Large Extra Dimensions from Flux Compactifications

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Based on:

hep-th/0502058 (V. Balasubramanian, P. Berglund, JC, F. Quevedo)

hep-th/0505076 (JC, F. Quevedo, K. Suruliz)

String Phenomenology 2005
Munich.

Review: GKP + KKLT

GKP (2001):

- Compactify F-theory / IIB orientifolds with non-trivial G_4 / G_3 flux
- This stabilises the complex structure moduli of 4-fold
  $\Rightarrow$ dilaton, complex structure and D7-brane moduli are stabilised in IIB
  (Kähler moduli are not stabilised)
- We will work in orientifold limit of F-theory (we need a technical result on the α' corrections only available here)
- Described by $\mathcal{N}=1$ supergravity with

(GVW) $W = \int G_3 \wedge \Omega$ ← holomorphic (3,0) form on CY.
$\hookrightarrow G_3 = F_3 - \tau H_3$

dilaton-axion

$$K = -2\ln(V) - \ln\left(-i\int \Omega \wedge \bar{\Omega}\right) - \ln(-i(\tau - \bar{\tau}))$$

1. Review of IIB Flux Compactifications

2. Importance of α' corrections

3. A particular model, $\mathbb{P}^4_{[1,1,1,6,9]}$

4. Other corrections and the general picture

5. Moduli spectrum and soft term computations

6. Conclusion

Metric is a warped product

$$g_{10} = e^{-2A(y)} dx_4^2 + e^{2A(y)} \underbrace{\tilde{g}_{mn}\, dy^m dy^n}_{\text{CY metric}}$$

Kähler potential has no-scale form

$$\sum_{\substack{i,j \\ \text{(Kähler moduli)}}} G^{ij} \partial_i K \partial_j K = 3$$

$$\Rightarrow V = e^K \left(G^{ij} D_i W D_j W - 3|W|^2 \right) \quad \left(D_i W = \partial_i W + (\partial_i K) W \right)$$

$$= \sum_{\substack{\text{C.S moduli} \\ \text{dilaton}}} e^K \Big(\underbrace{G^{ij} D_i W D_j W}_{\substack{\text{stabilises dilaton} \\ \text{and CS moduli}}} \Big)$$

In this approximation, Kähler moduli are not stabilised.

Dilaton and complex structure moduli are stabilised by solving

$$D_i W = 0.$$

KKLT (2003)

- Integrate out dilaton and CS moduli and introduce a non-perturbative superpotential (gaugino condensation / D3 instantons Witten '96) for the Kähler moduli

$$W = W_0 + A e^{ia\rho}$$

$$K = -3\ln(-i(\rho-\bar{\rho})) \qquad [V \sim (\rho-\bar{\rho})^{3/2}]$$

- Solve $D_\rho W = 0$ to stabilise Kähler modulus.

- Find that

$$D_\rho W = 0 \Rightarrow \rho \sim \frac{1}{aA}\ln(W_0)$$

- If W_0 is small, ρ becomes large.

But: what about perturbative corrections?

Perturbative & Non-perturbative Corrections

$N=1$ SUGRA has

- Kähler potential : $K = K_0 + K_p + K_{np}$
  $\equiv K_0 + J$

- superpotential : $W = W_0 + W_{np}$
  $\equiv W_0 + \Omega$

- (gauge kinetic function)

- Scalar potential:

$$V = e^K [\underbrace{G^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2}_{V_F}] + \text{D-terms}$$

$$V_F = \underbrace{V_0}_{\text{tree-level}} + \underbrace{V_J}_{\text{perturbative correction}} + \underbrace{V_\Omega}_{\text{non-perturbative correction}} + \cdots$$

These Issues in IIB Flux Compactifications

$$W = W_0 + \overbrace{\sum_i A_i e^{i a_i \rho_i}}^{\Omega}$$

$$K = K_{cs} - 2\ln\left(\mathcal{V} + \frac{\xi}{2 g_s^{3/2}}\right) = \overbrace{K_{cs} - 2\ln(\mathcal{V})}^{K_0} - \overbrace{\frac{\xi}{\mathcal{V} g_s^{3/2}}}^{J}$$

(BBHL, 2002)

((Candelas et al))

F-term potential is

$$V_F = e^K \Big[\overbrace{G^{\rho_i \bar{\rho}_j}\big(\partial_{\rho_i} W \partial_{\bar{\rho}_j} \bar{W} + (\partial_{\rho_i} W (\partial_{\bar{\rho}_j} K) \bar{W} + c.c)\big)}^{V_\Omega}\Big] + \frac{\overbrace{3 e^K |W|^2 \xi}^{V_J}}{\mathcal{V} g_s^{3/2}}$$

We want to ask when $|V_\Omega| > |V_J|$.

Let's look at a particular model: take a one-parameter Calabi-Yau, and let's use the geometry of the quintic.

Quintic

- $\mathcal{V} = \frac{5}{6} t^3 \Rightarrow \sigma = \frac{5t^2}{2}, \; \mathcal{V} = \frac{\sqrt{2}}{3\sqrt{5}} \sigma^{3/2}$

- $\rho = b + i\sigma$
- $\chi = -200 \Rightarrow \xi = +0.48$

Take $W = W_0 + e^{ia\rho}$

$K = -2 \ln\left(\mathcal{V} + \frac{\xi}{2 g_s^{3/2}}\right)$

Then

$$V = e^K \Bigg[\overbrace{\frac{4\sigma^2 a^2}{3} e^{-2a\sigma} - 4a\sigma e^{-a\sigma} W_0}^{V_\Omega} + \overbrace{\frac{9\sqrt{5}\, W_0^2}{4\sqrt{2}\,\sigma^{3/2} g_s^{3/2}}}^{V_J} \Bigg]$$

and, if $g_s = \frac{1}{10}$, $a = 2\pi$ (D3-brane instantons)

then we need $W_0 \sim 10^{-75}$ for there to be any region of moduli space in which $V_\Omega > V_J$.

$\Rightarrow$ generally perturbative corrections must be included.

$\mathbb{P}^4_{[1,1,1,6,9]}$ Example

- Study scalar potential for a particular model, $\mathbb{P}^4_{[1,1,1,6,9]}$.

- $h^{1,1} = 2, \quad h^{2,1} = 272$

- $$V = \frac{1}{9\sqrt{2}} \left(\tau_5^{3/2} - \tau_4^{3/2} \right)$$

τ_4 and τ_5 appear nonperturbatively in the superpotential (DOF) $\Rightarrow$

$$W = W_0 + A_4 e^{i a_4 \rho_4} + A_5 e^{i a_5 \rho_5}$$

$(\rho_4 = b_4 + i\tau_4, \; \rho_5 = b_5 + i\tau_5)$

$$K = K_{cs} - 2 \ln \left(V + \frac{\xi}{2 g_s^{3/2}} \right)$$

$$\left(\xi = -\frac{\zeta(3)\chi(M)}{2(2\pi)^3} \approx 1.31 \right)$$

- The geometry is known and so we can compute the scalar potential explicitly

- Examine the potential in the large volume limit ($\mathcal{V} = \frac{1}{9\sqrt{2}}(\tau_5^{3/2} - \tau_4^{3/2}) \gg 1$, $\tau_5 \gg \tau_4 > 1$)

- The scalar potential has the form

$$V = \frac{\lambda\sqrt{\tau_4}(a_4 A_4)^2 e^{-2a_4\tau_4}}{\mathcal{V}} - \frac{\mu \tau_4 W_0 (a_4 A_4) e^{-a_4\tau_4}}{\mathcal{V}^2} + \frac{\nu \xi W_0^2}{\mathcal{V}^3}$$

(– sign comes from axion)

- We can solve for the minimum analytically ($\frac{\partial V}{\partial \tau_4} = \frac{\partial V}{\partial \tau_5} = 0$)

- To see the structure, take the limit

$$\mathcal{V} \to \infty$$

$$a_4 A_4 e^{-a_4\tau_4} \approx \frac{W_0}{\mathcal{V}}$$

The potential then becomes

$$V_F = W_0^2 \left(\frac{\lambda' \sqrt{\ln \mathcal{V}} - \mu' \ln \mathcal{V} + \nu'}{\mathcal{V}^3} \right)$$

Note

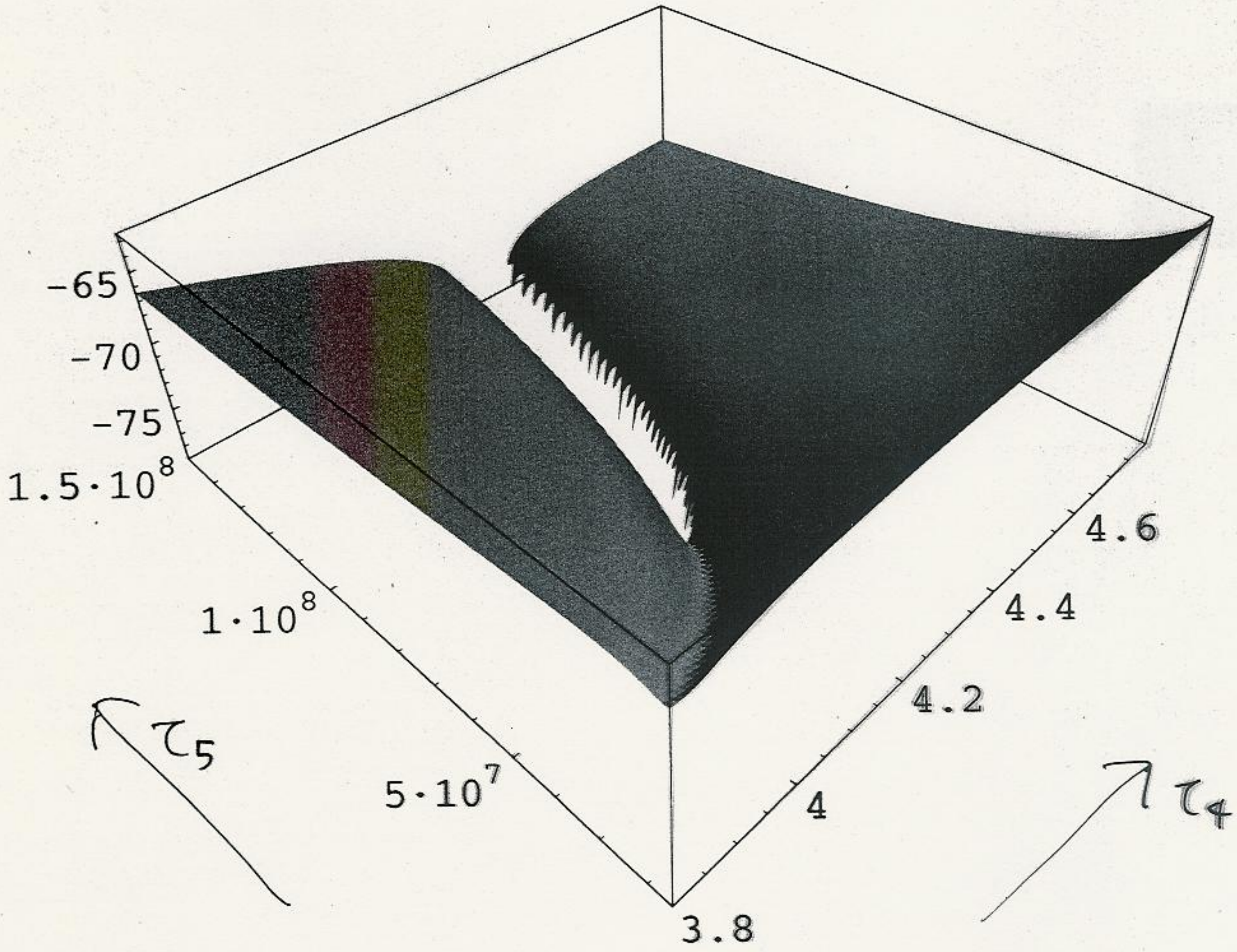
- The potential approaches zero from below as $\mathcal{V} \to \infty$

- The minimum of the potential is at exponentially large volume (as $\ln \mathcal{V}_{min} \sim \frac{\nu'}{\mu'}$) $\left(\mathcal{V} \propto W_0 e^{\frac{2\pi}{g_s N}}\right)$

- The minimum is a non-susy AdS minimum

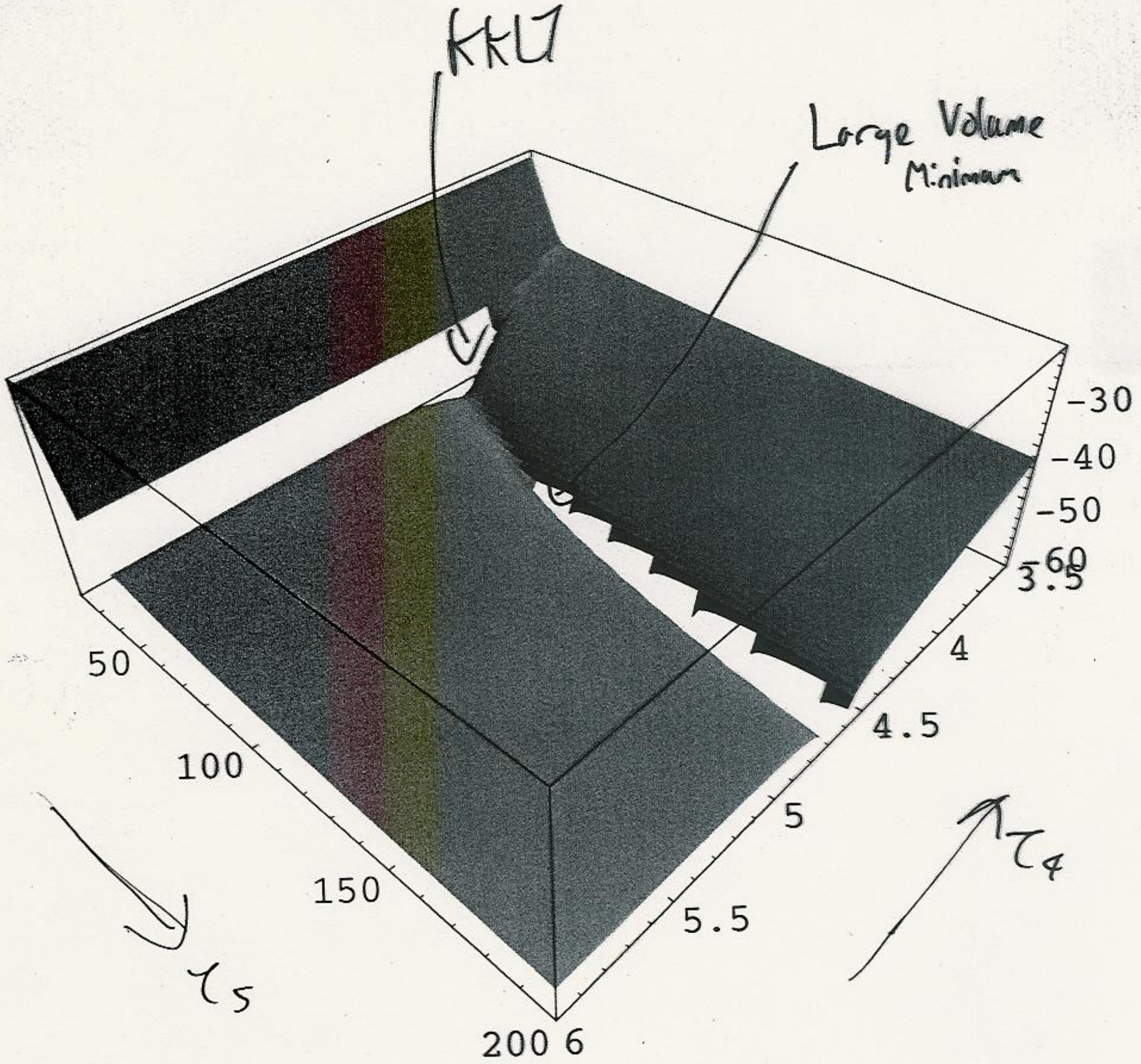
- V_F (minimum) $= -O\left(\frac{1}{\mathcal{V}^3}\right)$

- Can verify the above analytically / numerically

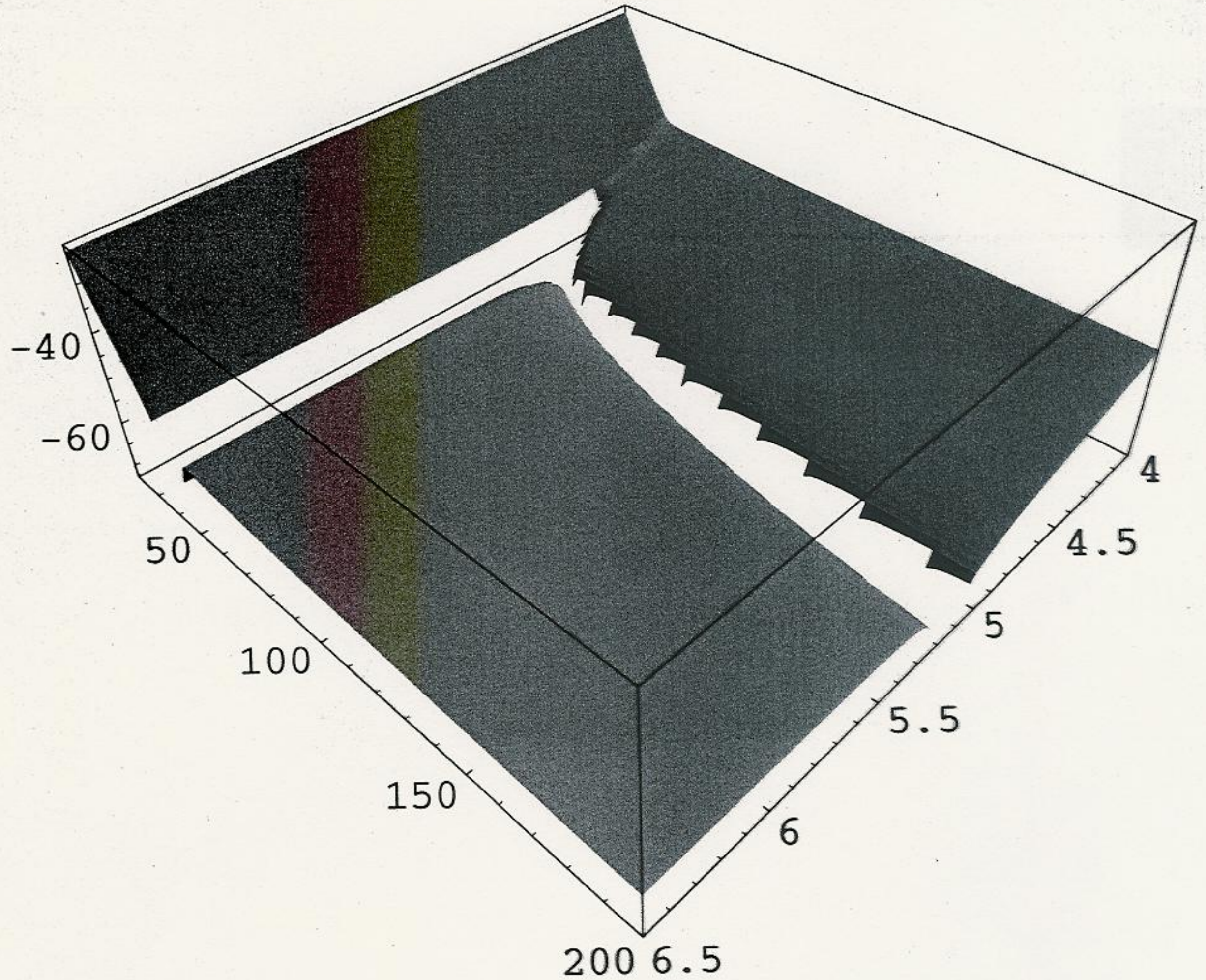


$\ln(V)$ is plotted: the void is where ~~ln~~ V is negative.

$(a_4 = \frac{2\pi}{10},\ g_s = \frac{1}{10},\ W_0 = 1)$



W_0 ~ 10^{-10} : the large volume minimum reduces in size and approaches KKLT.



$W_0 \sim 10^{-11}$, the two minima have merged

- Higher α' corrections do not destabilise the solution. To see this, look at the contributions to the scalar potential from various 10D terms.

(Tree level)

$$\int \dots G_3 \cdot \bar{G}_3 \dots \longrightarrow \underbrace{\frac{1}{\mathcal{V}^2}}_{\substack{\text{Einstein}\\ \text{frame}}} \times \underbrace{\mathcal{V}}_{\substack{\text{internal}\\ \text{integral}}} \times \underbrace{\frac{1}{\mathcal{V}}}_{G^2} \longrightarrow \left.\frac{1}{\mathcal{V}^2}\right] \text{included}$$

(α'^3)

$$\left.\begin{array}{l}\int \dots G_3^2 R^3 \dots \\ \int \dots G_3 \bar{G}_3 R^3 \dots\end{array}\right. \longrightarrow \underbrace{\frac{1}{\mathcal{V}^2}}_{\text{E.F}} \times \underbrace{\mathcal{V}}_{\text{i.i.}} \times \underbrace{\frac{1}{\mathcal{V}}}_{G^2} \times \underbrace{\frac{1}{\mathcal{V}}}_{R^3} \longrightarrow \left.\frac{1}{\mathcal{V}^3}\right] \text{included}$$

$$\int \dots G_3^4 R^2 \dots \longrightarrow \underbrace{\frac{1}{\mathcal{V}^2}}_{\text{E.F}} \times \underbrace{\mathcal{V}}_{\text{i.i.}} \times \underbrace{\frac{1}{\mathcal{V}^{2/3}}}_{R^2} \times \underbrace{\frac{1}{\mathcal{V}^2}}_{G_3^4} \longrightarrow \frac{1}{\mathcal{V}^{11/3}}$$

α'^4)

$$\int \dots G_3^4 R^3 \dots \longrightarrow \underbrace{\frac{1}{\mathcal{V}^2}}_{\text{E.F}} \times \underbrace{\mathcal{V}}_{\text{i.i.}} \times \underbrace{\frac{1}{\mathcal{V}^2}}_{G^4} \times \underbrace{\frac{1}{\mathcal{V}}}_{R^3} \longrightarrow \frac{1}{\mathcal{V}^4}$$

- We can argue likewise for warping effects.
- α' corrections to brane actions are already included in GKP.

. We must check whether this minimum remains a minimum of the full potential including the complex structure moduli
(Choi et al)

. Near the large volume minimum, the full potential looks like

$$V = \underbrace{\frac{G^{a\bar{b}} D_a W D_{\bar{b}} \bar{W}}{\mathcal{V}^2}}_{V_{\text{complex-structure}}} - \underbrace{O\left(\frac{1}{\mathcal{V}^3}\right)}_{V_{\text{kähler}}}$$

As $\mathcal{V} \gg 1$, $D_a W \neq 0$ would increase the scalar potential and so the minimum of the potential lies at $D_a W = 0$.

$\Rightarrow$ The solution is tachyon-free.

- Bulk g_s corrections are under control
  - the tree level (10D) IIB action receives no perturbative g_s corrections
  - at α'^3, the R^4 term gets one-loop corrections: but these are known exactly (as are the nonperturbative corrections) (Green, Sethi)

- D3s do not need to be included (if present, then ~~either~~ can use any correction as an uplift term) (Berg, Haack, Kors) (Bonis Körs s huth)

- D7 moduli are essentially complex structure moduli (F-theory perspective) and so should not affect the Kähler moduli terms – but analytic expressions would be very nice.

General Picture (so long as $h^{2,1} > h^{1,1}$)

In the $\mathbb{P}^4_{[1,1,1,6,9]}$ model, we may have divisor volumes

$$\tau_s \sim 10^8 \quad \text{(or larger)}$$
$$\tau_4 \sim 4$$

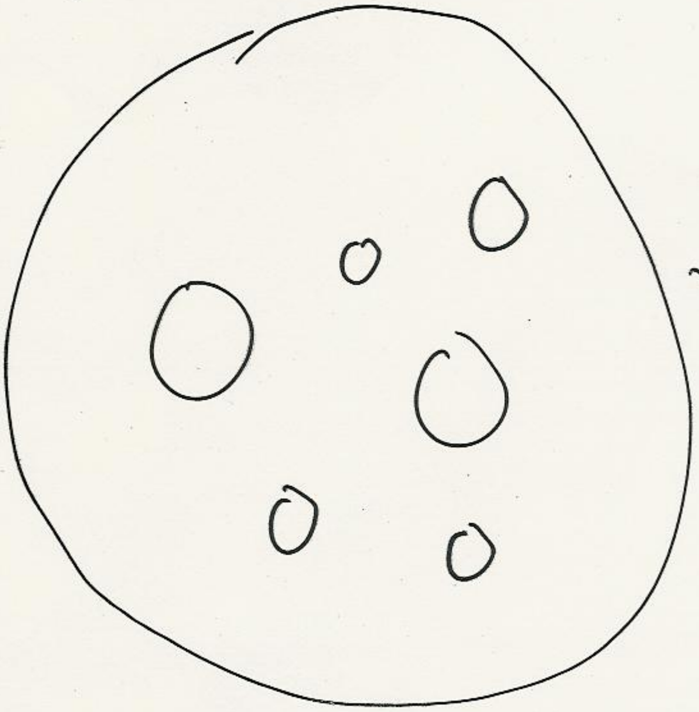
We can show that for more general Calabi-Yaus this structure remains:

$$\begin{array}{ll} 1 & \tau_b \ggg 1 \\ h^{1,1}-1 & \tau_{s,i} > 1,\ O(1) \quad (\sim \ln \tau_b) \end{array}$$

Using the result that

$$\frac{\partial^2 V}{\partial \tau_i \partial \tau_j}$$

has signature $(\overset{1}{+}, \overbrace{-,-,-,\dots -}^{h^{1,1}-1})$ we get a 'Swiss cheese' picture ((Candelas, de la Ossa)



$V \sim R^3 - r_1^3 - r_2^3 \dots - r_n^3$

Summary so far

- Stabilise all moduli to get Large Extra Dimensions$_{(ADD)}$

- Minimum is non-susy – basically inherits the no-scale structure.

- Minimum is automatically tachyon-free

- Large volume $(\mathcal{V} \gg 1)$ gives control over extra α' corrections

Spectra + Soft Terms

- We have an explicit minimum, and so can compute the spectrum and soft terms. (Madrid, München, Hamburg)
- As $\mathcal{V} \sim W_0 e^{\frac{2\pi a}{g_s N}}$, by tuning g_s we can tune the internal volume
  $\Rightarrow$ can tune the string scale.

MODULI SPECTRUM

Scale	Mass	GUT (GeV)	Intermediate (GeV)	TeV (GeV)
M_P	M_P	2.4×10^{18} GeV	2.4×10^{18} GeV	2.4×10^{18}
M_s	$\frac{g_s}{\sqrt{\mathcal{V}}} M_P$	1×10^{15} GeV	1×10^{11} GeV	1×10^{3}
M_{KK}	$\frac{g_s}{\mathcal{V}^{2/3}} M_P$	3×10^{14} GeV	10^{9} GeV	100 MeV
$m_{3/2}$	$\frac{g_s^2}{\mathcal{V}} M_P$	10^{12} GeV	10 TeV	10^{-3} eV
$M_{c.s}, m_\tau$, $M_{\text{small Kähler}}$	$\frac{g_s^2}{\mathcal{V}} M_P$	10^{12} GeV	10 TeV	10^{-3} eV
$M_{\text{large Kähler}}$	$\frac{g_s^2}{\mathcal{V}^{3/2}} M_P$	10^{10} GeV	10 MeV	10^{-15} eV
M_{b_a}	$\exp(-\mathcal{V}^{2/3}) M_P$	10^{-300} eV	$\exp(-10^{8})$ eV	$\exp(-10^{18})$ GeV

- The minimum is non-supersymmetric AdS
- The no-scale structure is inherited and we get 'volume-dominated' soft susy breaking.

$$D_s W = 0 \rightarrow F^s \sim \frac{1}{\mathcal{V}^2}$$

$$D_U W = 0 \rightarrow F^U = 0$$

$$D_{T_{small}} W \sim \frac{1}{\mathcal{V}} \rightarrow F^{T_{small}} \sim \frac{1}{\mathcal{V}}$$

$$D_{T_{big}} W \sim \frac{1}{\mathcal{V}^{2/3}} \Rightarrow F^{T_{big}} \sim \frac{1}{\mathcal{V}^{1/3}}$$

This structure survives the different uplift mechanisms.

We can also compute soft terms and D3 / D7 brane masses.

D3 soft parameters

Scale	'Mass'	GUT	Intermediate
Scalar m_α	$\frac{g_s^2}{\mathcal{V}^{7/6}} M_p$	10^{12} GeV	10^2 GeV
Gaugino M_a	$\frac{g_s^2}{\mathcal{V}^2} M_p$	10^8 GeV	10 eV
A-term	$g_s^2/\mathcal{V}^{4/3} M_p$	10^{11} GeV	1 GeV
μ-term	$g_s^2/\mathcal{V}^{4/3} M_p$	10^{11} GeV	1 GeV
B term	$g_s^2/\mathcal{V}^{7/6} M_p$	10^{12} GeV	10^2 GeV

D7 branes

Scalar m_α	$\frac{g_s^2}{\mathcal{V}} M_p$	10^{12} GeV	10 TeV
Gaugino M_a	$\frac{g_s^2}{\mathcal{V}} M_p$	10^{12} GeV	10 TeV
A-term	$g_s^2/\mathcal{V}\, M_p$	10^{12} GeV	10 TeV
μ-term	$g_s^2/\mathcal{V}\, M_p$	10^{12} GeV	10 TeV
B-term	$g_s^2/\mathcal{V}\, M_p$	10^{12} GeV	10 TeV

Conclusions

- Can for the first time realise Large Extra Dimensions scenario in string theory
- Very little tuning required (no need to have $W_0 \ll 1$)
- Perturbative corrections must generally be included
- Can stabilise all moduli and compute the moduli spectrum and soft terms

— need to find a way to include the SM / chiral matter

— would like better control of open string g_s corrections

— would like an expression for α' correction in F-theory