MODULI STABILIZATION, THRESHOLDS & OBLIQUE MAGNETIC FLUXES

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see also Antoniadis Maillard + Kumar hep-th/0412008-0505260

Plan, Motivations & conclusions:

- Moduli stabilization problem in string compactifications: a new calculable! tool

- Type I strings on "magnetized tori": parallel vs oblique fluxes
- Moduli stabilization (NS-NS & RR Sectors)

- Gauge couplings and thresholds

- Open problems:

  - More realistic models (eg. T^6/Γ, Gepner, ..)
  - Open/closed string mixing (wilson line moduli
  - Yukawa couplings (non-abelian twist correlators
  - Large Extra dimensions, susy breaking (combination with other fluxes/gaugings)
  - Subtle Freed Witten anomalies!

MAGNETIZED T^2: $R = \frac{1-F}{1+F} \in \underline{\underline{SO(2)}}$

Only "parallel" fluxes $\quad Z = X^1 + iX^2$

$$\partial_- Z\big|_a = \underbrace{e^{2i\beta_a}}_{R_C} \partial_+ Z\big|_a \qquad \beta_a = \arctan(q_a f_a)$$

$$F^a = F^a_{12}\, dX^1 dX^2 = \frac{i}{2} f_a\, dZ d\bar{Z} \qquad \underbrace{n \to n + \varepsilon_{ab}}_{\text{mode shift}}$$

Dirac quantization

$$\frac{1}{2\pi}\int_{\Sigma_2 = n_a \cdot T^2} \mathcal{F}^a = m^a \quad \longleftrightarrow \quad f_a = \frac{m_a}{n_a}\frac{\alpha'}{r_1 r_2}$$

m_a : magnetic number $\quad n_a$: wrapping number

$$W^i_{(a)\alpha} = \frac{\partial X^i}{\partial \sigma^\alpha_{(a)}} \qquad n_a = \det(W_a) \in \mathbb{Z}$$

$$m_a \in \mathbb{Z} + B \cdot n_a \qquad \Omega\text{: quantized } B$$

Magnetic shifts $\rightarrow$ tachyons /no susy

$$M^2 \cong M_0^2 + (2n+1)|qF| + 2S\,qF + \ldots \quad \text{Bachas}$$

Degeneracy of Landau level, Dirac index

$$I_{ab} = \frac{1}{2\pi}\int_{\Sigma_2}\left(q_a \mathcal{F}_a + q_b \mathcal{F}_b\right) = C_1\left(\mathcal{E}_a^{q_a} \otimes \mathcal{E}_b^{q_b}\right)$$

$$= \frac{W_a W_b}{2\pi}\int_{T^2}\left(q_a F_a + q_b F_b\right) = q_a m_a n_b - q_b m_b n_a$$

MAGNETIZED T^3: OBLIQUE FLUXES

3 independent components

$$\vec{f} = (F_{yz}, F_{zx}, F_{xy}) \leftrightarrow F_{ij} = \varepsilon_{ijk} f^k$$

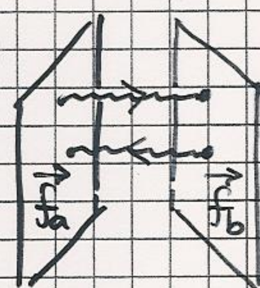
At least one "unmagnetized" direction

$$R\,\vec{u} = \vec{u} \qquad R^{\perp}\vec{v}_{\pm} = e^{\pm 2i\beta}\vec{v}_{\pm}$$

$$\beta = q \arctan(|\vec{f}|) \qquad q = \pm 1 \ (\Omega)$$

Neutral, singly/doubly charged strings ✓

What about dy-charged strings?



$$\vec{f}_a \neq \lambda \vec{f}_b \leftrightarrow [R_a, R_b] \neq 0$$

Boundary state: $R_{ab} = R_a^{q_a} \cdot R_b^{q_b}$

$$|B\rangle_a = \exp -\sum_{i,j;n} a^{i,+}_{n,+} R^{(a)}_{ij} \tilde{a}^{j,+}_{n,-} |0\rangle$$

$${}_a\langle B| e^{-tH_{cl}} |B\rangle_b \sim \sum_k \mathrm{tr}(R_a R_b^{-1})^k c_k$$

Use spin ½ representation

$$R_{ab} = R_a^{q_a} R_b^{q_b} = (c_a - i q_a \hat{f}_a \cdot \vec{\sigma} s_a)(c_b - i q_b \hat{f}_b \cdot \vec{\sigma} s_b)$$

$$= \underbrace{(c_a c_b - q_a q_b s_a s_b \hat{f}_a \cdot \hat{f}_b)}_{\cos(\beta_{ab})} - i(q_a s_a c_b \hat{f}_a + q_b s_b c_a \hat{f}_b + q_a q_b s_a s_b \hat{f}_a \times \hat{f}_b) \cdot$$

MAGNETIZED T^4

Use $SO(4) \sim SU(2) \times SU(2)$ decomposition:

$$F = F^+ + F^- = \frac{1}{2}\left(f_r^+ \eta^r_{ij} + f_r^- \bar{\eta}^r_{ij}\right) dx^i dx^j$$

$$R(F) = \frac{1-F}{1+F} = R_+ R_- = R_- R_+$$

where

$$R_\pm = \frac{1 \mp |f_+|^2 \pm |f_-|^2 - 2F^\pm}{\sqrt{1 + 2(|f_+|^2 + |f_-|^2) + (|f_+|^2 - |f_-|^2)^2}}$$

$$\tan(2\beta_\pm) = \frac{2|f_\pm|}{1 \mp |f_+|^2 \pm |f_-|^2}$$

Non abelian composition rule as before

$$\vec{\sigma} \leftrightarrow \vec{\Sigma} = i\vec{\eta} \;/\; \vec{\Sigma}^* = i\vec{\eta}^*$$

Magnetic shifts:

- bosons (internal vectors): $\pm\beta^L_{ab} \pm \beta^R_{ab}$
- fermions: $(\frac{1}{2},0) \to \pm\beta^L_{ab}$ $\quad (0,\frac{1}{2}) \to \pm\beta^R_{ab}$

Supersymmetry / BPS condition

$\forall a$: $\beta^a_L = 0$ $(F^a_+ = 0)$ or $\forall a$ $\beta^a_R = 0$ $(F^a_- = 0)$

NB: no "unmagnetized" directions generically

SCALAR POTENTIAL

$$V_D(\phi,\omega,\hat{G}) = \tau_D e^{-\phi} \sum_a |w_a| \hat{N}_a \sqrt{\omega^2 + \omega C_1^2(F_a) + C_2^2(F_a)}$$

overall volume $\omega = \sqrt{G}$, $\underset{(10)}{G_{ij}} = \underset{(1)}{\omega^{1/2}} \underset{(9)}{\hat{G}_{ij}}$

$$C_1^2(F_a) = \frac{1}{2} \hat{G}^{ij} \hat{G}^{kl} F^a_{ik} F^a_{jl}$$

$$C_2^2(F_a) = \frac{1}{8} \left(\varepsilon^{ijkl} F^a_{ij} F^a_{kl}\right)^2$$

Different behaviors for $C_1^2 \gtrless 2|C_2|$

Susy : $C_1^2(F_a) = 2|C_2(F_a)| \quad \forall a$

$\rightarrow \quad V_D = \tau_D e^{-\phi} \sum_a \hat{N}_a W_a \left(\omega + |C_2^{(a)}|\right)$

bound-state at threshold !

Ω-plane , tadpole cancellation

Minimize V $C_2^{(a)}(\mathfrak{I}, \hat{G})$ fixed, susy: $J \wedge F^a = \Omega \wedge F^a = 0$

Fix all the moduli except ϕ, ω, C_{IJ} C_{IJ}

Non linear instantons — Marino Moore Minasian Strominger, Rabadan, Marchesano, Shiu, ...

$$\sqrt{\det(\mathbb{1} + \mathcal{H})} = \sqrt{(1+h_1^2)(1+h_2^2)} = \sqrt{(1 \pm h_1 h_2)^2 + (h_1 \mp h_2)^2}$$

$h_1 = \pm h_2$ "linear" (a).s.d. $\qquad h_1 = \pm \frac{1}{h_2}$ non linear

MAGNETIZED T^6 (AT LAST!)

A priori 15 components

$$F_{ij} \;:\; 3\, F^{(2,0)}_{[IJ]} \quad 9\, F^{(1,1)}_{I\bar{J}} \quad 3\, F^{(0,2)}_{[\bar{I}\bar{J}]}$$

$$R = \frac{1-F}{1+F} \in SO(6) \quad \text{in general}$$

Restrict to susy configurations

- $F^{(2,0)} = 0$ Holomorphic bdle
- $F^a \wedge F^a \wedge F^a = 3 \underbrace{J\wedge J \wedge F^a}_{\text{1-loop in heterotic (Honecker....}}$ stability

Then $R = U_3 \oplus U_{3^*} \in SU(3)$ $\det U_3 = 1$

$$U_3 = \frac{\mathbb{1} - iH}{\mathbb{1} + iH} \qquad H = h_0 \mathbb{1} + h_r \lambda^r = h_0 \mathbb{1} + \hat{H}$$

Gell Mann matrices $\lambda_r^+ = \lambda_r$ $\mathrm{tr}\, \lambda^r \lambda^s = 3\delta^{rs}$

$$\lambda^r \lambda^s = \delta^{rs} \mathbb{1} + (i f^{[rst]} + d^{rst})\, \lambda_t$$

$$\rho_k^U = e^{2i\beta_k} = \frac{1 - ih_0 - i\mu_k}{1 + ih_0 + i\mu_k} \qquad \mu^3 - \frac{3}{2}|h|^2 \mu_k = dhhh$$

$$\mu_k^{\hat{H}} = \frac{e^{2\pi i k/3}}{\sqrt[3]{2}} \sqrt[3]{dhhh + i\sqrt{\tfrac{1}{2}|h|^6 - (dhhh)^2}} + \text{c.c.} \in \mathbb{R}$$

NB $|h|^6 \geq 2(dhhh)^2$ $\qquad |h|^6 \equiv \left(\sum_r h_r h_r\right)^3$

DY-CHARGED STRINGS

$$R_{ab} = R_a^{q_a} R_b^{q_b} = \left(\frac{1 - i q_a H_a}{1 + i q_a H_a}\right)\left(\frac{1 - i q_b H_b}{1 + i q_b H_b}\right)$$

$$= \left(\frac{1 - i q_a h_a^0}{1 + i q_a h_a^0}\right)\left(\frac{1 - i q_b h_b^0}{1 + i q_b h_b^0}\right) M_a^{+} M_a^{-1} M_b^{-1} M_b^{+} \underbrace{M_a^{+} M_a^{\dagger}}_{\mathbb{1}}$$

$$M_a = 1 + i q_a \hat{h}_a \cdot \lambda \qquad \hat{h}_a = \frac{h_a}{1 - i q_a h_a^0} \in \mathbb{C}$$

SU(3) composition rule

$$\hat{h}_{ab}^r = \frac{q_a \hat{h}_a^r + q_b \hat{h}_b^r + (f^r_{st} - i d^r_{st}) \hat{h}_a^s \hat{h}_b^t q_a q_b}{1 + q_a q_b \hat{h}_a \cdot \hat{h}_b} \rightarrow \hat{\mu}_{ab}$$

$$\rho_{ab}^{(k)} = e^{2i\beta_{ab}^{(k)}} =$$

$$= \frac{(1 - i q_a h_0^a)(1 - i q_b h_0^b)\left[|1 + q_a q_b \hat{h}_a \cdot \hat{h}_b|^2 + i (1 + q_a q_b \hat{h}_a \hat{h}_b)^2 \mu_a \ldots\right.}{(1 + i q_a h_0^a)(1 + i q_b h_0^b)\left[|1 + q_a q_b \hat{h}_a \hat{h}_b|^2 - i (1 + q_a q_b \hat{h}_a^* \hat{h}_b^*) \mu_a \ldots\right.}$$

Multiplicities

$$I_{ab} = -\frac{1}{3!(2\pi)^3} \int_{\Sigma_6} (q_a \mathcal{F}_a + q_b \mathcal{F}_b)^3 = C_{3,ab}$$

$$= -\frac{W_a W_b}{3!(2\pi)^3} \int_{T^6} (q_a F_a + q_b F_b)^3$$

$$= \prod_i \left(q_a m_a^i n_b^i + q_b m_b^i n_a^i\right)$$

$T^6 = T^2 \times T^2 \times T^2$ + "parallel" fluxes

SCALAR POTENTIAL (FOR $\langle \varphi_{op} \rangle = 0$!)

In $d=6$ $\quad H^a_{\hat{i}\hat{j}} = E^i_{\hat{i}} E^j_{\hat{j}} F^a_{ij}$ (frame components)

$$\frac{\sqrt{\det(G_a + \mathcal{F}_a)}}{\sqrt{\det(G_a)}} = 1 - \frac{1}{2}\operatorname{tr} H_a^2 + \frac{1}{4}(\operatorname{tr} H_a^2)^2 - \frac{1}{4}\operatorname{tr} H_a^4$$

$$\underbrace{- \frac{1}{6}\operatorname{tr}(H_a^6) + \frac{1}{8}\operatorname{tr}(H_a^2)\operatorname{tr}(H_a^4) - \frac{1}{48}\left[\operatorname{tr}(H_a^2)\right]^3}_{\frac{1}{8(3!)^2}(\varepsilon H_a H_a H_a)^2}$$

Skew diagonalizing H_a:

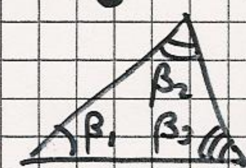
$$\sqrt{\det(1+H_a)} = \sqrt{\prod_1^3 (1+h_i^2)} = \sqrt{(1-h_1h_2-h_2h_3-h_3h_1)^2 + (h_1h_2h_3-h_1-h_2-h$$

Marchesano, Shiu, A-M, Dudas Timirgaziu NEGATIVE INDUCED TENSIONS! RR CHARGES

- For holomorphic bdles $F^a_{(2,0)} = 0 \ \forall a$

$$\rightarrow \frac{|W|}{3!}\sqrt{\left(\int J\wedge J\wedge J - 3\int F^a\wedge F^a\wedge J\right)^2 + \left(\int F^a\wedge F^a\wedge F^a - 3\int J\wedge J\wedge F^a\right)^2$$

Susy $\quad h_1^a h_2^a h_3^a = h_1^a + h_2^a + h_3^a \qquad h_i = \tan\beta$

Super-magnetic triangle 

- Susy preserving moduli stabilization

$dz^I = dx^I + \tau^I{}_J dy^J$: $\quad F^a_{20} = 0 \leftrightarrow \tau^T f_{xx}\tau - \tau^T f_{xy} - f_{xy}\tau_y = $

Need 6 stacks of branes $\xrightarrow{\text{e.g.}} \tau^I{}_J = i\delta^I{}_J$

$F^a \wedge F^a \wedge F^a = 3 J\wedge J\wedge F^a$ $\quad$ fix Kähler class J

Need 3 more stacks $\quad$ Antoniadis, Maillard

ANTONIADIS - MAILLARD MODEL

Stacks	magnetic and wrapping numbers	magnetic fields
$N_0 = 3$	$(m_{ij}, n_{ij}) = (0,1)$	$H_{ij} = 0$
$N_1 = 1$	$(m_{x_1y_2}, n_{x_1y_2}) = (1,-1)$	$H_{x_1y_2} = -\sqrt{2}$
	$(m_{x_2y_1}, n_{x_2y_1}) = (1,-1)$	$H_{x_2y_1} = -\sqrt{2}$
	$(m_{x_3y_3}, n_{x_3y_3}) = (0,-1)$	$H_{x_3y_3} = 0$
$N_2 = 1$	$(m_{x_1y_3}, n_{x_1y_3}) = (1,-1)$	$H_{x_1y_3} = -2$
	$(m_{x_3y_1}, n_{x_3y_1}) = (1,-1)$	$H_{x_3y_1} = -2$
	$(m_{x_2y_2}, n_{x_2y_2}) = (0,-1)$	$H_{x_2y_2} = 0$
$N_3 = 1$	$(m_{x_1x_2}, n_{x_1x_2}) = (1,-1)$	$H_{x_1x_2} = -\sqrt{2}$
	$(m_{y_1y_2}, n_{y_1y_2}) = (1,-1)$	$H_{y_1y_2} = -\sqrt{2}$
	$(m_{x_3y_3}, n_{x_3y_3}) = (0,-1)$	$H_{x_3y_3} = 0$
$N_4 = 2$	$(m_{x_1y_1}, n_{x_1y_1}) = (0,-1)$	$H_{x_1y_1} = 0$
	$(m_{x_2x_3}, n_{x_2x_3}) = (1,-1)$	$H_{x_2x_3} = -1$
	$(m_{y_2y_3}, n_{y_2y_3}) = (1,-1)$	$H_{y_2y_3} = -1$
$N_5 = 1$	$(m_{x_1x_3}, n_{x_1x_3}) = (1,-1)$	$H_{x_1x_3} = -2$
	$(m_{x_2y_2}, n_{x_2y_2}) = (0,-1)$	$H_{x_2y_2} = 0$
	$(m_{y_1y_3}, n_{y_1y_3}) = (1,-1)$	$H_{y_1y_3} = -2$
$N_6 = 2$	$(m_{x_1y_1}, n_{x_1y_1}) = (0,-1)$	$H_{x_1y_1} = 0$
	$(m_{x_2y_3}, n_{x_2y_3}) = (1,-1)$	$H_{x_2y_3} = -1$
	$(m_{x_3y_2}, n_{x_3y_2}) = (1,-1)$	$H_{x_3y_2} = -1$
$N_7 = 2$	$(m_{x_1y_1}, n_{x_1y_1}) = (-2,1)$	$H_{x_1y_1} = -\frac{1}{\sqrt{2}}$
	$(m_{x_2y_2}, n_{x_2y_2}) = (0,1)$	$H_{x_2y_2} = 0$
	$(m_{x_3y_3}, n_{x_3y_3}) = (1,1)$	$H_{x_3y_3} = \frac{1}{\sqrt{2}}$
$N_8 = 2$	$(m_{x_1y_1}, n_{x_1y_1}) = (-1,1)$	$H_{x_1y_1} = -\frac{1}{2\sqrt{2}}$
	$(m_{x_2y_2}, n_{x_2y_2}) = (1,1)$	$H_{x_2y_2} = \sqrt{2}$
	$(m_{x_3y_3}, n_{x_3y_3}) = (-1,1)$	$H_{x_3y_3} = -\frac{1}{\sqrt{2}}$
$N_9 = 1$	$(m_{x_1y_1}, n_{x_1y_1}) = (0,1)$	$H_{x_1y_1} = 0$
	$(m_{x_2y_2}, n_{x_2y_2}) = (-1,1)$	$H_{x_2y_2} = -\sqrt{2}$
	$(m_{x_3y_3}, n_{x_3y_3}) = (2,1)$	$H_{x_3y_3} = \sqrt{2}$

sectors	eigenvalues	unmagnetized directions
$U_4^{q_a} U_1^{q_b}$	$\rho = 1$ and $\rho^{\pm} = (-2 \pm i\sqrt{5})/3$	$(-\frac{iq_a q_b}{\sqrt{2}}, q_a, 1)$
$U_4^{q_a} U_3^{q_b}$	$\rho = 1$and $\rho^{\pm} = (-2 \pm i\sqrt{5})/3$	$(\frac{q_a q_b}{\sqrt{2}}, q_a, 1)$
$U_6^{q_a} U_1^{q_b}$	$\rho = 1$and $\rho^{\pm} = (-2 \pm i\sqrt{5})/3$	$(-\frac{q_a q_b}{\sqrt{2}}, -iq_a, 1)$
$U_6^{q_a} U_3^{q_b}$	$\rho = 1$and $\rho^{\pm} = (-2 \pm i\sqrt{5})/3$	$(-\frac{iq_a q_b}{\sqrt{2}}, -q_a, 1)$
$U_2^{q_a} U_4^{q_b}$	$\rho = 1$ and $\rho^{\pm} = (-4 \pm i\,3)/5$	$(\frac{iq_a}{2}, q_b, 1)$
$U_2^{q_a} U_6^{q_b}$	$\rho = 1$ and $\rho^{\pm} = (-4 \pm i\,3)/5$	$(\frac{iq_a}{2}, -iq_b, 1)$
$U_5^{q_a} U_4^{q_b}$	$\rho = 1$ and $\rho^{\pm} = (-4 \pm i\,3)/5$	$(-\frac{q_a}{2}, q_b, 1)$
$U_5^{q_a} U_6^{q_b}$	$\rho = 1$ and $\rho^{\pm} = (-4 \pm i\,3)/5$	$(-\frac{q_a}{2}, -iq_b, 1)$
$U_1^{q_a} U_2^{q_b}$	$\rho = 1$ and $\rho^{\pm} = (-13 \pm i\sqrt{56})/15$	$(2iq_b, -\sqrt{2} q_a q_b, 1)$
$U_1^{q_a} U_5^{q_b}$	$\rho = 1$ and $\rho^{\pm} = (-13 \pm i\sqrt{56})/15$	$(-2q_b, -i\sqrt{2} q_a q_b, 1)$
$U_3^{q_a} U_2^{q_b}$	$\rho = 1$ and $\rho^{\pm} = (-13 \pm i\sqrt{56})/15$	$(2iq_b, i\sqrt{2} q_a q_b, 1)$
$U_3^{q_a} U_5^{q_b}$	$\rho = 1$ and $\rho^{\pm} = (-13 \pm i\sqrt{56})/15$	$(-2q_b, -\sqrt{2} q_a q_b, 1)$

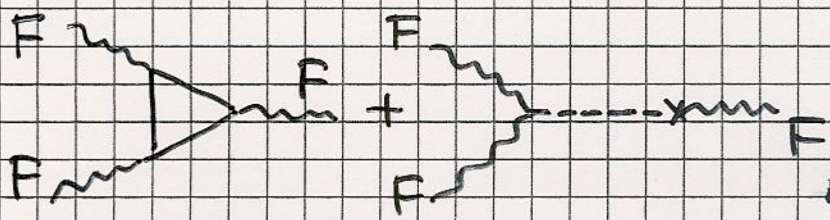
sectors	eigenvalues	unmagnetized directions
$U_1^{q_a} U_3^{q_b}$	$\rho = 1$ and $\rho^{\pm} = \frac{1}{9}(1 \pm 4i\sqrt{5})$	$(0, 0, 1)$
$U_2^{q_a} U_5^{q_b}$	$\rho = 1$ and $\rho^{\pm} = \frac{1}{25}(9 \pm 4i\sqrt{34})$	$(0, 1, 0)$
$U_4^{q_a} U_6^{q_b}$	$\rho = 1$ and $\rho^{\pm} = \pm i$	$(1, 0, 0)$
$U_7^{q_a} U_8^{q_b}$	$\rho_1 = -(-7 + 16 q_a q_b + 2\sqrt{2} i (2q_b + 7q_a))/27$ $\rho_2 = -(1 - 2i\sqrt{2} q_b)/3$ $\rho_3 = (1 + 8 q_a q_b + 2\sqrt{2} i (q_a - q_b))/9$	for $q_a \neq q_b$ no unmagnetized directions $(0, 0, 1)$ for $q_a = q_b$
$U_7^{q_a} U_9^{q_b}$	$\rho_1 = -\frac{1}{3}(1 + 2i\sqrt{2} q_b)$ $\rho_2 = -(1 + 8 q_a q_b + 2\sqrt{2} i (q_a - q_b))/9$ $\rho_3 = \frac{1}{3}(1 - 2i\sqrt{2} q_a)$	no unmagnetized directions
$U_8^{q_a} U_9^{q_b}$	$\rho_1 = \frac{1}{9}(7 - 4i\sqrt{2} q_a)$ $\rho_2 = (1 + 8 q_a q_b - 2\sqrt{2} i (q_a - q_b))/9$ $\rho_3 = (-1 + 8 q_a q_b + 2\sqrt{2} i (q_a + q_b))/9$	for $q_a \neq q_b$ no unmagnetized directions $(0, 1, 0)$ for $q_a = q_b$

sectors	eigenvalues	unmagnetized directions
$U_7^{q_a}U_1^{q_b}$	$\rho_1 = \frac{1}{3}(1+2i\sqrt{2}q_a)$ $\rho_{2,3} = \frac{1}{9}(-2+i\sqrt{2}q_a \pm 5\sqrt{(-1+2i\sqrt{2}q_a)})$	no unmagnetized directions
$U_7^{q_a}U_2^{q_b}$	$\rho_1 = 1$ and $\rho_{2,3} = \frac{1}{15}(-3 \pm 6i\sqrt{6}$	(0,1,0)
$U_7^{q_a}U_3^{q_b}$	$\rho_1 = \frac{1}{3}(1+2i\sqrt{2}q_a)$ $\rho_{2,3} = \frac{1}{9}(-2+i\sqrt{2}q_a \pm 5\sqrt{(-1+2i\sqrt{2}q_a)})$	no unmagnetized directions
$U_7^{q_a}U_4^{q_b}$	$\rho_1 = \frac{1}{3}(1-2i\sqrt{2}q_a)$ $\rho_{2,3} = \pm\frac{\sqrt{-1-2i\sqrt{2}q_a}}{\sqrt{3}}$	no unmagnetized directions
$U_7^{q_a}U_5^{q_b}$	$\rho_1 = 1\ ,\ \rho_{2,3} = \frac{1}{15}(-3 \pm 6i\sqrt{6})$	(0,1,0)
$U_7^{q_a}U_6^{q_b}$	$\rho_1 = (\frac{1}{3}(1-2i\sqrt{2}q_a)$ $\rho_{2,3} = \pm\frac{\sqrt{-1-2i\sqrt{2}q_a}}{\sqrt{3}}$	no unmagnetized directions
$U_8^{q_a}U_1^{q_b}$	$\rho_1 = \frac{1-2i\sqrt{2}q_a}{3}$ $\rho_{2,3} = \frac{1}{27}(-2-i\sqrt{2}q_a \pm \sqrt{(-241-482i\sqrt{2}q_a)})$	no unmagnetized directions
$U_8^{q_a}U_2^{q_b}$	$\rho_1 = -\frac{1-2i\sqrt{2}q_a}{3}$ $\rho_{2,3} = \frac{1}{3}(-1+i\sqrt{2}q_a \pm \sqrt{(2+4i\sqrt{2}q_a)})$	no unmagnetized directions
$U_8^{q_a}U_3^{q_b}$	$\rho_1 = \frac{1-2i\sqrt{2}q_a}{3}$ $\rho_{2,3} = \frac{1}{27}(-2-i\sqrt{2}q_a \pm \sqrt{(-241-482i\sqrt{2}q_a)})$	no unmagnetized directions
$U_8^{q_a}U_4^{q_b}$	$\rho_1 = \frac{1}{9}(7-4i\sqrt{2}q_a)$ $\rho_{2,3} = \pm\frac{1}{3}\sqrt{(-7-4i\sqrt{2})}$	no unmagnetized directions
$U_8^{q_a}U_5^{q_b}$	$\rho_1 = -\frac{1}{3}(1-2\sqrt{2}iq_a)$ $\rho_{2,3} = \frac{1}{3}(-1+i\sqrt{2} \pm \sqrt{2+i4\sqrt{2}q_a})$	no unmagnetized directions
$U_8^{q_a}U_6^{q_b}$	$\rho_1 = \frac{1}{9}(7-4i\sqrt{2}iq_a)$ $\rho 2,3 = \pm\frac{1}{3}\sqrt{(-7-4i\sqrt{2}iq_a)}$	no unmagnetized directions
$U_9^{q_a}U_1^{q_b}$	$\rho_1 = -\frac{1}{3}(1-2\sqrt{2}iq_a)$ $\rho_{2,3} = \frac{1}{9}(-1+i\sqrt{2}q_a \pm \sqrt{26}\sqrt{(1+2\sqrt{2}iq_a)})$	no unmagnetized directions
$U_9^{q_a}U_2^{q_b}$	$\rho_1 = -\frac{1}{3}(1+2\sqrt{2}iq_a)$ $\rho_{2,3} = \frac{1}{15}(-3-3\sqrt{2}iq_a \pm \sqrt{6}\sqrt{(11-22i\sqrt{2}q_a)})$	no unmagnetized directions
$U_9^{q_a}U_3^{q_b}$	$\rho_1 = -\frac{1}{3}(1-2\sqrt{2}iq_a)$ $\rho_{2,3} = \frac{1}{9}(-1+i\sqrt{2}q_a \pm \sqrt{26}\sqrt{(1+3\sqrt{2}iq_a)})$	no unmagnetized directions
$U_9^{q_a}U_4^{q_b}$	$\rho_1 = 1\ ,\ \rho_{2,3} = \pm i$	$(1,0,0)$
$U_9^{q_a}U_5^{q_b}$	$\rho_1 = -\frac{1}{3}(1+2\sqrt{2}iq_a)$ $\rho_{2,3} = \frac{1}{15}(-3-3\sqrt{2}iq_a \pm \sqrt{6}\sqrt{(11-22i\sqrt{2}q_a)})$	no unmagnetized directions
$U_9^{q_a}U_6^{q_b}$	$\rho_1 = 1\ ,\ \rho_{2,3} = \pm i$	$(1,0,0)$

R-R MODULI "Petite Bouffe"

$C_{I\bar{J}}$ ($\underline{\underline{9}}$) eaten by anomalous U(1)'s

Generalized GSS mechanism



$\epsilon_{ijk} A^i \wedge A^j \wedge F^k$

$= 0 \pm$ CS coupling

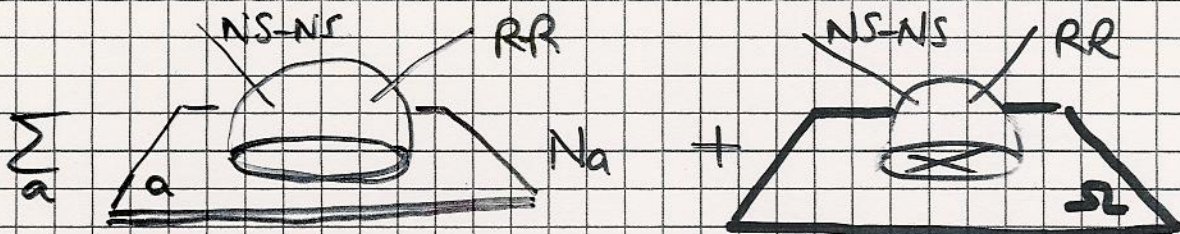
Anastasopoulos, Kiritsis, Dud...

$C_{I\bar{J}} \leftrightarrow \tilde{C}_{\mu\nu, K\bar{L}M\bar{N}}$ $\quad \mathcal{L}_{WZ} = \left(C_0 F \wedge F + \tilde{C}_2 \wedge F \right)$

$C_{IJ} / C_{\bar{I}\bar{J}}$ (3+3) complex structure

non derivative mixing with $\delta G_{IJ}, \delta G_{\bar{I}\bar{J}}$

- $H_3 = dC_2 + e^{-\phi} \omega_3$ $\quad dC_2 \wedge * \omega_3 e^{-\phi} \sim C_2 \wedge d(*\omega_3) e$
- Alternatively, 2-pt amplitude



s, t poles $\qquad$ only t poles

$s = p_1 D p_1$ (open) $\qquad t = (p_1 + p_2)^2$ (closed)

- t poles cancel (tadpole cancellation)
- s pole open/closed mixing
- remnant : non derivative mixing for $F \neq 0$!

GAUGE COUPLINGS

- Tree level   Blumenhagen, Görlich, Stieberger, ...

$$\frac{1}{g_a^2} = |W_a| e^{-\phi} \sqrt{\det(G + F_a)}$$

á la Bachas, Fabre ; Antoniadis, Bachas

- one-loop running and thresholds : Dudas

Character valued partition function

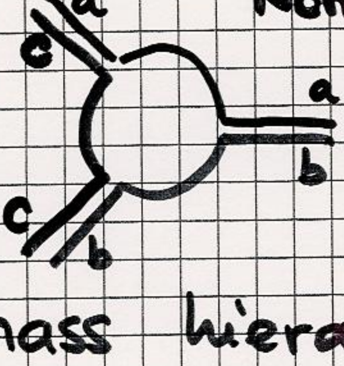
$$K(0) \sum_\alpha{}' C_\alpha \frac{\Theta_\alpha^4(0)}{\eta^{12}} \to K(\varepsilon) \sum_\alpha{}' C_\alpha \prod_{i=1}^4 \frac{\Theta_\alpha(\varepsilon_{ab}^i \tau)}{\Theta_1(\varepsilon_{ab}^i \tau)}$$

zero mode contributions

$\varepsilon_{ab}^{sp.t.}$ small and arbitrary (M_4 non compac

ε_{ab}^{int} "quantized" ($\leftarrow$ mode shift, ...)

- Yukawa couplings & triangle :



Non-abelian!

Twist field correlator :

$$\varepsilon_{ab}^{(i)} \varepsilon_{bc}^{(i)} \varepsilon_{ca}^{(i)} = \varepsilon_{ab}^{(i)} + \varepsilon_{bc}^{(i)} + \varepsilon_{ca}^{(i)}$$

mass hierarchy?

- **T-dual to intersecting branes <u>with</u> fluxes**

Ibanez, Uranga, Marchesano, Shiu, Cvetic, Langacker, Papadimit

Antoniadis, Kiritsis, Tomaras ; Blumenhagen, Görlich, Lüst, ...

PARTITION FUNCTION

4 contributions

$$\mathcal{T} + \mathcal{K} = \frac{1}{2} \mathrm{Tr}_{cl} \, (1+\Omega) \, q^{H} \bar{q}^{\tilde{H}}$$ unoriented closed strings are unaffected

$$\mathcal{A} + \mathcal{M} = \frac{1}{2} \mathrm{tr}_{op} \, (1+\Omega) \, q^{H_{op}} \rightarrow \mathcal{A}(F) + \mathcal{M}(F)$$

Annulus $\quad \tau_A = it/2 \quad$ (up to $\int \frac{dt}{t} \times$)

- neutral: $\quad \mathcal{A}_{00} = \frac{1}{2} N_0^2 \, \frac{V_x}{t^2} \, \Lambda^{KK}_{00}(\tau_A) \, \mathcal{Q}(\tau_A)$

$N=4 \quad \mathcal{Q}(\zeta^I|\tau) = \frac{1}{2} \sum_{\alpha\beta} e^{2\pi i(\alpha+\beta)} \frac{\theta[{}^{\alpha}_{\beta}]}{\eta^3} \prod_{I=1}^{3} \frac{\theta[{}^{\alpha}_{\beta}](\zeta^I)}{\theta_1(\zeta^I)}$

- dipole: $\quad \mathcal{A}_{a\bar{a}} = N_a \bar{N}_a \, \frac{V_x}{t^2} \, \Lambda_{a\bar{a}} \, \mathcal{Q}(\tau_A)$

$\Lambda_{a\bar{a}}$: generalized (KK) momenta $\quad P_L = R_a P_R$

- singly charged

$$\mathcal{A}^{[N=1]}_{0a} = N_0 N_a \, \frac{V_x}{t^2} \, I_{0a} \, \mathcal{Q}(\epsilon^I_{0a}\tau_A|\tau_A) \qquad \prod_I \epsilon \neq 0$$

$$\mathcal{A}^{[N=2]}_{0a} = N_0 N_a \, \frac{V_x}{t^2} \, I^{\perp}_{0a} \, \Lambda^{u}_{0a} \, \mathcal{Q}(\epsilon^{I\neq u}_{0a}\tau_A|\tau_A) \qquad \epsilon^u = 0$$

u = unmagnetized direction(s) $\quad R_a u = u$

$$I_{0a} = C_3(\epsilon^{q_a}_a) \qquad\qquad I^{\perp}_{0a} = C^{\perp}_2(\epsilon^{q_a}_a)$$

• doubly charged strings

$$A_{aa}^{[N=1]} = \frac{1}{2} N_a^2 \; 2^3 I_{oa} \; Q(2\epsilon_{oa}^I \tau_A | \tau_A) \quad (\pi\epsilon \neq 0)$$

$$A_{aa}^{[N=2]} = \frac{1}{2} N_a^2 \; 2^2 I_{oa}^{\perp} \Lambda_{oa} \; Q(2\epsilon_{oa}^I \tau_A | \tau_A) \quad (\epsilon^u = 0$$

Möbius strip projection $\tau_M = \frac{it}{2} + \frac{1}{2} = \tau_A + \frac{1}{2}$

$$M_{oo}^{[N=4]} = -\frac{1}{2} N_o \Lambda_{oo}(\tau_A) \; \hat{Q}(0|\tau_M)$$

$$M_{aa}^{[N=1]} = -\frac{1}{2} N_a \hat{I}_{aa} \; \hat{Q}(\epsilon_{aa}^I \tau_A | \tau_M) \quad (\text{no } u\text{'s})$$

$$M_{aa}^{[N=2]} = -\frac{1}{2} N_a \hat{I}_{aa}^{\perp} \Lambda_{aa}^{\perp}(\tau_A) \; \hat{Q}(\epsilon_{aa}^I \tau_A | \tau_M)$$

$$\hat{I}_{aa} = I_{aa} = 2^3 I_{oa} \, , \quad \hat{I}_{aa}^{\perp} = I_{aa} = 2^2 I_{oa}$$

($B_{NS-NS} = 0$ for simplicity ! No wilson lines)

dy-charged strings $R_a R_b \neq R_b R_a$!

$$A_{ab}^{[N=1]} = I_{ab} N_a N_b \; Q(\epsilon_{ab}^I \tau_A | \tau_A) \quad (\text{no } u\text{'s})$$

$$A_{ab}^{[N=2]} = I_{ab}^{\perp} N_a N_b \Lambda_{ab}^u \; Q(\epsilon_{ab}^{I \neq u} \tau_A | \tau_A)$$

Λ_{ab}^u : generalized momenta $R_a^{q_a} R_b^{q_b} u = u$

$$p_L = R_a^{q_a} p_R \qquad p_L = R_b^{+q_b} p_R \qquad p_L = R_a^{q_a} R_b^{-q_b} p_R$$

[consistent with T-duality]

TADPOLE CANCELLATION

Transverse / closed string / tree level channel

$$\left(\tilde{K}+\hat{A}+\hat{M}\right)_{m=0} = 0$$

Cai, Polchinski, Itoyama
MB Pradisi Sagnotti, ...

RR sector → anomalies   NS-NS sector → instabilit

- RR sector : up to $\int_0^\infty dl$ (no momentum flow)

$$\tilde{K}_{m=0} = \frac{1}{2}\, 2^5 \cdot 8_s$$

$$\tilde{A}_{m=0} = \frac{1}{2}\,\frac{8_s}{2^5} \sum_{\substack{ab \\ q_a q_b}} N_a N_b W_a W_b \prod_I \cos\pi\epsilon^I_{ab}\, \|(1+q_a H_a)(1+q_b \ldots$$

$$\tilde{M}_{m=0} = -2\,\frac{1}{2}\, 8_s \sum_{a, q_a} N_a W_a \prod_I \cancel{\cos\pi\epsilon^I_{a0}}\, \|\cancel{1+q_a H_a}\|$$

* $I_{ab} = V(T^6)\, W_a W_b\, \mathrm{Pfaff}(q_a H_a + q_b H_b) =$

$$= \prod_I \sin(\pi\epsilon^I_{ab})\, \frac{1}{V(T^6)} \sqrt{\det(G_a+\mathcal{F}_a)(G_b+\mathcal{F}_b)}$$

* $$\prod_I 2\cos\pi\epsilon^I_{ab} = \frac{2^3\sqrt{\det(1+q_a q_b H_a H_b)}}{\sqrt{\det(1+q_a H_a)(1+q_b H_b)}} = \mathrm{tr}_s\, \mathcal{U}_a^{q_a} \mathcal{U}_b^{q_b}$$

where

$$\mathcal{U}^q = A\mathrm{Exp}\left(-\frac{q}{2} H_{\hat{i}\hat{j}} \Gamma^{\hat{i}\hat{j}}\right) \Big/ \sqrt{|1+qH|}$$

$$Q^{ij}_a = \frac{1}{8}\, \varepsilon^{ijklmn} H^a_{kl} H^a_{mn}$$

$$0 = \tilde{A}_{m=0} + \tilde{K}_{m=0} + \tilde{M}_{m=0} = \Big(\sum_{a q_a} N_a W_a - 32\Big)^2 + \sum_{ij}\Big(\sum_{a q_a} N_a W_a Q^{ij}_a \ldots$$

NS-NS SECTOR:

$$\tilde{K}_{m\varepsilon o} = \frac{1}{2} 2^5 \cdot 8v$$

$$\tilde{A}_{m\varepsilon o} = \frac{1}{2} \cdot \frac{1}{2^5} \sum_{\substack{ab \\ q_a q_b}} N_a W_a N_b W_b \left[2 + \sum_I 2 \cos(2\pi \epsilon_{ab}^I) \right]$$

$$\cdot \sqrt{\det(1+q_a H_a)} \sqrt{\det(1+q_b H_b)}$$

$$\tilde{M}_{m\varepsilon o} = -2 \cdot \frac{1}{2} \sum_{a q_a} N_a W_a \sqrt{\det(1+q_a H_a)} \left[2 + 2 \sum_I \cos 2\pi \epsilon \right.$$

$*$ $$2 \sum_I \cos(2\pi \epsilon_{ab}^I) = \mathrm{Tr}_v (R_a^{q_a} R_b^{q_b})$$

$$= \mathrm{Tr}_v \left(\frac{1-q_a H_a}{1+q_a H_a} \frac{1-q_b H_b}{1+q_b H_b} \right)$$

$$\tilde{K}_{m\varepsilon o} + \tilde{A}_{m\varepsilon o} + \tilde{M}_{m\varepsilon o} = \left(\sum_{a, q_a} N_a W_a - 32 \right)^2 +$$

$$+ \sum_{ij} \left(\sum_{a q_a} N_a W_a \tau_a^{ij} \right)^2 = 0$$

where $$\tau^a_{\hat{i}\hat{j}} = E^i_{\hat{i}} E^j_{\hat{j}} \frac{\partial V_a}{\partial G^{ij}}$$

$$V_a = \sqrt{\det(G + F_a)}$$

$\tau^a_{ij} = 0 \longrightarrow$ moduli stabilization

(Modulo open/closed string mixing!!)

T-DUALITY

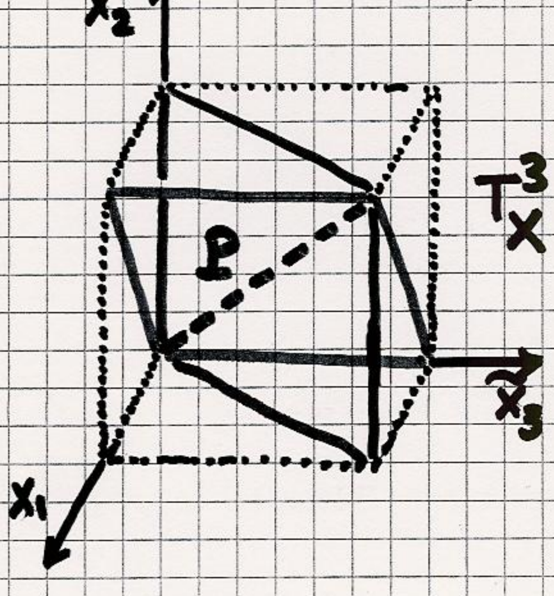
For a given pair of branes with oblique fluxes there is a T-duality mapping the pair into intersecting branes.

T depends on the choice of a and b so a configuration with many stacks cannot be mapped into a configurations of intersecting branes!

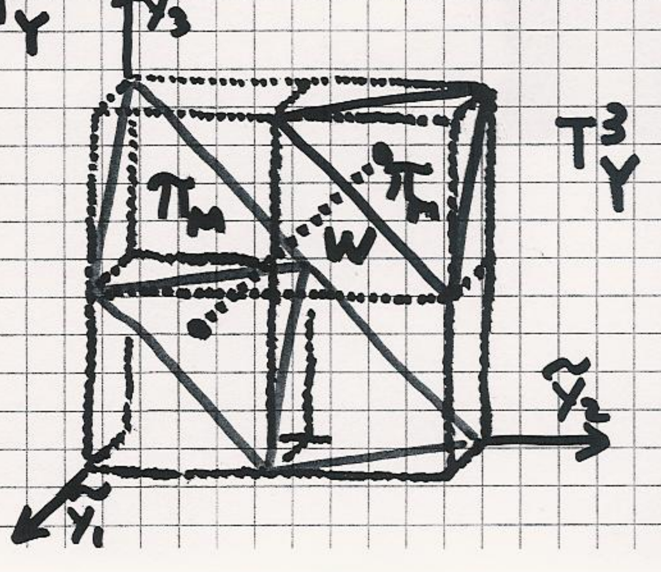
E.g. 4-5 sector of AM model

$F_5 = -dx^1 \wedge dx^3 - dy^1 \wedge dy^3 \quad F_4 = -dx^2 \wedge dx^3 - dy^2 \wedge dy^3$

$T = T_{y^1} T_{y^2} T_{x^3} \longrightarrow D6_4, D6_5$



$T^6 = T^3_X \times T^3_Y$



ONE-LOOP THRESHOLDS

Background field method

Bachas, Fabre, Antoniadis, Dudas, ...

$$F_{\mu\nu} = f\, \delta^2_{[\mu}\delta^3_{\nu]}\, Q \qquad Q \in \mathbb{CP}$$

$Tr(Q)=0 \quad tr(Q^2)=1/2$ (non anomalous!)

$$\frac{4\pi}{g^2_Q} = \frac{4\pi^2}{g^2_Q}\Big|_{tree} + \frac{b_Q}{4\pi}\log\frac{M_s^2}{\mu^2} + \Delta_Q$$

Δ_Q UV_{op} (IR_{cl}) finite

$\mathcal{N}=4$ sectors do not contribute (Kiritsis rev)

$\mathcal{N}=2$ sectors $\to \frac{1}{2}$ BPS States

$\mathcal{N}=1$ sectors also massive oscillator modes!

$$\Delta_Q = \int\frac{dt}{4t}\, B_Q(t)$$

$$B^A_Q\Big|_{\mathcal{N}=1} = \frac{i}{\pi}\sum_{ab} I_{ab}\, Tr_{N_aN_b}(Q_a+Q_b)^2 \sum_I \frac{\vartheta_1'(\epsilon^I_{ab}\tau_A|\tau_A)}{\vartheta_1(\epsilon^I_{ab}\tau_A|\tau_A)}$$

(similarly for B^M_Q)

$$B^A_Q\Big|_{\mathcal{N}=2} = \frac{1}{2}\sum_{ab} I^{\perp}_{ab}\, Tr_{N_aN_b}(Q_a+Q_b)^2\, \Lambda^{u}_{ab}(it)$$

EXPLICIT RESULTS

Relevant integrals (Stieberger, ...)

- $$\int_0^\infty d\ell \sum_{k=1}^{\infty} 2\zeta(2k)\, \epsilon^k \left[E_{2k}(i\ell) - 1 \right] =$$
  $$-\pi \log\left[\frac{\Gamma(1-\epsilon)}{\Gamma(1+\epsilon)}\right] + \cancel{2\pi\epsilon}\, \gamma_E$$
  $=0$ after $\sum_I \epsilon^I = 0$

- $$\int_0^\infty \frac{dt}{t} \sum_{k_1 k_2}{}' \, e^{-\pi t \frac{|k_1 + U k_2|^2}{V_2 U_2}} = \gamma_E - \log\left(4\pi V_2 U_2 |\eta(U)|\right.$$

E.g. $SO(6)_o$ and $SU(2)_7$ of AM model

$$b_{SO(6)_o} = 4\left(2\sum_{b\neq 8} |I_{oa}^{\perp}| N_b + |I_{o8}| N_8\right)\cdot \frac{1}{2} = 60$$

$$b_{SU(2)_7} = \ \ldots \ = +68$$

All β-functions are positive: vectors $\in \mathcal{N}=4$!

$$\Delta^{\mathcal{N}=2}_{SO(6)} = -4\sum_a N_a I_{oa}^{\perp}(\log V_a + \rho)\cdot\frac{1}{2} = -39.88$$

$$\Delta^{\mathcal{N}=1}_{SO(6)} = -4\,|I_{o8}|\, N_8\, \frac{1}{2} \sum_I \log\frac{\Gamma(1-\epsilon_8^I)}{\Gamma(1+\epsilon_8^I)} = 0.07$$

Similarly for $SU(2)_7$