Volume Stabilization and Brane Inflation at One Loop

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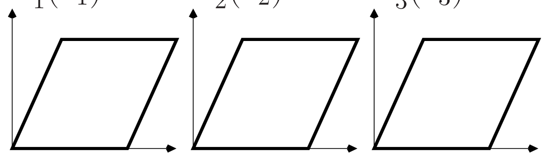
Outline

1. One-loop Kähler potential in a $\mathcal{N} = 1$ orientifold
   - Orientifold $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with D3/D7-branes
   - Classical moduli space and gauge couplings
   - One-loop string threshold corrections
   - Vertex operators for moduli fields
   - An $\mathcal{N} = 2$ truncation: one-loop prepotential
2. Volume stabilization at one-loop
   - Moduli stabilization with fluxes and gaugino condensation
   - No-scale violation at one-loop
   - One-loop corrections to KKLT minima
3. Brane inflation at one-loop
   - D3/D7-brane inflation with shift symmetry
   - Violation of shift symmetry at one-loop
   - Slow rolling at one-loop

Orientifold $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with D3/D7-branes

- Relevant compactification: "IIB orientifold on $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$" [Berkooz, Leigh; and others]

$\mathbb{T}^6 :\quad \mathbb{T}^2_1(z_1) \qquad \mathbb{T}^2_2(z_2) \qquad \mathbb{T}^2_3(z_3)$



$$\begin{array}{llll} \Theta_1 : & z_1 \mapsto -z_1 & z_2 \mapsto -z_2 & \\ \Theta_2 : & z_1 \mapsto -z_1 & & z_3 \mapsto -z_3 \\ \Theta_3 : & & z_2 \mapsto -z_2 & z_3 \mapsto -z_3 \end{array}$$

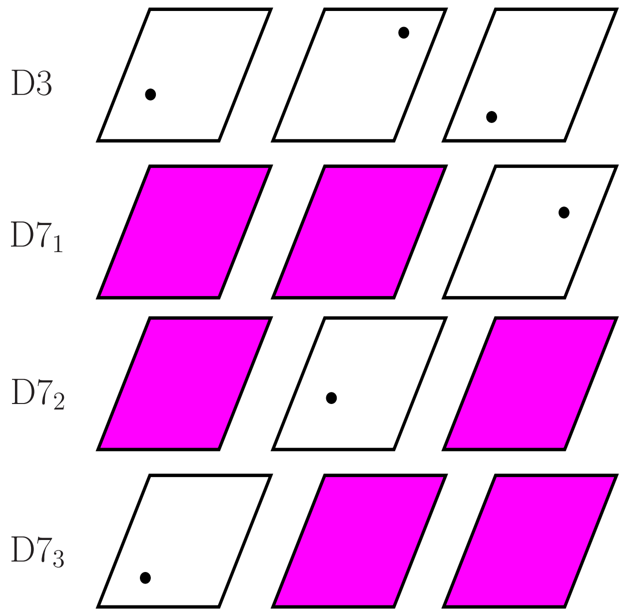
Identification under

$$\text{orbifold group}: \qquad \mathbb{Z}_2 \times \mathbb{Z}_2 = \{1, \Theta_1, \Theta_2, \Theta_3 = \Theta_1\Theta_2\}$$

$$\text{orientifold}: \qquad \Omega R_6 (-1)^{F_R} \ , \quad R_6 : z_I \mapsto -z_I \ , \quad I = 1,2,3$$

Each element Θ_I has a fixed 2-torus $\mathbb{T}^2_I$, transverse $\mathbb{T}^4_I$.

- Model has D3- and (three types of) D7-branes on $\mathbb{T}^4_I$



The classical moduli space

- Untwisted moduli which are considered

$$\begin{aligned} \text{dilaton} &: \quad S \sim e^{-\Phi} + iC \\ \mathbb{T}^4_I/\mathbb{T}^2_I &: \quad S_I \sim e^{-\Phi}\mathrm{vol}(\mathbb{T}^4_I) + iC_4|_{\mathbb{T}^4_I} + \cdots \\ & \quad\;\; U_I \sim \text{cpl. str. of } \mathbb{T}^2_I \\ \text{positions of D3} &: \quad A_I = U_I a_{2I} - a_{2I-1} \end{aligned}$$

  Moduli not considered are: moduli of D7, twisted moduli.

- Classical Kähler potential [Atoniadis, Bachas, Fabre, Partouche, Taylor]

$$\mathcal{K}^{(0)} = -\ln\left[(S+\bar{S})[(S_I+\bar{S}_I)(U_I+\bar{U}_I) - \frac{1}{2}(A_I+\bar{A}_I)^2]\right]$$

  for moduli space

$$\left[\frac{SU(1,1)}{U(1)}\right]_S \times \prod_{I=1}^{3}\left[\frac{SO(3,2)}{SO(3)\times SO(2)}\right]_{S_I,U_I,A_I}$$

  and classical gauge kinetic functions

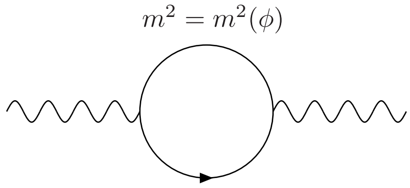
$$f^{(0)}_{\mathrm{D3}} = S\ , \quad f^{(0)}_{\mathrm{D7}_I} = S_I$$

- This defines the $\mathcal{N}=1$ effective action (up to superpot.).

One-loop string threshold corrections

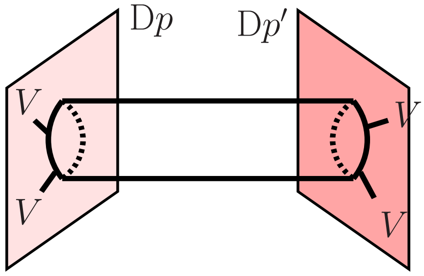
- Threshold correction in string theory:

Field theory:

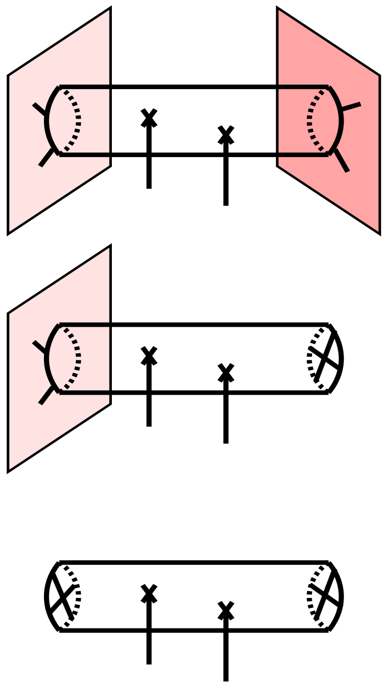


String theory:

Cylinder diagram



Also need to add Möbius strip, Klein bottle, torus diagrams. [Bachas, Fabre; Antoniadis, Bachas, Dudas; and others]



And sum over all boundaries: D3- and D7_I-branes

Technical remarks

- Important technicality: open/closed string vertex operators

$$\mathcal{S} = \frac{1}{2\pi\alpha'}\int_\Sigma d^2z \underbrace{G_{ij}\partial X^i \bar\partial X^j}_{\text{funct. of } S_I, U_I, A_I} + i\int_{\partial\Sigma} d\tau \underbrace{a_i \partial_\tau X^i}_{\text{funct. of } U_I, A_I}$$

Vertex operators of U_I, A_I have bulk plus boundary pieces!

- With this get "classical" metric from sphere+disc:

$$\mathcal{K}^{(0)}_{U_I\bar U_I} = \underbrace{\frac{1}{(U+\bar U)^2}}_{\mathcal{O}(1)} + \underbrace{\frac{(A+\bar A)^2}{(U+\bar U)^3(S_I+\bar S_I+\cdots)}}_{\mathcal{O}(e^{\Phi})}$$
$$+\frac{1}{4}\underbrace{\frac{(A+\bar A)^4}{(U+\bar U)^4(S_I+\bar S_I+\cdots)^2}}_{\mathcal{O}(e^{2\Phi})}$$

"annulus" term from modified vertex operator on disc/sphere.

- One-loop moduli dependence from KK sums

$$\pi^2\sqrt{G_I}\int_0^\infty dl\, l \sum_{\vec n} e^{-l\tilde M^2_{\vec n}} e^{2\pi i \vec a \vec n} = \frac{1}{\sqrt{G_I}} E_2(A_I, U_I) + \cdots$$

with

$$E_s(A,U) = \sum_{n,m\neq(0,0)} \frac{(U+\bar U)^s}{|n+mU|^{2s}} \exp\left[2\pi i \frac{A(n+mU)+\bar A(n-m\bar U)}{U+\bar U}\right]$$

When $A_I = 0$ this becomes Eisenstein series $E_2(U_I)$.

One-loop Kähler potential

- Results for complete one-loop 2-point functions

$$\langle V_{\phi_I} V_{\bar{\phi}_I} \rangle_{\mathcal{K}+\mathcal{A}+\mathcal{M}} = \partial_{\phi_I} \partial_{\bar{\phi}_I} \left[k \frac{\mathcal{E}_2(A_I, U_I)}{(S+\bar{S})(S_I+\bar{S}_I)} \right] \Bigg|_{\phi_I = S_I, U_I, A_I}$$

with: $\mathcal{E}_2(A_I, U_I) = $ sum of $E_2(A_I, U_I)$ over all diagrams

$$\mathcal{E}_2(A,U) = 16 \big[E_2(A,U) + E_2(-A,U) \big] - \big[E_2(2A,U) + E_2(-2A,U) \big]$$

From here read off one-loop Kähler potential

$$\mathcal{K}^{(1)} = k \sum_{I=1}^{3} \frac{\mathcal{E}_2(A_I, U_I)}{(S+\bar{S})(S_I+\bar{S}_I)}$$

- In earlier work have obtained one-loop gauge couplings [Berg, Haack, Körs]

$$f^{(0)}_{\mathrm{D}7_I} = S_I \ , \quad f^{(1)}_{\mathrm{D}7_I} = k' \ln \big[\vartheta_1(A_I, U_I) \big] + \cdots$$

- Together this suffices to define the one-loop correction to the potential in models with volume stabilization via gaugino condensation (KKLT). Furthermore, the corrections can be essential for the inflaton dynamics in models of brane-inflation models built on top of KKLT.

Side-remark: $\mathcal{N}=2$ special geometry

- One can "truncate" $\mathbb{T}^6/\mathbb{Z}_2^2$ to three copies of $\mathbb{T}^2 \times \mathbb{T}^4/\mathbb{Z}_2$

$$\underbrace{\frac{\mathbb{T}^6}{\mathbb{Z}_2 \times \mathbb{Z}_2}}_{\mathcal{N}=1} \;=\; 3 \text{ copies of } \underbrace{\frac{\mathbb{T}^4}{\mathbb{Z}_2} \times \mathbb{T}^2}_{\mathcal{N}=2} \;+\; \text{corrections}$$

  But: for contributions to $\mathcal{K}^{(1)}$ corrections vanish!

- Constraints from $\mathcal{N}=2$ special geometry on $\mathcal{K}$:

$$\mathcal{K} = -\ln\left[2\mathcal{F} + 2\bar{\mathcal{F}} - \sum_N (\phi_N + \bar{\phi}_{\bar{N}})(\mathcal{F}_N + \bar{\mathcal{F}}_{\bar{N}})\right]$$

  with (one-loop exact) prepotential $\mathcal{F}(\phi)$, $\mathcal{F}_N = \partial_{\phi_N}\mathcal{F}$.

- In any of the three $\mathcal{N}=2$ sectors have

$$\mathcal{K}^{(0)} + \mathcal{K}^{(1)} =$$

$$-\ln\Big[\underbrace{(S+\bar{S})[(S_I+\bar{S}_I)(U_I+\bar{U}_I) - \frac{1}{2}(A_I+\bar{A}_I)^2]}_{\text{classical}} + \underbrace{k(U_I+\bar{U}_I)\mathcal{E}_2(A_I,U_I)}_{\text{quantum}}\Big]$$

[Compare to Harvey, Moore; and others]

  and

$$\begin{aligned}
(U+\bar{U})\mathcal{E}_2 &= 2f + 2\bar{f} - (U+\bar{U})(\partial_U f + \partial_{\bar{U}}\bar{f}) - (A+\bar{A})(\partial_A f + \partial_{\bar{A}}\bar{f}) \\
f(A,U) &= 16\Big[h(A,U) + h(-A,U)\Big] - \Big[h(2A,U) + h(-2A,U)\Big] \\
h(A,U) &= \frac{2\pi^4}{8i}\left[\frac{1}{90}U^3 - \frac{1}{3}UA^2 + \frac{2}{3}A^3\right] \\
&\quad + \frac{\pi}{4}Li_3(e^{2\pi i A}) + \frac{\pi}{4}\sum_{m>0}\Big[Li_3(e^{2\pi i(mU-A)}) + Li_3(e^{2\pi i(mU+A)})\Big]
\end{aligned}$$

Supergravity potentials: no-scale

- General form of any $4d$ $\mathcal{N}=1$ potential

$$\mathcal{V}(\phi,\bar{\phi}) = e^{\mathcal{G}}\left[\mathcal{G}^{N\bar{M}}\mathcal{G}_N\bar{\mathcal{G}}_{\bar{M}} - 3\right] + \Re(f_{ab}(\phi))D_aD_b$$

with definitions

$$\begin{aligned}
\mathcal{G} &= \mathcal{K}(\phi,\bar{\phi}) + \ln|W(\phi)|^2 \ , \\
\mathcal{G}_N &= \partial_{\phi^N}\mathcal{G} \ , \quad \mathcal{G}_{N\bar{M}} \ = \ \partial_{\phi^N}\partial_{\bar{\phi}^{\bar{M}}}\mathcal{G} \ = \ \partial_{\phi^N}\partial_{\bar{\phi}^{\bar{M}}}\mathcal{K}
\end{aligned}$$

- Completely determined by three (sets of) functions

$$\begin{aligned}
\mathcal{K}(\phi,\bar{\phi}) &= \mathcal{K}^{(0)} + \mathcal{K}^{(1)} + \sum_{n=2}^{\infty}\mathcal{K}^{(n)} + \mathcal{K}_{\text{nper}} \\
W(\phi) &= W_{\text{cl}} + W_{\text{nper}} \\
f_a(\phi) &= f_a^{(0)} + f_a^{(1)} + (f_a)_{\text{nper}}
\end{aligned}$$

Blue = what we know (at least in part)

- Classical no-scale property of the potential

$$\mathcal{K}(\phi,\bar{\phi}) = \mathcal{K}^{(0)} \ , \quad W(\phi) = W_{\text{cl}}(U_I)$$

leads to

$$\mathcal{G}^{\Sigma\bar{\Gamma}}\mathcal{G}_\Sigma\bar{\mathcal{G}}_{\bar{\Gamma}} = 3 \ , \quad \phi_\Sigma = S_I, A_I \text{ not } U_I$$

and

$$\mathcal{V}(\phi,\bar{\phi}) = \mathcal{V}(U_I,\bar{U}_I) \geq 0$$

a positive definite potential, and flat in S_I (the volume).

Breaking the no-scale property

- Setting U_I, A_I to constants, the one-loop Kähler potential

$$\mathcal{K}^{(0)} + \mathcal{K}^{(1)} = -\ln\left[\prod_{I=1}^{3}(S_I+\bar{S}_I)\right] + \sum_{I=1}^{3}\frac{k_I}{S_I+\bar{S}_I}$$
$$k_I = k\left\langle \frac{\mathcal{E}_2(A_I,U_I)}{S+\bar{S}} \right\rangle$$

- There is another known (tree-level) α' corr. to Kähler pot. [Becker, Becker, Haack, Louis; Balasubramanian, Berglund, Conlon, Quevedo]

$$\mathcal{K}^{(0)}_{\alpha'} = -\ln\left[\prod_{I=1}^{3}(S_I+\bar{S}_I)\right] + \frac{\xi}{\left[\prod_{I=1}^{3}(S_I+\bar{S}_I)\right]^{1/2}}$$

At large volume, $S_I+\bar{S}_I \gg 1$, $\mathcal{K}^{(1)}$ is dominant correction!

- Both corrections break no-scale!
- Set all $S_I = v_6^{2/3}$, $k_I = k$, and use $\mathcal{K} = \mathcal{K}^{(0)}_{\alpha'} + \mathcal{K}^{(1)}$ to get

$$\mathcal{V}(v_6^{2/3}) = \left[3\frac{(v_6+kv_6^{1/3}-\xi)^2}{v_6^2+2kv_6^{4/3}-\frac{5}{2}\xi v_6} - 3\right]|W_{\rm cl}|^2 \xrightarrow{v_6=0} 0$$

which (apparently) does not lead to minima.

Moduli stabilization with fluxes and KKLT

- Superpotential from background fluxes (in orientifolds)

[Gukov, Vafa, Witten; Taylor, Vafa; many others]

$$W_{\mathrm{cl}}(S, U_I) = \int \Omega_3 \wedge G_3$$

Sufficiently generic fluxes can fix all U_I. (Decoupling?)

- Superpotential from gaugino condensation

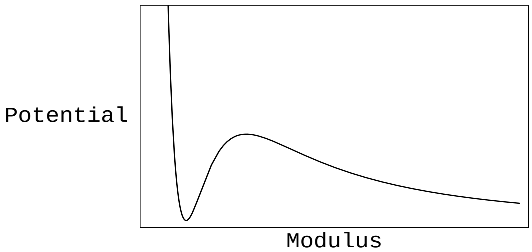
$$W_{\mathrm{nper}}(S_I, U_I, A_I) = \sum_{I=1}^{3} \Lambda^3_{\mathrm{D7}_I} e^{-b f_{\mathrm{D7}_I}(S_I, U_I, A_I)}$$

- D-term from D3-brane tension ("anti-branes")

$$\mathcal{V}_D(S_I, \bar{S}_I, ...) \sim \left[\prod_{I=1}^{3} (S_I + \bar{S}_I + \cdots) \right]^{-3}$$

- With these ingredients can get meta-stable minima: KKLT

[Kachru, Kallosh, Linde, Trivedi; many others]



Positive vacuum energy through combining D-terms and W_{nper}.

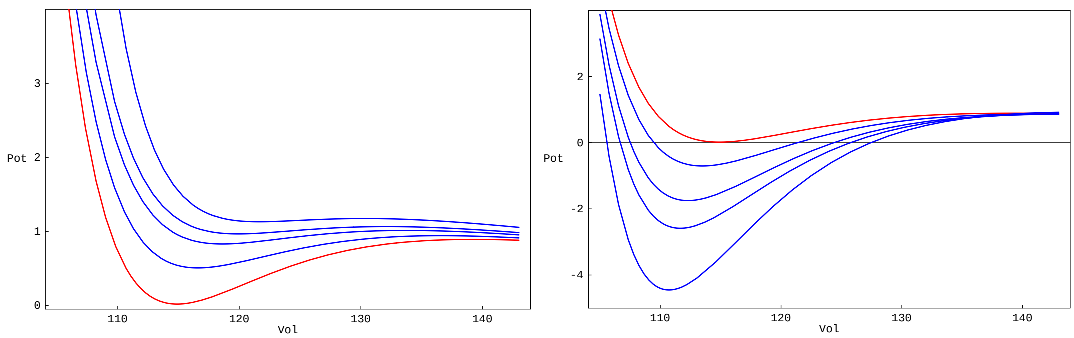
- Only survivors: $A_I \longrightarrow$ inflaton candidates of brane inflation

Impact of one-loop corrections on KKLT minima

- Use same parameter as KKLT
- Set all $S_I + \bar{S}_I \sim \mathrm{vol}(\mathbb{T}^4_I) = v_6^{2/3}$ equal.
- Add one-loop correction to Kähler potential:

$$f_{\mathrm{D7}_I} = f^{(0)}_{\mathrm{D7}_I}(S_I) = v_6^{2/3}\,, \quad \mathcal{K} = \mathcal{K}^{(0)} + \mathcal{K}^{(1)} = \mathcal{K}^{(0)} + k\, v_6^{-2/3}$$

Potential $\mathcal{V}(v_6^{2/3}) \times 10^{15}$ looks like (note $\mathcal{K}^{(0)} \sim 10$)



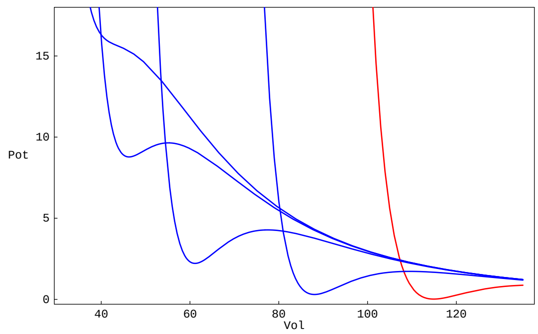
$k \in \{0, 50, 100, 130, 180\}$ $\qquad$ $k \in \{0, -50, -100, -130, -180\}$

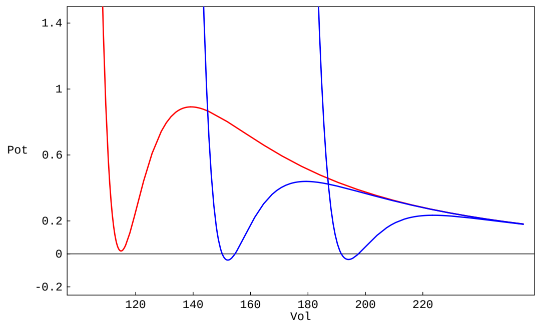
Impact of one-loop correction on KKLT minima

- Same for one-loop gauge couplings:

$$\mathcal{K} = \mathcal{K}^{(0)} \ , \quad f_{\mathrm{D7}_I} = f^{(0)}_{\mathrm{D7}_I}(S_I) + f^{(1)}_{\mathrm{D7}_I}(U_I, A_I) = v_6 + f^{(1)}_{\mathrm{D7}_I}$$

Potential $\mathcal{V}(v_6^{2/3}) \times 10^{15}$ looks like





$f^{(1)}_{\mathrm{D7}_I} \in \{0, 25, 50, 65, 71\}$ $\qquad$ $f^{(1)}_{\mathrm{D7}_I} \in \{0, -35, -75\}$

The inflaton mass of D3/D7-inflation

- Model has classical shift symmetry for $\Im(A_I)$ [Hsu, Kallosh, Prokushkin; Kallosh, Linde; Firouzjahi, Tye; and others]

$$\delta A_I = ic \;\Rightarrow\; \delta\mathcal{V} = 0$$

since

$$\mathcal{K}^{(0)} = -\sum_{I=1}^{3} \ln\left[(S_I + \bar{S}_I)(U_I + \bar{U}_I) - \frac{1}{2}(A_I + \bar{A}_I)^2\right]$$

$$W = W_{\rm cl}(U_I) + \sum_{I=1}^{3} \Lambda_I^3 e^{-b[f^{(0)}_{{\rm D7}_I} + \cdots]}\,, \quad f^{(0)}_{{\rm D7}_I} = S_I$$

Can $\Im(A_I)$ satisfy slow rolling without fine-tuning?

- Corrections violate shift symmetry: non-trivial dynamics [Berg, Haack, Körs]

$$\mathcal{K} = \mathcal{K}^{(0)} + \mathcal{K}^{(1)} + \cdots$$

$$f_{{\rm D7}_I} = f^{(0)}_{{\rm D7}_I} + f^{(1)}_{{\rm D7}_I} = S_I + f^{(1)}_{{\rm D7}_I}(A_I, U_I)$$

In particular: $f^{(1)}_{{\rm D7}_I}$ holomorphic in A_I.

- Compute slow-roll parameter

$$\eta = \frac{\mathcal{V}''}{\mathcal{V}} \overset{!}{\lesssim} 0.01$$

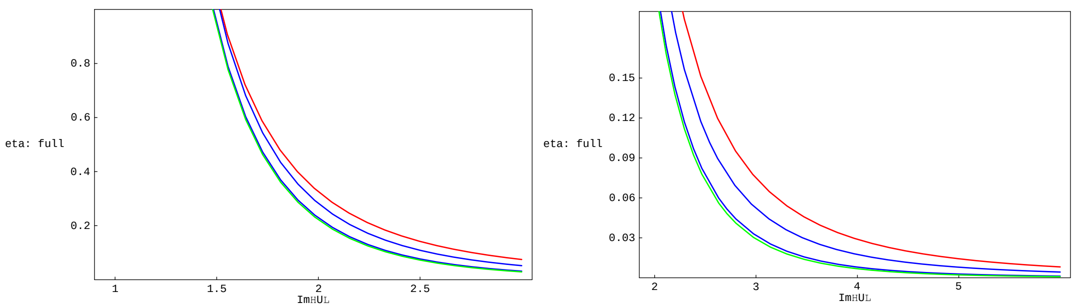
for inflaton candidate $\Im(A_I)$ as function of moduli/parameters.

The inflaton mass of D3/D7-inflation

- The η parameter of $\Im(A)$ for

$$b = 0.1 \ , \quad \Lambda_I = \Lambda \ , \quad W_0 = -10^{-4}$$
$$S = 10 \ , \quad S_I = 100 \ , \quad U_I = iu_2 \ , \quad A_I = i\phi$$

as a function of u_2: $\eta = \eta(u_2)$; and for various Λ



$$\Lambda \in \left\{ \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{1}{10} \right\}$$

- Slow-roll is possible ($u_2 \gtrsim 5$), but very parameter-dependent. Varying the expectation value of $S + \bar{S}$ (string coupling) or the value for Λ_I has significant impact on the range of values fo η.