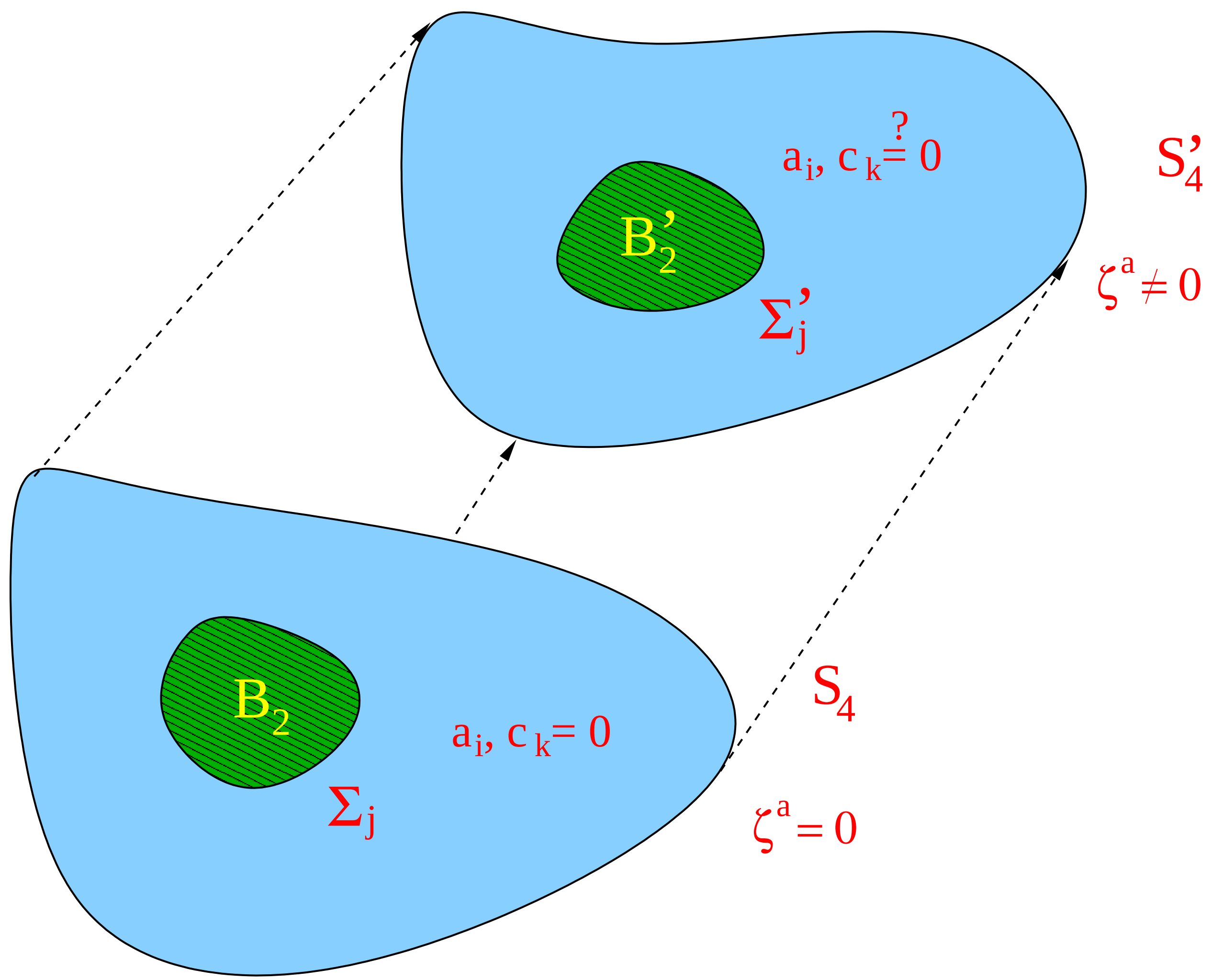
$a_i, c_k \overset{?}{=} 0$
S'_4
B'_2
$\zeta^a \neq 0$
Σ'_j
B_2
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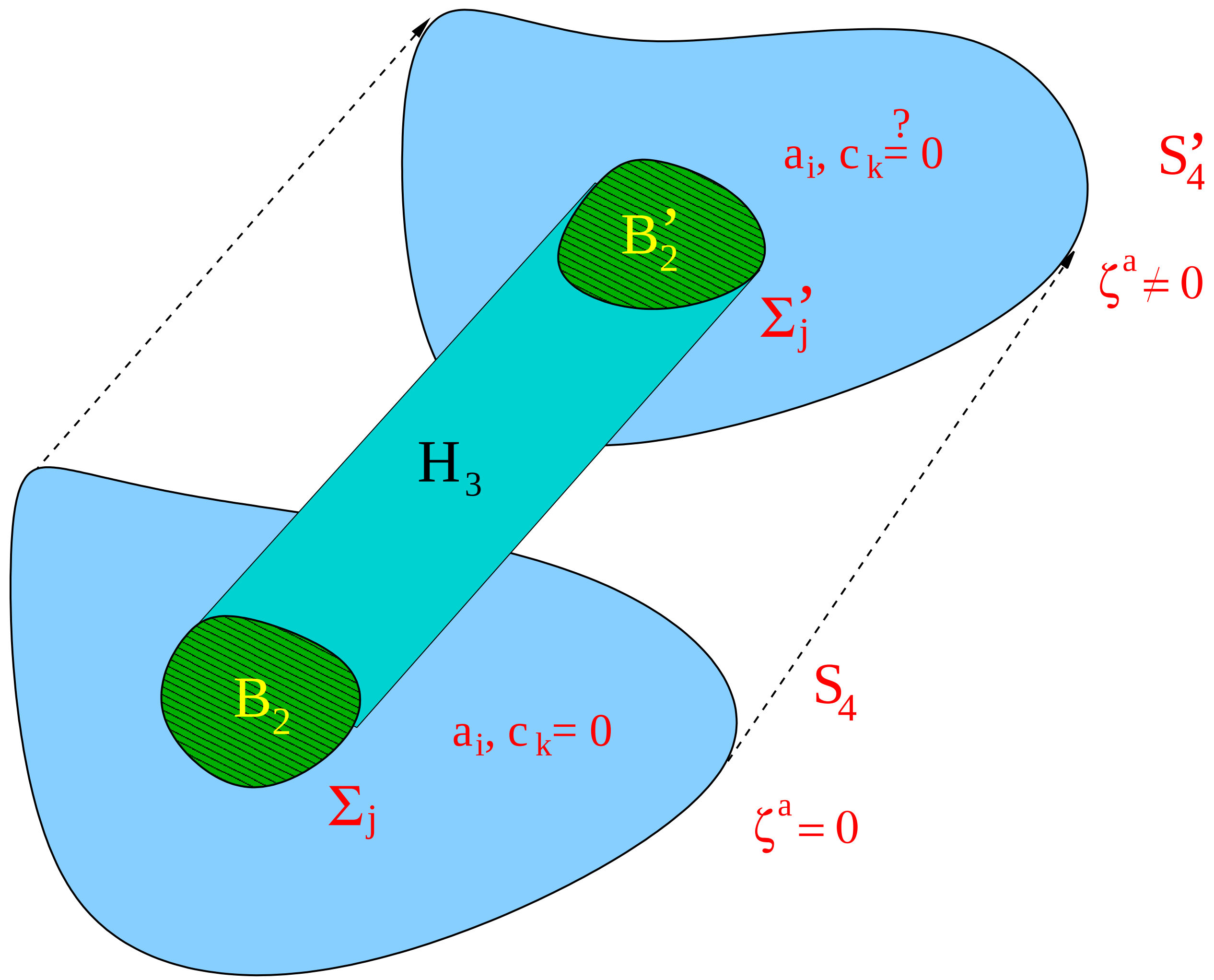

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- Supersymmetry of the background implies

$\longrightarrow G_3 \wedge J = 0 \quad \Rightarrow \quad c_J = \text{const.}$

$\longrightarrow G_3$ is (2,1) $\quad \Rightarrow \quad a_i =$ holomorphic on $\{\zeta^a\}$

D7-branes III

D7-brane moduli

- Old moduli space:

$$\mathcal{M}(D7) = \begin{cases} \text{Geom. deform.} & \longrightarrow \ \mathcal{M}(\mathcal{S}_4) = \{\zeta^i\} \quad \text{dim } = h^{0,2}(\mathcal{S}_4) \\ \text{Wilson lines} & \longrightarrow \ \mathcal{M}(\mathcal{F}) = \{\xi^j\} \quad \text{dim } = h^{0,1}(\mathcal{S}_4) \end{cases}$$

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- $\mathcal{F}$ is now coupled to $\mathcal{M}(\mathcal{S}_4)$ via a non-trivial H_3, so supersymmetry restricts the possible deformations $\{\zeta^i\}$

$$a_1(\zeta^1, \ldots, \zeta^{h^{0,2}}) = 0,$$
$$\vdots$$
$$a_{h^{0,2}}(\zeta^1, \ldots, \zeta^{h^{0,2}}) = 0,$$

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$\longrightarrow$ Reduced moduli space: subvariety $\mathcal{M}^{G_3}(\mathcal{S}_4) \subset \mathcal{M}(\mathcal{S}_4)$, typically zero dimensional

$\longrightarrow$ Wilson lines untouched

So far so good. . .

- We are interested in knowing which is the moduli space of D-branes in flux compactifications

- In order to work it out, we have deduced the supersymmetry conditions for D-branes in manifolds with $SU(3)$ structure, via κ-symmetry

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- We are interested in knowing which is the moduli space of D-branes in flux compactifications

- In order to work it out, we have deduced the supersymmetry conditions for D-branes in manifolds with $SU(3)$ structure, via κ-symmetry

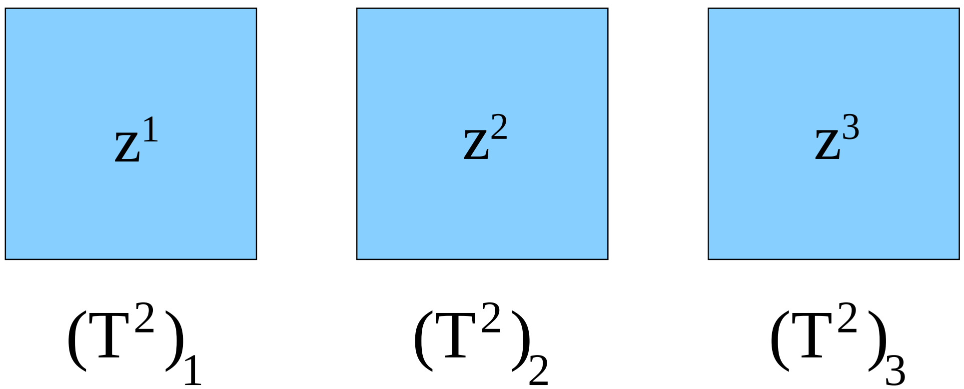
- For a D7-brane wrapping a divisor $\mathcal{S}_4$, we have discovered that $\mathcal{F}$ depends non-trivially on the space of deformations $\mathcal{M}(\mathcal{S}_4)$

- Most of these deformations spoil the supersymmetry conditions, which effectively reduces the moduli space to $\mathcal{M}^{G_3}(\mathcal{S}_4) \subset \mathcal{M}(\mathcal{S}_4)$

- $\mathcal{M}^{G_3}(\mathcal{S}_4)$ is the solution to $h^{0,2}$ equations with $h^{0,2}$ unknowns
  $\rightarrow$ All geometrical moduli are typically lifted

A simple example

- Let us consider a toroidal compactification



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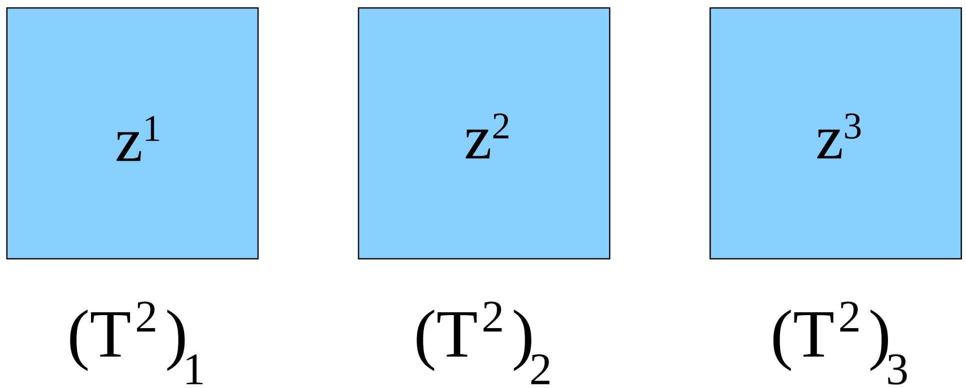

with constant fluxes

$$F_3 = 4\pi^2\alpha' N \left(dx^1 \wedge dx^2 \wedge dy^3 + dy^1 \wedge dy^2 \wedge dy^3\right)$$
$$H_3 = 4\pi^2\alpha' N \left(dx^1 \wedge dx^2 \wedge dx^3 + dy^1 \wedge dy^2 \wedge dx^3\right)$$

which create a superpotential

$$W = \int \Omega \wedge G_3 \propto (1 + \tau_1\tau_2) \cdot (1 + \tau_3\tau),$$

$$\Downarrow$$

$$\tau_1\tau_2 = -1, \qquad \tau_3\tau = -1$$

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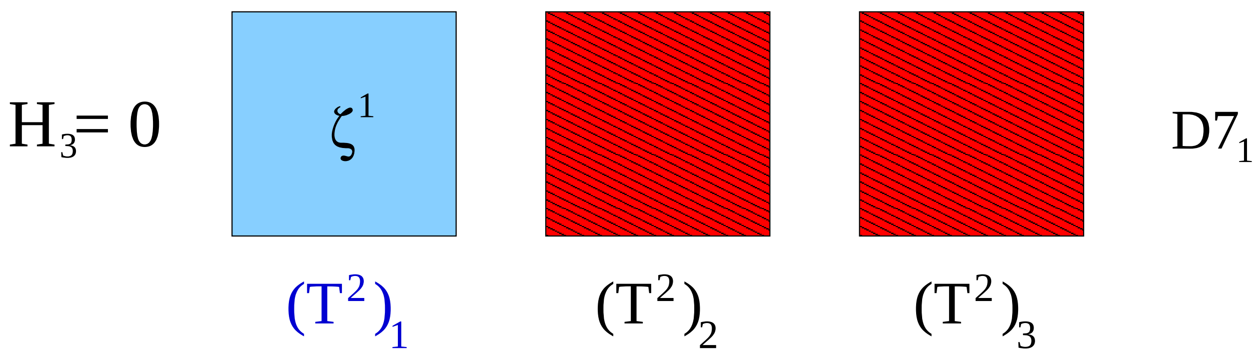

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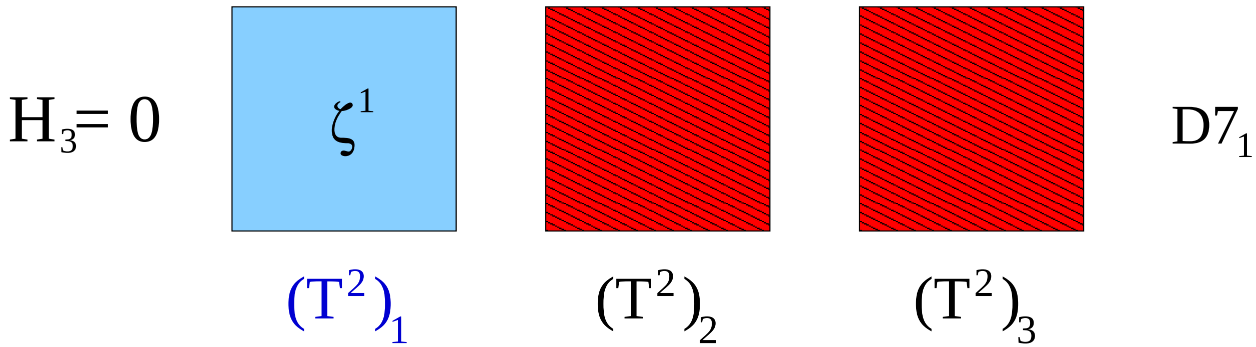

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$$H_3 = 4\pi^2\alpha' \, N \left(dx^1 \wedge dx^2 \wedge dx^3 \, + \, dy^1 \wedge dy^2 \wedge dx^3\right)$$

$$B_2 = 4\pi^2\alpha' \, N \left(x^1 dx^2 \wedge dx^3 + y^1 dy^2 \wedge dx^3\right)$$

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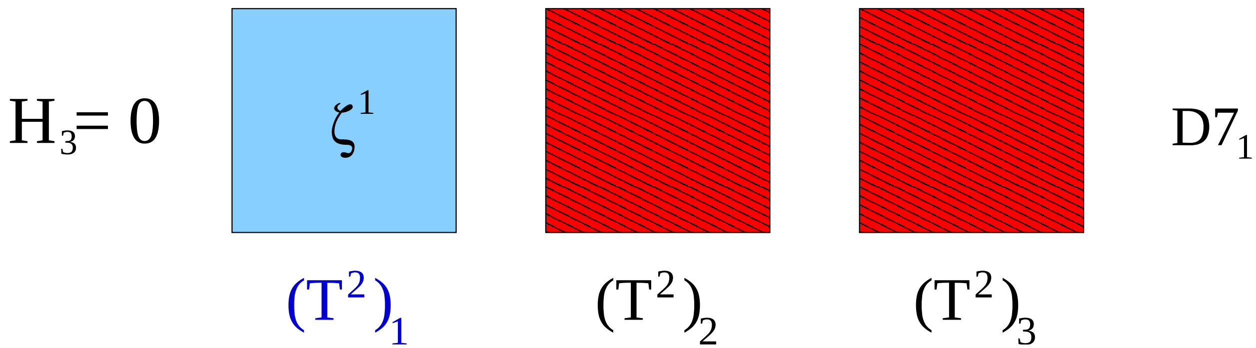

with a $D7_1$-brane, $h^{0,2}(D7_1) = 1$, $\mathcal{M}(D7_1) = (T^2)_1$

$$H_3 = 4\pi^2\alpha' N \left(dx^1 \wedge dx^2 \wedge dx^3 + dy^1 \wedge dy^2 \wedge dx^3\right)$$

$$B_2|_{D7_1} = 4\pi^2\alpha' N \left(\frac{\tau_2}{\tau_2-\bar{\tau}_2}\zeta^1 d\bar{z}^{\bar{2}} + \frac{\tau_1}{\tau_1-\bar{\tau}_1}\bar{\zeta}^{\bar{1}} dz^2\right) \wedge \frac{\mathrm{Im}\,(\tau_3 d\bar{z}^{\bar{3}})}{\mathrm{Im}\,\tau^3}$$

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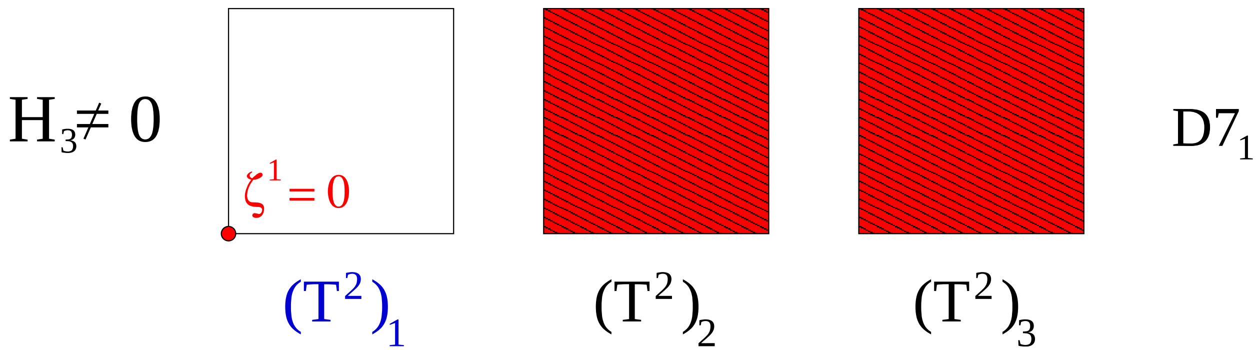

with a D7$_1$-brane, $h^{0,2}(D7_1) = 1$, $\mathcal{M}(D7_1) = (T^2)_1$

$$H_3 = 4\pi^2\alpha' N \left(dx^1 \wedge dx^2 \wedge dx^3 + dy^1 \wedge dy^2 \wedge dx^3\right)$$

$$B_2|_{D7_1} = 4\pi^2\alpha' N \left(\frac{\tau_2}{\tau_2 - \bar{\tau}_2}\zeta^1 d\bar{z}^{\bar{2}} + \frac{\tau_1}{\tau_1 - \bar{\tau}_1}\bar{\zeta}^{\bar{1}} dz^2\right) \wedge \frac{\mathrm{Im}\,(\tau_3 d\bar{z}^{\bar{3}})}{\mathrm{Im}\,\tau^3}$$

$$\Downarrow$$

$$a_1(\zeta^1) = N\zeta^1\tau_2\tau_3 \quad \Rightarrow \quad \zeta^1 = 0 \quad \left(\Rightarrow B_2|_{D7_1} = 0\right)$$

Finding the discretum

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- In general we have to consider

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$$\begin{aligned} B|_{D7_1} &= 4\pi^2\alpha' \, N \left(\zeta_x^1 \, dx^2 \wedge dx^3 + \zeta_y^1 \, dy^2 \wedge dx^3\right) \\ F_{D7_1} &= 2\pi \left(n_1 \, dx^2 \wedge dx^3 + n_2 \, dy^2 \wedge dx^3 + \ldots\right) \end{aligned}$$

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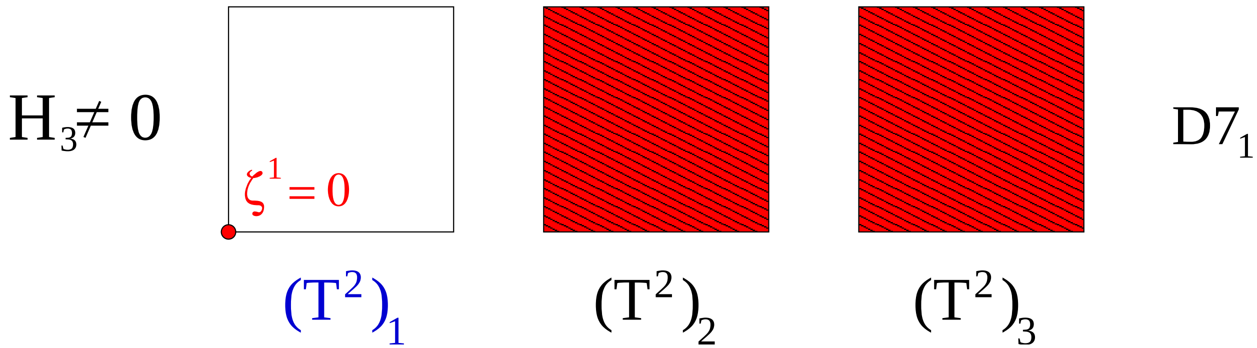
- Additional solutions

$$\begin{aligned} \zeta_x^1\, N &= -n_1 \in \mathbb{Z} \\ \zeta_y^1\, N &= -n_2 \in \mathbb{Z} \end{aligned}$$

Finding the discretum



$$\mathcal{M}^{G_3}(D7_1) \subset \mathcal{M}(D7_1)$$

$$\{\zeta^1 = 0\} \subset (T^2)_1$$

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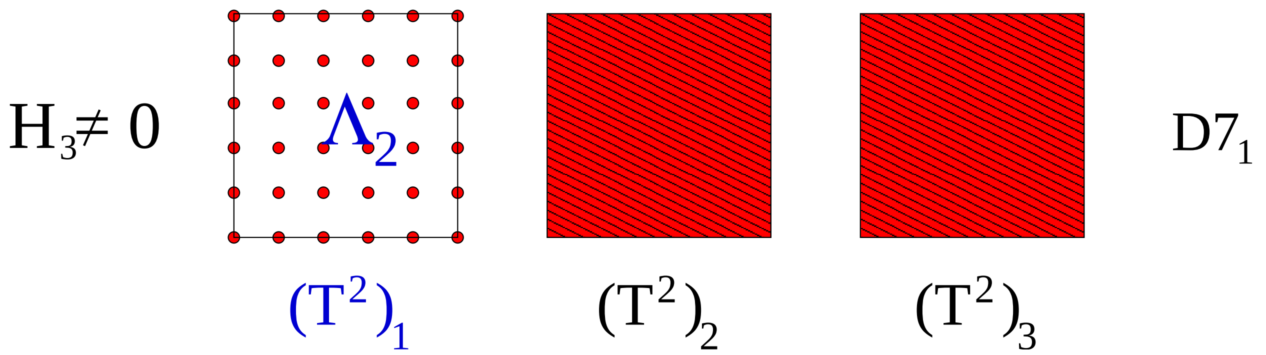

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$$\downarrow$$

$$\Lambda_2 \subset (T^2)_1$$

$$\Lambda_2 = \left\{ \frac{m_x + \tau_1 m_y}{N} \middle| \, m_x, m_y \in \mathbb{Z} \right\} / \Lambda$$

Opening the Landscape

- D7-brane moduli lifting implies a open string reduced moduli space which locally looks like

$$\mathcal{M}^{g_3}(\mathcal{S}_4), \quad \text{dim}_{\mathbb{C}} = h^{0,2} - q$$

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- We can actually characterize this new moduli space by

$$\mathcal{M}^{G_3}(\mathcal{S}_4) = \left\{ \mathcal{S}_4 \in [\mathcal{S}_4] \,|\, B_2|_{\mathcal{S}_4} \in A_p^{(1,1)}(\mathcal{S}_4,) \cup H_p^2(\mathcal{S}_4, \mathbb{Z}) \right\},$$

Open String Quantum Numbers

• The spectrum of BPS D7-branes with $U(1)$ bundles is now described in terms of the quantum numbers

i) $[\mathcal{S}_4] \in H_4(\mathcal{M}_6, \mathbb{Z})$, $\quad (\mathcal{S}_4$ hol., $[H_3|_{\mathcal{S}_4}] = 0)$

ii) $[F] \in H^{(1,1)}(\mathcal{S}_4, \mathbb{R}) \cap H^2(\mathcal{S}_4, \mathbb{Z})$

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$i) + ii)$ Considered by [Blumenhagen et al.] in Intersecting D-brane statistics

$iii)$ New

$\rightarrow$ Not connected by continuous deformations

$\rightarrow$ Can be understood in terms of $B_2|_{\mathcal{S}_4}$ being well-quantized

Conclusions

• The supersymmetry conditions for D7-branes in flux compactifications has allowed us understand moduli lifting from a general and geometrical point of view

• The fact that all (geometrical) moduli are lifted does not mean that the D7-brane can be located at only one point

$\Rightarrow$ There is a discretum of D7-brane positions

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• The fact that all (geometrical) moduli are lifted does not mean that the D7-brane can be located at only one point

$\Rightarrow$ There is a discretum of D7-brane positions

• This defines an Open String Landscape directly connected to the physics of $D = 4$ non-Abelian gauge theories and chiral matter. Any statistical prediction derived from it should have an interpretation in terms of particle physics

More Conclusions

- This Open String Landscape should affect the counting of type IIB vacua once that we include D7-branes into the story

[Douglas] [Ashok, Douglas] [Denef, Douglas]
[Giryavets, Kachru, Tripathy]

- It should be also taken into account when performing statistics of D-brane configurations in a given background

[Blumenhagen et al.]

- Finally, one should be able to give a general description of this new landscape in terms of F/M-theory

[Lüst, Mayr, Reffert, Stieberger]

[Uranga's Talk]