Blatt 9

1) $\mathcal{L} = \bar{Q}_v (i v\cdot D) Q_v - \frac{1}{2m_Q} \bar{Q}_v D_\perp^2 Q_v$ ← Kinetic op.

$- c_F(\mu) \frac{g}{4m_Q} \bar{Q}_v \sigma^{\mu\nu} G_{\mu\nu} Q_v$ ← magnetic mom. op.

$\left(+ \sum_q \bar{q}\, i \not{D} q - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a \right)$

$$D_\mu = \partial_\mu + i g (\tilde{\mu}^\epsilon) A^a_\mu T^a$$

$\frac{1}{m_Q}$ - Feynman-Regeln:

$$D_\perp^2 = (D - (D\cdot v) v)^2 = D^2 - 2(D\cdot v)^2 + (D\cdot v)^2 =$$

$$= D^2 - \cancel{(D\cdot v)^2} \quad (\text{EOM: } (v\cdot D) Q_v = 0)$$

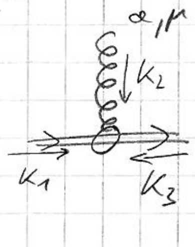
$$\Rightarrow \bar{Q}_v D^2 Q_v = \overset{1}{\bar{Q}_v} \left[\partial^2 + i g \{ \overset{2}{A}_\mu, \partial^\mu \} - g^2 A^2 \right] \overset{3}{Q_v}$$

$$\rightarrow \bar{Q}_v \left(-K_3^2 + g A_\mu (K_3^\mu - K_1^\mu) - g^2 A^2 \right) Q_v$$

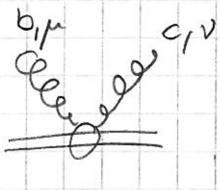
$$\bar{Q}_v (\sigma^{\mu\nu} G_{\mu\nu}) Q_v = \overset{1}{\bar{Q}}_v \sigma^{\mu\nu} \left((\partial_\mu \overset{2}{A^a_\nu}) - (\partial_\nu A^a_\mu) - g f^{abc} A^b_\mu A^c_\nu \right) T^a \overset{3}{Q_v}$$

$$\rightarrow \bar{Q}_v \sigma^{\mu\nu} \underbrace{\left(-i K_{2\mu} A^a_\nu + i K_{2\nu} A^a_\mu \right.}_{2 i \sigma^{\mu\nu} K_{2\nu} A^a_\mu} \left. - g f^{abc} A^b_\mu A^c_\nu \right) T^a Q_v$$

$\Longrightarrow$



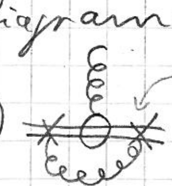
$$: \; i \frac{g}{2m_Q} (K_3^\mu - K_1^\mu) T^a + \frac{c_F(\mu)}{2m_Q} g\, \sigma^{\mu\nu} K_{2\nu} T^a$$

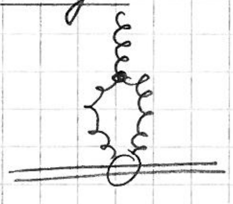


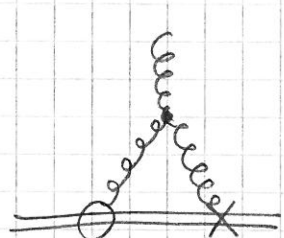
$$: \; - i \frac{g^2}{m_Q} \underset{\uparrow\,(T^a)^2}{C_F} \delta^{bc} g^{\mu\nu} + i \frac{c_F(\mu)}{2m_Q} g^2 f^{abc} T^a \sigma^{\mu\nu}$$

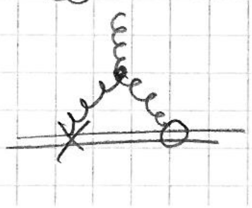
1-loop Renormierung:

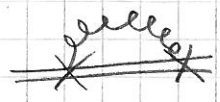
Diagramme:

1) 

2) 

3) 

4) 

5)  (Feldrenormierung)

magnetic mom. op. ist Spin-abhängig ($\sigma^{\mu\nu}$),
Kinetic $\left(\frac{1}{m_Q}\right)$ op. <u>nicht</u> $\Rightarrow$ Können <u>nicht</u> mischen!

Color-Struktur:

1) $T^a T^b T^a = T^a\left(T^a T^b - [T^a, T^b]\right) =$
$= C_F T^b - i f^{abc} T^a T^c =$
$= C_F T^b + \frac{1}{2} f^{abc} f^{acd} T^d =$
$= \underline{\underline{C_F T^b}} - \frac{1}{2} C_A T^b$

[mit $T^a T^a = C_F \mathbb{1}$; $f^{abc} f^{abd} = C_A \delta^{cd}$;
$T^a T^b = \frac{1}{2}\left(\frac{1}{3}\delta^{ab} + d^{abc} T^c + i f^{abc} T^c\right)$]

2) $f^{abc} T^c f^{abd} = C_A T^d$

3) $f^{abd} T^a T^b = f^{abd} \frac{i}{2} f^{abc} T^c = \frac{i}{2} C_A T^d$

4) $\quad$ "

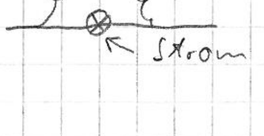
5) $T^a T^a = \underline{C_F \mathbb{1}}$

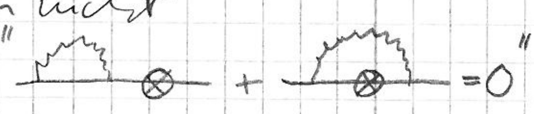
Background field gauge: $Z_g = Z_A^{-1/2}$
$\Rightarrow$ gA^μ <u>nicht</u> renormiert!

Renormierung des mag. mom. op:
Kein Mischen mit Kin. op $\Rightarrow$ Benutze "$\sigma^{\mu\nu}$"-Terme der $\frac{1}{m_Q}$-Feynman Regeln!

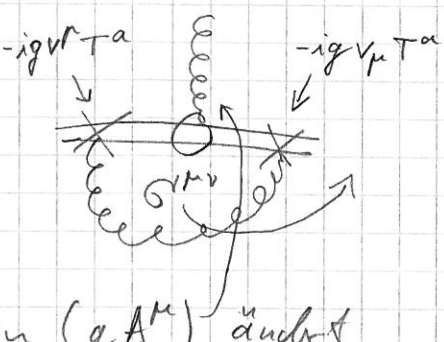
<u>Behauptung</u>: anomale Dimension von $C_F(\mu) \propto C_A$
$\Leftrightarrow$ C_F-Terme heben sich auf.

<u>Beweis</u>:

Strom-Renormierung in QED/QCD
(VLS $\rightarrow$ "Current renormalization" in "Composite Operators & Operator Mixing"): 

→ Dort: (Axial-)Vektor-Strom nicht renormiert (1-loop): "  "

Hier: • Dirac-Struktur ($\sigma^{\mu\nu}$) ändert nichts am (QED-) Argument:



• Auslaufendes Gluon (gA^μ) ändert nichts, da gA^μ nicht renormiert wird (d.h. keinen Counterterm erhält)

• Color-Struktur (T^b) für C_F-Term gleich:

C_F-Term von $(T^a T^a) T^b$ = C_F-Term von $T^a T^b T^a$
(WFN-renormalization)

⇒ C_F-Terme tragen nicht zur Renormierung des mag. mom. op.'s bei.

⇒ $c_F(\mu) \propto C_A$ (Beh.)

Blatt 9

2) hep-ph/9708306 (Bauer/Manohar)

a) Eq. (13): $\frac{dc_F}{d\ln\mu^2} = C_A c_F \frac{\alpha_s}{4\pi}$

$$\Rightarrow \frac{dc_F}{c_F} = C_A \frac{\alpha_s}{4\pi}\left(\frac{d\alpha_s}{\alpha_s^2}\frac{4\pi}{\beta_0}\right) = \frac{C_A}{\beta_0}\frac{d\alpha_s}{\alpha_s}$$

$$\Rightarrow c_F(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_s(m)}\right]^{-\frac{C_A}{\beta_0}} \quad ; c_F(m) = 1$$

b) $O_D = \frac{g}{8m^2}\,\bar{h}\left(D_\mu G^{\mu\nu}\right) v_\nu h \quad ; \quad h \hat{=} Q_v$

$$O_1^{hl} = \frac{g^2}{8m^2}\sum_i \bar{h}\,T^A h\;\bar{q}_i \not{v}\, T^A q_i =$$

$$= \frac{g^2}{8m^2}\,\bar{h}\,T^A h\, v_\nu \underbrace{\sum_i \bar{q}_i \gamma^\nu T^A q_i}_{} =$$

EOM: $\frac{1}{g}\left(D_\mu G^{\mu\nu}\right)^A - v^\nu \bar{h} T^A h$

$$= \underbrace{\frac{g}{8m^2}\,\bar{h}\left(D_\mu G^{\mu\nu}\right) v_\nu h}_{O_D} + \underbrace{\frac{g^2}{8m^2}\left(\bar{h}T^A h\right)^2}_{\text{Im single heavy-quark-Sector weglassen!}}$$

Anomale Dimension:

$$\mu\frac{d}{d\mu}\begin{pmatrix} O_1 \\ \vdots \\ O_n \end{pmatrix} = \Gamma \begin{pmatrix} O_1 \\ \vdots \\ O_n \end{pmatrix}$$

$O_1 = O_D \quad ; \quad O_6 = O_1^{hl}$

Eliminiere $O_1^{hl} \Rightarrow O_1 = O_6$

Addiere 1. und 6. Spalte von Γ und lasse 6. Zeile & Spalte weg $\Rightarrow \Gamma'$ im Paper Eq. (31)

O_{Km}, O_{KL}, O_{mm} sind zeitgeordnete Produkte
$\hat{=}$ alle mgl. Einsetzungen der O_K, O_m in Diagramme! $\Rightarrow c_{Km} = c_K(-c_F)$

d) $\mu \frac{dc_S}{d\mu} \overset{\text{Eq.(15); } c_K \equiv 1}{=} \frac{g^2}{16\pi^2} 4 C_A c_F(\mu) =$

$$= \frac{g^2}{4\pi^2} C_A \underbrace{\left(\frac{\alpha_S(\mu)}{\alpha_S(m)}\right)^{-\frac{C_A}{\beta_0}}}_{z^{-C_A}} \quad ; \quad c_S(m) = 1$$

$$\Rightarrow c_S(\mu) \overset{(*)}{=} 2 z^{-C_A} - 1 = 2 c_F(\mu) - 1 \quad (\text{R.P.J})$$

$$\mu \frac{dc_D}{d\mu} = \frac{g^2}{16\pi^2}\left(\frac{13}{3} C_A c_D - \left(\frac{20}{3} C_A + \frac{32}{3} C_F\right) - \frac{1}{3} C_A c_F^2\right)$$

$c_D(m) = 1$

$$\Rightarrow c_D(\mu) \overset{(**)}{=} z^{-2C_A} + \left(\frac{20}{13} + \frac{32}{13}\frac{C_F}{C_A}\right)\left(1 - z^{-\frac{13}{6}C_A}\right)$$

$$(*) \quad d\ln\mu = \frac{d\alpha}{-\frac{\beta_0}{2\pi}\alpha^2} = -\frac{2\pi}{\alpha}\frac{dz}{z}$$

$$dc_S = -2 C_A z^{-C_A-1} dz$$

$(**)$ $\quad A = \frac{13}{3} C_A \;;\; B = \left(\frac{20}{3} C_A + \frac{32}{3} C_F\right) \;;\; C = \frac{C_A}{3}$

$$\frac{d c_D}{d x} = \frac{\alpha}{4\pi}\left(A c_D - B - C z^{-2C_A}\right) \qquad ; x = \ln\mu$$

$$\underbrace{c_D' - \alpha\frac{A}{4\pi} c_D}_{f(x)} = \underbrace{-\frac{\alpha}{4\pi}\left(B + C z^{-2C_A}\right)}_{g(x)}$$

lin. diff. eq. of 1. ord.

$$\Rightarrow c_D = e^{-a(x)}\left(\int g(x) e^{a(x)} dx + \text{const.}\right)$$

$$a(x) = \int f(x)\, dx = -\frac{A}{4\pi}\int \alpha \, d\ln\mu = \frac{A}{2\beta_0}\int \frac{d\alpha}{\alpha}$$

$$= \frac{A}{2\beta_0} \ln\frac{\alpha(\mu)}{\alpha(m)} \qquad \Rightarrow e^{a(x)} = z^{\frac{A}{2}}$$

$$\int g(x) e^{a(x)} dx = \frac{1}{2}\int \left(B + C z^{-2C_A}\right) z^{\frac{A}{2}} \frac{dz}{z}$$

$$\overset{A=\frac{13}{3}C_A}{=} \frac{1}{A}\left[13\, C\left(z^{C_A/6} - 1\right) + B\left(z^{\frac{13 C_A}{6}} - 1\right)\right]$$

$$c_D = z^{-\frac{A}{2}} \frac{1}{A}\left[13\, C\left(z^{\frac{C_A}{6}} - 1\right) + B\left(z^{\frac{13 C_A}{6}} - 1\right)\right] + z^{-\frac{A}{2}} \text{const.}$$

$$c_D(z=1) \stackrel{!}{=} 1 \quad \Rightarrow \underline{\text{const} = 1}$$

$$\Rightarrow c_D = z^{-2C_A} + \left(\frac{20}{13} + \frac{32}{13}\frac{C_F}{C_A}\right)\left(1 - z^{-\frac{13}{6}C_A}\right) \quad \checkmark$$