Factorization for charmless hadronic B decays
NLO spectator scattering

Sebastian Jäger (LMU München)
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1. Physics of factorization. Factorization formulas
2. Calculation of the pieces. SCET representation
3. Phenomenological uses and tests of factorization
Exclusive modes in B-physics

Interesting observables, from “easy” to hard

- **leptonic:** $B \to l\nu$  
  $\langle l\nu|Q|B\rangle \propto m_l f_B$  
  need: decay constant $f_B$ (lattice QCD, QCD sum rules)

- **semileptonic and radiative:** $B \to \pi l\nu$, $B \to V\gamma$, $B \to \gamma l\nu$  
  $\langle l\nu\pi|Q|B\rangle \propto f_{B\pi}(q^2)$  
  $B \to \pi l\nu$: measurements at various $q^2$ exist  
  need: form factor $f_{B\pi}(q^2)$ (QCD sum rules, lattice extrapolations)

- **nonleptonic:** $B \to \pi\pi, \pi K, \ldots$  
  $96+$ final states, many measured (rates + CP asymmetries)  
  need: full hadronic matrix elements $\langle M_1 M_2|Q|B\rangle$. 
This talk: two-body hadronic final states

weak phase (CP odd)

\[ A(\bar{B} \to M_1 M_2) = e^{-i\gamma} T_{M_1 M_2} + P_{M_1 M_2} \]

strong amplitudes (CP even)

Branching fractions, CP asymmetries sensitive to |P|, |T|, \(\gamma\) and strong (rescattering) phase \(\text{arg}(P/T)\)

\[ T, P = |\lambda_{\text{CKM}}^{(P,T)}| \sum_i C_i(\mu) \langle M_1 M_2 |Q_i(\mu)| \bar{B} \rangle \]

hadronic matrix element (scale dependent)

matrix elements currently inaccessible on the lattice

“naive factorization”: \(\langle M_1 M_2 |Q| \bar{B}\rangle^{\text{fact}} = f_{M_2} f^{BM}_+ (0)\)

\(\Rightarrow\) wrong (no) scale dependence; no CP asymmetries
Factorization: weak scale

Strong hierarchy $M_W \gg M_B, p_B, p_\pi, \ldots$ implies

$$W \xrightarrow{+} W \ldots = \sum_i C_i \left( \ldots \right)$$

$$\langle f|i \rangle_{\text{SM}} = C_1 \langle f|Q_1|i \rangle_{\text{QCD}} + C_2 \langle f|Q_2|i \rangle_{\text{QCD}} + \cdots$$

- $C(M_W, \ldots; \alpha_s, \ln(\mu^2/M_W^2))$: heavy particles, gluons far off shell. Computed with arbitrary (partonic) external states, expanding in $p/M_W$ (OPE).
- $\langle f|Q_i(\mu)|\bar{B} \rangle$ contain all dynamics below factorization scale $\mu$
- Assume that factorization continues to hold for hadronic states:

$$\mathcal{A}(\bar{B} \rightarrow f) = \underbrace{C_i(\mu)}_{\text{Wilson coefficient}} \underbrace{\langle f|Q_i(\mu)|\bar{B} \rangle}_{\text{hadronic matrix element}}$$
Factorization: general features

Factorization of weak and strong interactions was possible because of two disparate scales in the problem.

\[
\begin{align*}
\text{weak scale} & \quad >> \quad \text{B physics scales, QCD scale} \\
\downarrow & \quad \quad \downarrow \\
\mathcal{H}_{\text{eff}} & \quad \quad \text{soft QCD (partly nonpert.)}
\end{align*}
\]

1. Amplitudes factorize: \( A_{i \to f} = \sum C_i(\mu)\langle Q_i(\mu)\rangle \)

2. Below hard scale, have effective Hamiltonian \( \mathcal{H}_{\text{eff}} = \sum C_i(\mu)Q_i(\mu) \)

3. \( C_i(\mu) \) contain all short-distance physics, independent of states

4. \( \langle Q_i(\mu)\rangle \) contain all long-distance physics, independent of short distance physics (\( W \), top, almost any new physics)

5. Change of factorization scale \( \mu \) by renormalization group evolution
   - resums large logarithms \( \ln m_b/M_W \) through anomalous dimensions of \( Q_i \)
QCD factorization formula


“nonfactorizable”
gluons are hard

no involvement
of spectator

with involvement of spectator

perturbative; small strong phases
form factor, non-perturbative

light-cone distribution amplitudes (LCDA), non-perturbative
\[ A(\bar{B} \rightarrow M_1 M_2) = f_+^{BM_1}(0) f_{M_2} T^I \star \phi_{M_2} \]

\[ + f_B f_{M_1} f_{M_2} T^{II} \star \phi_B \phi_{M_1} \phi_{M_2} \]

"YOU WANT PROOF? I'LL GIVE YOU PROOF!"
Digression: heavy-to-light form factor

The hierarchy $m_B^2 \gg q^2, \Lambda_{QCD}^2$ implies

$$F_i^{B\rightarrow M}(q^2) = C_i(q^2) \xi_{BM}(q^2) + \int \frac{d\omega}{\omega} \int dv \frac{T_i(q^2; \omega, v)}{\phi_B^+(\omega)\phi_M(v)}$$

$$+ \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$$


- $C_i = \mathcal{O}(\alpha_s^0)$ perturbative
- $T_i = \mathcal{O}(\alpha_s)$ perturbative “spectator” scattering kernel
- similar to factorization in other hard QCD processes (e.g. $\pi$ electromagnetic form factor Efremov, Radyushkin; Brodsky, Lepage 1980)
- power corrections generally do not factorize, need new methods
Origin of factorization: heavy quark limit

- \( m_B - m_b \sim \mathcal{O}(\Lambda_{\text{QCD}}) \ll m_b \) and \( m_\pi, m_K, \ldots \sim \mathcal{O}(\Lambda_{\text{QCD}}) \ll m_b \)
- can describe initial and final states by low-virtuality modes
  - heavy quark: \( p_b = m_b (n_+ + n_-)/2 + \mathcal{O}(\Lambda_{\text{QCD}}) \), \( n_\pm^2 = 0 \)
  - spectator: \( p_{qs} = \mathcal{O}(\Lambda_{\text{QCD}}) \)
  - collinear quark: \( p_{qc} = u m_b n_+/2 + \mathcal{O}(\Lambda_{\text{QCD}}) \), \( u = \mathcal{O}(1) \)
- From these, can form invariants of different virtualities
  - \( p_b \cdot p, q \sim m_b^2 \), \( q_s \cdot p_{1,2} \sim \Lambda m_b \equiv \lambda^2 m_b^2 \), \( p^2, q_s^2 \sim \Lambda^2 \)
- Draw all diagrams and expand in \( \lambda = \sqrt{\Lambda/m_b} \)
Effective theory representation: SCET


do not factorize straightforwardly but see (*)

manifest decoupling @ leading power

full QCD $\rightarrow$ SCET\(_I\) $\rightarrow$ SCET\(_{II}\)

all modes

hard-collinear, collinear, soft modes

$\bar{q}\gamma^{\mu}b = C^A \bar{\xi} h_v + C^B * \bar{\xi} \mathbb{D} h_v = C^A \bar{\xi} h_v + C^B * J * \bar{\xi} * \bar{q}_s h_v$

hard scattering kernels become Wilson coefficients
Operators in the effective theory

At leading power, diagrams without/with spectator interaction match only onto

\[ J^A(s) = \bar{\xi}(sn_+)n^{+}_+h_v(0) \]

\[ J^B(s_1, s_2) = \frac{1}{m_b} \bar{\xi}(s_1n_+)n^{+}_+i\slashed{D}_\perp(c)(s_2n_+)h_v(0) \]

collinear quark field HQET heavy quark

where

\[ n^{+}_-\xi = 0 \quad \bar{\psi}h_v = h_v \quad iD_\perp = i\partial_\perp + gA_\perp \]

\[ J^B \] factorizes further in going to SCET\( \Pi \):

\[ T(\mathcal{L}_{SCET_1}J^B) \rightarrow J(s,t)[\bar{\xi}(sn_+)\frac{n^{+}_+}{2}\xi(0)] [\bar{q}_s(tn_-)h_v(0)] \]

The two brackets define twist-2 light-cone distribution amplitudes

\[ \text{F.T.} \left[ \langle \pi | \bar{\xi}(sn_+)\frac{n^{+}_+}{2}\xi(0) | 0 \rangle \right] \propto f_\pi \phi_\pi(u) \quad \text{F.T.} \left[ \langle \bar{B} | \bar{q}_s(tn_-)h_v(0) | 0 \rangle \right] \propto \hat{f}_B \phi_B(\omega) \]
Back to nonleptonic decays

\[ O^I = C_A(s)J^A(s) + C_B(s_1, s_2)J^B(s_1, s_2) \]
\[ O^{II} = \frac{1}{m_b} \left( \bar{\chi}(t_{n-}) \frac{\gamma}{2} \gamma_5 \chi(0) \right) J^B(s_1, s_2) \]

- \( J^A \) and \( J^B \) same SCET currents as in form factor case
- Second meson factorizes off at scale \( O(m_b) \), therefore power corrections are \( O(1/m_b) \), not \( O(1/\sqrt{\Lambda m_b}) \)

With our operator basis, \( O^I \) is proportional to the QCD (not SCET) form factor \( \Rightarrow \) can use theory (sum rules, lattice) or experimental input.
Recovering the BBNS formula

\[ \mathcal{H}_{\text{SCET}_1} = \int du \, T^1(u) O^1(u) + \int du \, dv' \, H^{\text{II}}(u, v') O^{\text{II}}(u, v') \]

gives

\[ \langle \pi \pi | \mathcal{H}_{\text{SCET}_1} | \bar{B} \rangle = T^1 \ast \phi_\pi \, f_+^{B\pi}(0) \quad + \quad H^{\text{II}} \ast \phi_\pi \ast \langle \pi | J_B(v') | \bar{B} \rangle \]

QCD form factor \quad \text{another (bilocal) form factor}

But we have already factorized the $B$-type form factor, thus the second matching step onto SCET$_{\text{II}}$ is simple. It gives the BBNS formula:

\[ \langle \pi \pi | \mathcal{H}_{\text{SCET}_1} | \bar{B} \rangle = T^1(\mu_h) \ast \phi_\pi(\mu_h) \, f_+^{B\pi}(0) \]

\[ + \quad T^{\text{II}} \]

\[ + \quad H^{\text{II}}(\mu_h) \ast U_\parallel(\mu_h, \mu_{hc}) \ast J(\mu_{hc}) \ast \phi_\pi(\mu_h) \ast \phi_\pi(\mu_{hc}) \ast \phi_{B+}(\mu_{hc}) \]
Cautious remarks

• In SCET the factorization formula seems to follow almost trivially.

• However, all the formalism (e.g. mode content) was based on the presumption that nonfactorizable strong interactions are indeed dominated by (hard) gluon exchange.

• This hypothesis deserves to be checked: Wilson coefficients IR finite? Convolutions finite? If not, this signals important soft physics - not clear (to me) how one could treat this in a perturbative framework.

• $1/m_b$ power corrections will, in general, not factorize. (Also, they might receive contributions from “genuine” nonperturbative, nonfactorizable physics.)
Status of higher-order computations

- $T^1$ known to $\mathcal{O}(\alpha_s)$ (1 loop)  
- $J$ known to $\mathcal{O}(\alpha_s^2)$  
- $H^{\Pi}$ now known at one loop  
  Kivel 2006

Motivations for going to NNLO

- reduce residual scale uncertainties
- $J$ alone at NLO gives scheme-dependent result
- check perturbativity of hard-collinear scale
- check factorization at higher orders
- understanding anomalously large $B_d \to \pi^0\pi^0$

There are several operators and diagram topologies (tree, penguin) 
which are separately scheme- and scale-independent.
NLO spectator scattering: current-current

- Compute diagrams analytically in dim reg as function of large momentum components
- Nonfactorizable IR divergences cancel nontrivially among many diagrams [e.g. $1/\epsilon \ln(1 - uv)$], otherwise factorization breakdown
- Proper UV renormalization (including evanescent operators) Buras, Weisz 1990
SCET side of matching

- Partonic QCD amplitude $\frac{1}{i}$ partonic SCET$_I$ amplitude

$$\langle \bar{q}qq|O_1^u|b\rangle_{\text{QCD}} = T^I \ast \langle |\bar{q}q|0\rangle \langle \bar{\chi}X|J_A|b\rangle + H^{II} \ast \langle |\bar{\chi}X|0\rangle \ast \langle qg|J_B(v')|b\rangle$$

- Diagrams factorize by construction of SCET$_I$

- All loops vanish in dim reg, amplitude is given by counterterms

$$Z(u, u'; v, v') = 1 + \frac{\alpha_s}{4\pi} \left( Z_{BL}(u, u') \delta(v - v') + \delta(u - u') Z_{||}(v, v') \right) + \ldots$$

(Brodsky-Lepage kernel and renormalization $Z_{||}$ of $J_B$)

- Already at tree level, evanescent Dirac structures appear, e.g.

$$D_2 = \frac{\eta^2}{2} \gamma_{\perp} \otimes \frac{\eta^2}{2} \gamma_{\mu} \gamma_{\perp} - 2 \frac{\eta^2}{2} \otimes \frac{\eta^2}{2} \gamma_{\mu} \perp$$

Need finite renormalization at one loop to ensure $\langle D_2 \rangle_{\text{phys}}^{(1)} = 0$
Penguin contractions

[Beneke, SJ 2006; see also Li,Yang 2005 (incomplete)]

- QCD side contains no IR divergences
- No SCET subtraction at this order (no tree terms)
- as with current-current diagrams, convolutions converge
- Several topologies contribute only for vector mesons
Phenomenological part
## Implementations of factorization

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<td>perturbative; identical kernels [up to basis]</td>
<td>fit to data (possible for LO hard kernels)</td>
</tr>
<tr>
<td>hardcollinear scale ($\sqrt{m_b\Lambda}$)</td>
<td>perturbative</td>
<td>introduce extra complex parameter (fit to data)</td>
</tr>
<tr>
<td>charm penguin</td>
<td>no special treatment (generally) small perturbative phase</td>
<td>introduce extra complex parameter (fit to data)</td>
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<td>QCD form factor, B &amp; light meson LCDA</td>
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<td>power corrections</td>
<td>calculate or model potentially large ones</td>
<td>largely omitted</td>
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“Topological” amplitudes

Define scheme and scale independent parameters

Physical amplitudes take simple forms, e.g.

\[ A_{B^0 \to \pi^+ K^-} = \sum_{p=u,c} \lambda_p A_{\pi K} \left[ \delta_{pu} \alpha_1 + \alpha_4^p + \alpha_{4,EW}^p + \beta_3^p - \frac{1}{2} \beta_{3,EW}^p \right] \]

CKM normalization (essentially form factor)

power correction (penguin annihilation), modeled

“tree” \( \alpha_1, \alpha_2 \sim T, C \)

(electroweak) penguins \( (\alpha_{3EW}, \alpha_{4EW}), \alpha_3, \alpha_4 \)
Topological tree amplitudes

With choice “G” obtain a better fit to $B \to \pi\pi\pi$ data. No “puzzle”

\[
\begin{align*}
10^6 \text{Br}(B^- \to \pi^-\pi^0) &= 5.5^{+0.3}_{-0.3}(\text{CKM})^{+0.5}_{-0.4}(\text{hadr.})^{+0.9}_{-0.8}(\text{pow.}) & \text{exp: } 5.7 \pm 0.4 \\
10^6 \text{Br}(\bar{B}^0 \to \pi^+\pi^-) &= 5.0^{+0.8}_{-0.9}(\text{CKM})^{+0.3}_{-0.5}(\text{hadr.})^{+1.0}_{-0.5}(\text{pow.}) & \text{exp: } 5.2 \pm 0.2 \\
10^6 \text{Br}(\bar{B}^0 \to \pi^0\pi^0) &= 0.73^{+0.27}_{-0.24}(\text{CKM})^{+0.52}_{-0.21}(\text{hadr.})^{+0.35}_{-0.25}(\text{pow.}) & \text{exp: } 1.31 \pm 0.21
\end{align*}
\]

[Scenario “G” also describes BR($B \to \pi K$) rather well]
Topological penguin amplitudes

![Diagrams showing small and significant corrections to QCD and EW amplitudes]

- Small correction to QCD penguin amplitude
- Significant correction to color-suppressed EW penguin amplitude

\( a_4, a_{10} \) are the leading-power parts of \( \alpha_4, \alpha_{4,\text{EW}} \)

The small correction to \( a_4 \) is due to a cancellation. Accidental?

\[
[a_4^u(\pi\pi)]_{HP} = [0.0074 + 0.0060i]_{C_AC_1} - [0.0073 + 0.0053i]_{C_FC_1} - 0.0002 + 0.0001i,
\]
Penguin-to-tree ratios

\[ \frac{P_{M_1 M_2}}{(C_{\pi \pi} + T_{\pi \pi})} \sim \hat{\alpha}_4^c(M_1 M_2)/(\alpha_1(\pi \pi) + \alpha_2(\pi \pi)) \]  
can be determined (modulus and phase) from available data.

\[ P_{M_1 M_2} \sim \hat{\alpha}_4^c(M_1 M_2) = a_4(M_1 M_2) \pm r_{\chi}^{M_2} a_6(M_1 M_2) + \beta_3^p(M_1 M_2) \]

leading power  
factorizable power correction  
annihilation (modeled)

predicted large for pseudoscalar \( M_2 \)  
predicted small for vector \( M_2 \)

A highly nontrivial check of the factorization framework
Conclusions & outlook

- NLO spectator scattering for nonleptonic B decays complete at leading power
- Perturbation series well behaved at scales $\mu = m_b$ and $\mu = \sqrt{m_b \Lambda}$
- Large effects in color-suppressed tree & penguin amplitudes. Small effects in color-allowed tree and QCD penguin amplitude
- Data on branching ratios well described, CP asymmetries for PP final states appear to require some power corrections to the (imaginary) part of penguin amplitudes
- motivates NLO computation of “scalar penguin” $a_6$