01/26/05 ①

$\boxed{\text{Heavy Quark Effective Theory}}$ (HQET) (general: $\bar{Q}q$ $\quad Q = b, c$)

Consider: B-mesons $\rightarrow$ bound state made primarily of $(\bar{b}q)$

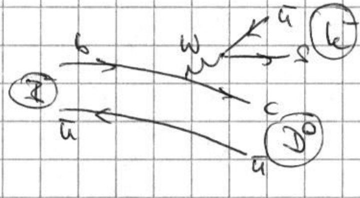
q: light quarks (u, d, s)

D-mesons $(\bar{c}q)$

* decays of B mesons mediated by weak interactions
  - $\Rightarrow$ most important instrument to measure CKM elements V_{ub}, V_{cb}, CP phase
  - $\rightarrow$ rare decays could be sensitive for new physics

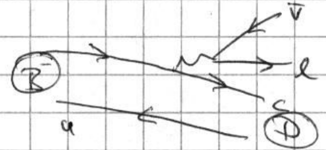
e.g.

$B^- \rightarrow D^0 K^-$



$\sim |V_{cb} V_{us}^*|^2$

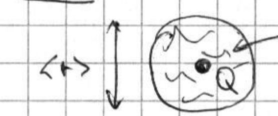
$B^- \rightarrow D l \bar{\nu}$



$\sim |V_{cb}|^2$

* Scales involved in the problem

Λ_{QCD}: bound state dynamics (non perturbative)



$\langle r^{-1} \rangle \sim \Lambda_{QCD} \ll m_Q$

$\boxed{M_W \gg m_b \gg \Lambda_{QCD}}$

Q acts primarily like a static color source

$M_B \approx m_b$: energy transfer in decays (perturbative)

M_W: scale where weak decay process arises (perturbative)

* sequence of EFT's:

$\boxed{SM}$ $\quad \mu \gtrsim M_W$

$\downarrow$

$\boxed{\mathcal{L}_{QCD}^{(u,d,s,c,b)} + \mathcal{L}_{weak}^{eff} \text{ ("Fermi theory")}}$ $\quad \mu \sim M_W$ "full QCD"

$\downarrow$

EFT for mesons made from one heavy quark Q + light dof's.

$\boxed{HQET}$ $\quad \mu \sim m_b$

EFT for heavy quarks $(m_Q \rightarrow \infty)$ where the heavy quarks are not fully integrated out (low E fluctuations of Q remain in theory) (similar to proton for the H atom in nonrel. QM)

$\downarrow$

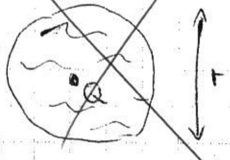
bound state dynamics $\quad \mu \sim 1\,\text{GeV}$

6th lecture

Heavy Quark Effective Theory

→ An EFT for heavy quarks ($m_Q \to \infty$) that are in fact not removed from the theory

consider: B-mesons : → bound state made primarily of $\bar{b} q$, q light quark
$q = u, d, s, (c)$



→ $\langle r^{-1} \rangle \sim \Lambda_{QCD} \ll m_Q$
bottom quark acts like a static color source
→ main dynamics made by light degrees of freedom in the heavy quark itself and by the light quarks and gluons

→ decays of the B mesons mediated by the weak interactions, experimental analyses of B decays can lead to measurements of CKM matrix elements

→ We have already talked about how the effects from scales above $\mu = m_b$ can be treated by switching ~~from~~ the SM to an EFT where M_W, M_Z, m_H, t are integrated out.

Now: we need to go further to $\mu < m_b$
There is:
- physics involving scales $O(m_b)$ → hard effects, static source, perturb.
- physics involving scales $O(\Lambda_{QCD})$ → soft effects, non-perturbative

→ both types of effects need to be separated!

Idea: EFT (one more!) → hard effects ⟹ Wilson coefficients of operators
soft effects ⟹ matrix elements of operators

- Degrees of freedom → ? heavy quark (somehow)
gluons → gauge invariance
- Power counting → expansion in $\frac{\Lambda_{QCD}}{m_Q}$ ✓
- Symmetries →

Somehow we have to take the limit: $m_Q \to \infty$ in

$$\mathcal{L}_{QCD} = \bar{Q}(i\not{D} - m_Q)Q + \text{light stuff} + \text{h.c.}$$

Note: Today all these issues are believed to be entirely understood!

→ Heavy Quark Effective Theory is established

→ I will tell you the basic issues and a few applications only
There are tons of important phenomenology I cannot mention at all!
e.g. Ligeti hep-ph/0302031 (CP!)
BaBar Physics Book → look at google!
CKM Workshop Yellow Report hep-ph/0304132

→ let's try to get some intuition on what's going on!

heavy quark Q: $\boxed{p^\mu = m_Q v^\mu + k^\mu}$; $m_Q \to \infty$

p^μ is only known up to $O(\Lambda_{QCD})$

$v^\mu = (1,0,0,0)$ in rest frame (meson velocity)

$k^\mu \sim O(\Lambda_{QCD})$

particle spinor: $u(p) = \sqrt{\frac{E+m}{2E}} \begin{pmatrix} \varphi \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \varphi \end{pmatrix}$

φ: Pauli spinor, $E = \sqrt{m^2+p^2}$

$\vec{p} \sim \vec{k}$ ($\vec{v} = (1,0,0,0)$)
→ lower component much smaller than upper one!

→ How does this help to get the EFT Lagrangian?

$$\text{propagator: } i\frac{\not{p}+m_Q}{p^2-m_Q^2+i\epsilon} = i\frac{m_Q\not{v}+m_Q+\not{k}}{2m_Q v\cdot k + k^2 + i\epsilon} = i\underbrace{\left(\frac{1+\not{v}}{2}\right)}_{\text{rest frame } \frac{1}{2}(1+\gamma_0) = \begin{pmatrix} I & 0 \\ 0 & 0\end{pmatrix}}\frac{1}{v\cdot k+i\epsilon} + O\left(\frac{1}{m_Q}\right)$$

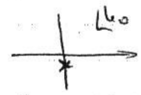
higher order

Fourier transform (in the rest frame)

mass suppression!

$$\int\frac{d^4k}{(2\pi)^4} e^{-ikx}\frac{i}{k_0+i\epsilon} = \delta^{(3)}(\vec{x})\int\frac{dk^0}{(2\pi)}\frac{i}{k_0+i\epsilon}e^{-ik_0 t}$$

$$= \Theta(t)\,\delta^{(3)}(\vec{x})$$



$t>0$: close in negative imaginary k_0 plane

$t<0$: in upper

$\boxed{\text{Static source, only travels in positive time direction}}$

no antiparticle propagation! ($\hat{=}$ particle propagation backwards in time)

→ Interaction with gluons: → propagator on each side:

$$= -ig_s\gamma^\mu T^a_{ij}$$

$$= -ig_s v^\mu T^a$$

$$\left(\frac{1+\not{v}}{2}\right)\gamma^\mu\left(\frac{1+\not{v}}{2}\right) = \cancel{\left(\frac{1+\not{v}}{2}\right)\left(\frac{1-\not{v}}{2}\right)\gamma^\mu} + \left(\frac{1+\not{v}}{2}\right)v^\mu$$

$$= \left(\frac{1+\not{v}}{2}\right)v^\mu\left(\frac{1+\not{v}}{2}\right)$$

$$\not{v}\not{v} = v^2 = 1 \; ; \quad \left(\frac{1+\not{v}}{2}\right)^2 = \left(\frac{1+\not{v}}{2}\right)$$

→ leading order Lagrangian: $\mathcal{L}_{HQET} = \bar{Q}_v\, i v\cdot D\, Q_v$ + light stuff

with $\left(\frac{1+\not{v}}{2}\right)Q_v = Q_v$

i.e. $\mathcal{L}_{HQET} \approx \bar{Q}_v\, iv\cdot D\, Q_v + \sum_{\text{light } q} \bar{q}(i\not{D}-m_q)q - \frac{1}{4}G_A^{\mu\nu}G^A_{\mu\nu}$

+ higher dim. terms $\left(O\left(\frac{1}{m_Q}, \frac{1}{m_Q^2}, \ldots\right)\right)$

6th

→ A direct derivation of $\mathcal{L}_{HQET}^{tree}$

take out large phase

Start from $\mathcal{L}_{QCD}$: $Q(x) = \sum_v e^{-i m_Q v \cdot x}\left[Q_v(x) + H_v(x)\right] \quad (v^2 = 1)$

$$\left(\frac{1+\slashed{v}}{2}\right) Q_v = Q_v$$
$$\left(\frac{1-\slashed{v}}{2}\right) H_v = H_v$$
$$\Rightarrow \slashed{v} Q_v = Q_v \quad ; \quad \slashed{v} H_v = -H_v$$

insert into $\mathcal{L}_{QCD} = \bar{Q}(x)\,(i\slashed{D} - m_Q)\,Q(x)$

massive fluctuation

(we only look a single v sector) (v.v' terms zero)

$$= \bar{Q}_v\, iv\cdot D\, Q_v - \bar{H}_v\,(iv\cdot D + 2m_Q)\,H_v + \underbrace{\bar{Q}_v\, i\slashed{D}\, H_v + \bar{H}_v\, i\slashed{D}\, Q_v}_{\bar{Q}_v\, i\slashed{D}_\perp H_v + \bar{H}_v\, i\slashed{D}_\perp Q_v}$$

$$X_\perp^\mu = X^\mu - X\cdot v\, v^\mu \quad , \quad v = (1,\vec{0}) \Rightarrow X_\perp^\mu = (0, \vec{x})$$

→ At tree level we can integrate out the massive fluctuation by using the equation of motion (Foldy - Wouthuysen)

$$(iv\cdot D + 2m_Q)\,H_v = i\slashed{D}_\perp Q_v \quad \Rightarrow \quad H_v = \frac{1}{iv\cdot D + 2m_Q}\, i\slashed{D}_\perp Q_v$$

$$\mathcal{L} = \bar{Q}_v\left(iv\cdot D + i\slashed{D}_\perp \frac{1}{iv\cdot D + 2m_Q}\, i\slashed{D}_\perp\right) Q_v$$

$$= \bar{Q}_v\left[iv\cdot D - \frac{1}{2m_Q}\slashed{D}_\perp\slashed{D}_\perp + O\!\left(\tfrac{1}{m^2}\right)\right] Q_v = \mathcal{L}_0 + \underset{O(\frac{1}{m})}{\mathcal{L}_1} + O\!\left(\tfrac{1}{m^2}\right)$$

$$\slashed{D}_\perp\slashed{D}_\perp = \gamma_\mu\gamma_\nu D_\perp^\mu D_\perp^\nu = \tfrac{1}{2}\{\gamma_\mu,\gamma_\nu\} D_\perp^\mu D_\perp^\nu + \tfrac{1}{2}[\gamma_\mu,\gamma_\nu] D_\perp^\mu D_\perp^\nu$$
$$= D_\perp^2 + \tfrac{1}{4}[\gamma_\mu,\gamma_\nu]\,[D_\perp^\mu, D_\perp^\nu]$$
$$= D_\perp^2 + \tfrac{1}{2} g\, \sigma_{\mu\nu} G^{\mu\nu}$$

$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$

$[D^\mu, D^\nu] = ig\, G^{\mu\nu}$

$\bar{Q}_v [\gamma_\mu,\gamma_\nu] v^\mu Q_v = 0$

$$\Rightarrow \boxed{\begin{aligned}\mathcal{L}_0 &= \sum_{Qv} \bar{Q}_v\, iv\cdot D\, Q_v\\ \mathcal{L}_1 &= \sum_{Qv} -\bar{Q}_v \frac{D_\perp^2}{2m_Q} Q_v - g\,\bar{Q}_v \frac{\sigma_{\mu\nu}G^{\mu\nu}}{4m_Q} Q_v\end{aligned}}$$

→ several heavy quarks $\mathcal{L}_0 = \sum_Q \bar{Q}_v\, iv\cdot D\, Q_v\,, \ldots$

after renormalization + higher order matching $\left\{ \boxed{\begin{aligned}&C_k(\mu)\\ &C_k(m_Q) = 1 + O(\alpha_s)\end{aligned}} \right.$

"kinetic energy of heavy quark"

"magnetic moment operator"

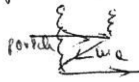
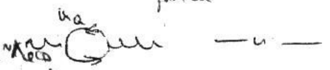
$\boxed{C_F(\mu)\,,\ C_F(m_Q) = 1 + O(\alpha_s)}$

Comments:

- Our manipulations are only true at tree level, but the resulting operators are the full set of operators even beyond tree level

$\boxed{\text{we already have all tree level matching conditions!}}$

} all operators have Wilson-coefficients!

- Off-shell antiparticle effects are integrated out!  does not exist
- fluons with $q \sim m_Q$ are also integrated out! 

} effects contained in short distance coefficients of operators

Q_v cannot propagate backwards in time!

Comment:

~~The choice of v^μ is arbitrary, any choice with $v^2=1$ does the job.~~
~~⇒ The more general formulation of the EFT has the form~~

$$\cancel{\mathcal{L}_0 = \sum_{Q,v} \bar{Q}_v \, i v\cdot D \, Q_v}$$

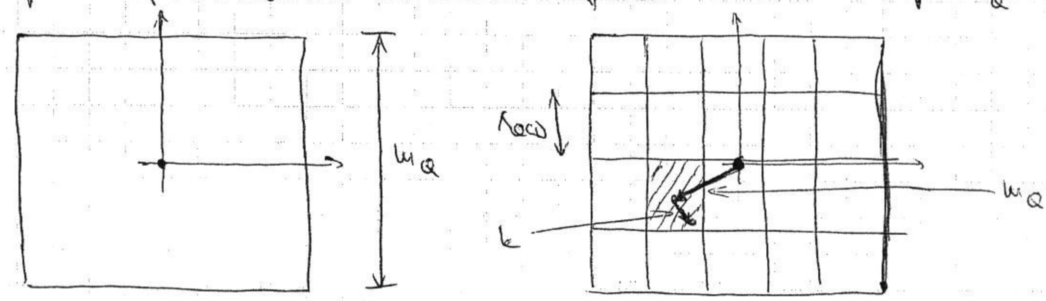
$$\cancel{\mathcal{L}_1 = \sum_{Q,v} -\bar{Q}_v \frac{D_\perp^2}{2m_Q} Q_v - g\bar{Q}_v \frac{\sigma_{\mu\nu}G^{\mu\nu}}{4m_Q} Q_v}$$

~~$\sum_v$: sum over all possible choices of v~~

Physical idea:

available momentum space in full QCD

available momentum space in HQET $\quad p = m_Q v + k$



Later we will see that there can be interactions generated by the weak interactions that generate a transition between heavy quark with different v's.

$Q_v(x)$
"label"
└ describes fluctuations only in the 4-dim momentum space box with side length $1/\Lambda_{QCD}$

The same is true for the gluon field $A^\mu(x)$, but we do not have to introduce a label for it because it looks the same in each box.

$\boxed{\text{Powercounting:}}$ $\quad S \sim \exp\Big[\underbrace{\int d^4x}_{(\Lambda_{QCD}^{-1})^4} \mathcal{L}\Big] \sim \Lambda_{QCD}^0$

$$\partial^\mu Q_v(x) \sim k^\mu Q_v(x) \sim \Lambda_{QCD} Q_v$$

$$\partial A^\mu(x) \sim \Lambda_{QCD} A^\mu$$

Scaling of fields: kinetic energy terms must scale as Λ_{QCD}^4 to have $S\sim\Lambda_{QCD}^0$

$$\Rightarrow \quad Q_v \sim \Lambda_{QCD}^{3/2}, \qquad A^\mu \sim \Lambda_{QCD}$$

} power counting is in powers of $\frac{1}{m_Q}$! (manifest powercounting)

Symmetries in HQET

→ gauge symmetry ✓ obvious, but: $Q_v^{(x)} \to U(x)\, Q_v(x)$
$\partial^\mu U(x) \sim \Lambda_{QCD}\, U(x)$
⇒ the phases of the gauge transformations fluctuate at length scales $x \sim 1/\Lambda_{QCD}$ only!
⇒ the gluons do not have fluctuations at energies $\sim m_Q$ anymore!

→ Lorentz invariance: rotations ✓
boosts → no! broken by fixed choice of v
I'll come back to this later!

→ P, T invariance ✓ (C, too if quark and antiquark are included)

Heavy Quark Symmetry $\mathcal{L}_{HQET} = \mathcal{L}_0 + \mathcal{L}_1 + \ldots$

$\mathcal{L}_0 = \sum_{Q,v} \bar{Q}_v\, iv\cdot D\, Q_v$ independent of m_Q & spin

$\mathcal{L}_1 = \sum_{Q,v}\left(-c_k \bar{Q}_v \frac{D_\perp^2}{2m_Q} Q_v - g c_F \bar{Q} \frac{\sigma_{\mu\nu}G^{\mu\nu}}{4m_Q} Q_v\right)$

~~flavor~~ spin, ~~flavor~~ Effect suppressed by $\frac{\Lambda}{m_Q} \ll 1$

let us just look at $\mathcal{L}_0$ for now, which describes the dominant physics for $m_Q \to \infty$ ("heavy quark limit")

- heavy quark flavor symmetry: dynamics unchanged by exchanging the quark flavors
  → $\mathcal{L}_0$ independent of m_Q
  } $U(N_h)$ ← # of heavy quarks

- heavy quark spin symmetry → $\mathcal{L}_0$ spin independent of spin of the heavy quark } $SU(2)$

Together: $U(2N_h)$

Spectroscopy for mesons ($Q\bar{q}$ or $\bar{Q}q$) (spin symmetry)

→ one can use the symmetries to classify the meson states into multiplets

"$Q\bar{q}$": (Q has quantum # of Q)
$\bar{q}$: not really just "$\bar{q}$": $\bar{q}$, any # of $q\bar{q}$ pairs, gluons } "light dof's"
but has # of a $\bar{q}$ (quantum)

total angular momentum: $\vec{J}$ conserved (trivial) $\vec{J}^2 = j(j+1)$

heavy quark spin $\vec{S}_Q$ conserved $\vec{S}_Q^2 = s_Q(s_Q+1)$

⇒ spin of all light dof's $\vec{S}_\ell = \vec{J} - \vec{S}_Q$ conserved $\vec{S}_\ell = s_\ell(s_\ell+1)$ → incl. angular momentum between Q and $\bar{q}$
(including relative angular momentum $\vec{L}$)

For each s_ℓ one has a symmetry doublet which is degenerate in the limit $m_Q \to \infty$ $j_\pm = s_\ell \pm \frac{1}{2}$

(light dof's are not aware of heavy quark ~~Q~~-spin at LO because $\mathcal{L}_0$ does not depend of heavy quark spin)

Parity P: $q, \bar{q}$ have opposite intrinsic parity

← parity

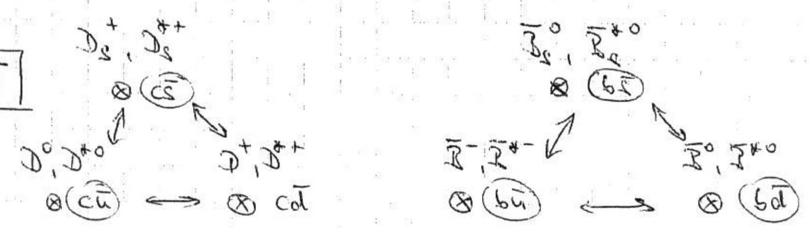
	s_ℓ^P	J^P	Mesons	
$L=0$	$\frac{1}{2}^-$	0^-	B, D	pseudoscalar
		1^-	B^*, D^*	vector
$L=1$	$\frac{1}{2}^+$	0^+	B_0^*, D_0^*	
		1^+	B_1^*, D_1^*	
	$\frac{3}{2}^+$	1^+	B_1, D_1	
		2^+	B_2^*, D_2^*	

leave out for lecture

in addition: we also have a symmetry for exchanging the light constituent (only in constituent quark model!)

$Q\bar{q} - Q(\bar{u}, \bar{d}, \bar{s}) \rightarrow SU(3)_V$ triplets : $\bar{B}^-, \bar{B}^0, \bar{B}^0_s$
D^0, D^+, D_s^+

e.g. $\boxed{J^\pi = 0^-}$



Applications: strong decays $\quad s_\ell \rightarrow s_\ell' + \pi$ (Pion)

decay of excited mesons into a pion and the meson ground state

e.g. $(D_2^*, D_1) \longrightarrow (D^*, D) + \pi$
$s_\ell = \frac{3}{2}^+ \qquad s_\ell = \frac{1}{2}^-$
$s_Q = \frac{1}{2} \qquad s_Q = \frac{1}{2}$

} all four decays related by angular mom. + parity conservation
(→ Clebsch-Gordon analysis)
(→ partial wave analysis)

Exercise:
⇒ $\Gamma(D_1 \rightarrow D\pi) : \Gamma(D_1 \rightarrow D^*\pi) : \Gamma(D_2^* \rightarrow D\pi) : \Gamma(D_2^* \rightarrow D^*\pi)$
$0 \quad : \quad 1 \quad : \quad 2.3 \quad : \quad 0.92$

$\frac{Br(D_2^* \rightarrow D\pi)}{Br(D_2^* \rightarrow D^*\pi)} \approx 2.5 \qquad$ experiment: $2.3 \pm 0.6 \qquad$ just from symmetry!

Reparametrization invariance → Luke/Manohar PLB 286, 348 (1992) hep-ph/9205228
Manohar hep-ph/9701294

One can, instead of $p = m_Q v + k$, choose v slightly different by an amount of order Λ_{QCD}
→ Does not change the physics! Gives another type of symmetry.

$$v \to v' = v + \frac{\epsilon}{m_Q} \qquad v, v' \text{ are 4-velocities}$$

$$k \to k' = k - \epsilon \qquad \to v^2 = v'^2 = 1 \Rightarrow \underline{\underline{v.\epsilon = 0}}$$ 3 parameter transformation

→ The relation $\not{v} Q_v = Q_v$ must always hold

$$Q_v \to Q_v + \delta Q_v \quad \to (\not{v} + \frac{\not{\epsilon}}{m_Q})(Q_v + \delta Q_v) = Q_v + \delta Q_v$$

$$\to (1-\not{v})\delta Q_v - \frac{\not{\epsilon}}{m_Q} Q_v = 0$$ to linear order $\frac{\epsilon}{m_Q}$

solution: $\delta Q_v = \frac{\not{\epsilon}}{2m_Q} Q_v$

⇒ Invariance under the transformations
$$\boxed{v \to v + \frac{\epsilon}{m_Q}, \quad Q_v \to e^{i\epsilon.x}\left(1 + \frac{\not{\epsilon}}{2m_Q}\right) Q_v}$$

$$\mathcal{L}_0' = \bar{Q}_v \left(1 + \frac{\not{\epsilon}}{2m_Q}\right) e^{-i\epsilon.x}\, i\left(v - \frac{\epsilon}{m_Q}\right).D\, e^{i\epsilon.x}\left(1 - \frac{\not{\epsilon}}{2m_Q}\right) Q_v$$ $\frac{1+\not{v}}{2}\not{\epsilon}\frac{1+\not{v}}{2} \sim v.\epsilon = 0$

$$= \mathcal{L}_0 + \bar{Q}_v\, i\frac{\epsilon.D}{m_Q} Q_v$$ $\epsilon.D = \epsilon.D_\perp$ because $v.\epsilon = 0$

$$\mathcal{L}_1' = \mathcal{L}_1 - c_k \bar{Q}_v\, i\frac{\epsilon.D_\perp}{m_Q} Q_v$$

⇒ $c_k \stackrel{!}{=} 1$ to all orders in perturbation theory

Reparametrization invariance mixes different orders in $\frac{1}{m_Q}$

Also: factors of v and D_μ occur only in the combination $\left(v + \frac{iD}{m_Q}\right)^\mu$

Dimension 5 contributions in $\mathcal{L}_1$

$\bar{Q}_v \frac{\sigma_{\mu\nu} G^{\mu\nu}}{4m_Q} Q_v$ breaks spin symmetry & flavor symmetry

$\bar{Q}_v \frac{D_\perp^2}{2m_Q} Q_v$ breaks flavor symmetry

→ symmetry of heavy quark symmetry broken

⇒ $\boxed{\text{We treat them and other dim} > 5 \text{ terms in } \mathcal{L}_{HQET} \text{ as perturbations!}}$

7th lecture | ref: Falk et al. NPB 343 (1990) 1 | 04/16/03 (57)

$\boxed{\text{Covariant representation of fields}}$ → $\boxed{\text{heavy quark symmetry in action!}}$

⇒ We still consider $m_Q \to \infty$ with the full heavy quark symmetry! ⇐
"Heavy Quark limit"

Note: We start from the states described by $\mathcal{L}_0$ which has the largest symmetry. The symmetry breaking interactions are treated as corrections (just like the Hamilton).

Problem: Our action contains quarks & gluons as dof's, but experiments only deal with hadrons (= nonpert. solutions of the action $\mathcal{L}_0$)

⇒ ~~Way out~~. The symmetries of our action are still present in the hadron properties.

→ We use symmetries to relate operators of quarks & gluons to hadron field operators (modulo nonperturbative factors)

Example: $S_\ell^P = \frac{1}{2}^-$ ground state B mesons: $\bar{B}_v, \bar{B}_v^*$ ⟷ $|Q(v)\rangle |Q^*(v)\rangle$ ← meson containing heavy quark with velocity v
D mesons: D_v, D_v^*

⇒ very pragmatic approach: states constructed from a heavy quark u_h and a light anti quark $\bar{v}_\ell$ ⇒ $\boxed{u_h \bar{v}_\ell}$

→ all symmetries of the states contained in $u_h \bar{v}_\ell$ shall be encoded in our meson field representation (1-to-1)

e.g.

- flavor sym. $\quad u_h \bar{v}_\ell \to U_{hh'} u_{h'} \bar{v}_\ell \quad$ (trivial)
- spin sym. $\quad u_h \bar{v}_\ell \to D(R)_h u_h \bar{v}_\ell \quad$ (rotation of heavy quark spin)
- Lorentz $\quad u_h \bar{v}_\ell(x) \to D(\Lambda) u_h \bar{v}_\ell (\Lambda^{-1} x) D(\Lambda)^{-1}$
- Parity $\quad u_h \bar{v}_\ell \to \gamma^0 u_h \bar{v}_\ell (x_0, -\vec{x}) \gamma^0$

$D(R), D(\Lambda)$ are γ-matrices; exact form not important now.

→ go to rest frame: $S^i = \frac{1}{2}\begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}$ spin operator, $S^i = \frac{i}{4} \epsilon^{ikm} [\gamma^k, \gamma^m]$?

spinor basis $u_{h\uparrow}^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$; $u_{h\uparrow}^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$; $v_{\ell\downarrow}^{(1)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$, $v_{\ell\downarrow}^{(2)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

$(\overline{\vec{S} v_\ell}) = v_\ell^+ \vec{S}^+ \gamma^0 = -\bar{v}_\ell \vec{S}$

$\Rightarrow \vec{S}(u_h \bar{v}_\ell) = (\vec{S} u_h) \bar{v}_\ell - u_h (\bar{v}_\ell \vec{S})$

$\Rightarrow \vec{S}\left[u_h^{(1)} \bar{v}_\ell^{(1)} + u_h^{(2)} \bar{v}_\ell^{(2)}\right] = 0$; $S^3 \left[u_h^{(1)} \bar{v}_\ell^{(1)} - u_h^{(2)} \bar{v}_\ell^{(2)}\right] = 0$

$S^3 \left[u_h^{(1)} \bar{v}_\ell^{(2)}\right] = + u_h^{(1)} \bar{v}_\ell^{(2)}$, $S^3 \left[u_h^{(2)} \bar{v}_\ell^{(1)}\right] = - u_h^{(2)} \bar{v}_\ell^{(1)}$

7th

04/16/03 (52)

γ-structures

Singlet (pseudoscalar): $\frac{1}{\sqrt{2}}\left(u_h^{(1)}\bar{v}_e^{(1)} + u_h^{(2)}\bar{v}_e^{(2)}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -I \\ 0 & 0\end{pmatrix}$ $\Big\}$ $\frac{1}{\sqrt{2}}\frac{1+\not v}{2}\gamma^5$

Triplet (vector): $\frac{1}{\sqrt{2}}\left(\dots - \dots \right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & \sigma_3 \\ 0 & 0\end{pmatrix}$

$u_h^{(1)}\bar{v}_e^{(2)} = \frac{1}{2}\begin{pmatrix} 0 & \sigma_1+i\sigma_2 \\ 0 & 0\end{pmatrix}$

$u_h^{(2)}\bar{v}_e^{(1)} = \frac{1}{2}\begin{pmatrix} 0 & \sigma_1-i\sigma_2 \\ 0 & 0\end{pmatrix}$ $\Big\}$ circular pol.

$\Big\}$ $\frac{1}{\sqrt{2}}\frac{1+\not v}{2}\not\epsilon$

$v=(1,\vec{0})$

ϵ^μ: polariz. vector

Meson field operators:

$$P_v^{(Q)}(0)\,|Q(v)\rangle = |0\rangle \qquad \text{pseudoscalar}$$

$$P_{v,\mu}^{*(Q)}(0)\,|Q^*(v)\rangle = \epsilon_\mu |0\rangle \qquad \text{vector}$$

$$\Rightarrow \boxed{H_v^{(Q)} = \frac{1+\not v}{2}\left[\not P_v^{*(Q)} + i P_v^{(Q)}\gamma^5\right]}$$

$\Big\}$ has the correct symmetry properties

standard convention

"spin-covariant field operator of $s_e^P = \frac{1}{2}^-$ bound state meson

standard normalization

Normalization of States

$|H(\vec{p})\rangle$ hadron with p

$|H(v,k)\rangle$ hadron with $p = v + \frac{k}{m_Q}$

Standard relativistic normalization

$$\langle H(\vec{p}\,')\,|\,H(\vec{p})\rangle = 2\underbrace{E_p}_{\text{dim}-1}(2\pi)^3\delta^{(3)}(\vec{p}-\vec{p}\,') \quad , E_p = \sqrt{m_H^2+\vec{p}^{\,2}}$$

$\rightarrow$ HQET: states are defined in limit $m_Q\to\infty$

solutions of $\mathcal{L}_0$ have different normalization

QCD: $\sum_\sigma u_\sigma(p)\bar{u}_\sigma(p) = \not p + m$

HQET: $\sum_\sigma Q_{v,\sigma}\bar{Q}_{v,\sigma} = \not v + 1$

$\Big\}$ $\underset{p=m_Q v+\dots}{|H(\vec{p})\rangle} = \sqrt{m_H}\left[|H(v)\rangle + O\left(\frac{1}{m_Q}\right)\dots\right]$

arise from $\mathcal{L}_1, \mathcal{L}_2, \dots$

$$\Rightarrow \underset{\text{dim}-\frac{3}{2}}{\langle H(v',k')\,|\,H(v,k)\rangle} = 2v^0\delta_{vv'}(2\pi)^3\delta^{(3)}(\vec{k}-\vec{k}')$$

$\rightarrow$ we check the consistency of these relation with our covariant representation of the meson ground states for $m_Q\to\infty$, in the rest frame $p = m_Q v$, $v=(1,\vec{0})$

$$\frac{1}{m_B}\langle \bar{B}^{(*)}(p)\,|\,\bar{b}\gamma^0 b(0)\,|\,\bar{B}^{(*)}(p)\rangle \stackrel{!}{=} 2 \qquad \Leftarrow \text{QCD relation}$$

just counts # of b quarks $=1$, $b,\bar{b}$ b quark annihilation production operators

let's see if we get there to check out our normalization and formalism

7th

04/16/03 T3

(external current)

→ start from the vector current in QCD, due to flavor symmetry for $m_Q \to \infty$ we can have two different heavy quark fields

$$\bar{Q}' \gamma^\mu Q \quad (QCD)$$

$$\downarrow \; m_Q \to \infty$$

I do not write out the Wilson coefficient!

$$\sum_{v,v'} \bar{Q}'_{v'} \gamma^\mu Q_v \, \overset{w}{C}(\mu) \, (HQET) \qquad w \equiv v \cdot v'$$

← Wilson coeff.

we can have $v \neq v'$ in general e.g. for a weak decay that changes the velocity we only had $v = v'$ here!

→ we need to relate the current to a current of our covariant meson fields for an explicit computation

general case: $\bar{Q}'_{v'} \Gamma Q_v$ → under heavy quark spin transformations

some Lorentz structure

$$Q_v \to D(R)_Q \, Q_v$$

$$\bar{Q}_v \to \bar{Q}_v \, D(R)_Q^{-1}$$

Standard trick: pretend that $\Gamma \to D(R)_{Q'} \Gamma D(R)_Q$ under spin trafo

⇒ $\bar{Q}'_{v'} \Gamma Q_v$ invariant

⇒ meson current:
- must have same Lorentz properties • parity, time reversal, etc.
- must have meson fields $H_v^{(Q)}$ and $H_{v'}^{(Q')}$
- must be invariant under spin trafo

⇒ $\bar{Q}'_{v'} \Gamma Q_v = Tr\left[X \, \bar{H}_{v'}^{(Q')} \Gamma H_v^{(Q)} \right]$

most general bispinor, depends on v, v'

parity: $X \to \gamma^0 X(-) \gamma^0$ $\qquad w \equiv v \cdot v'$

Skip:

→ $X = a_0 + a_1 \not{v} + a_2 \not{v}' + a_3 \not{v} \not{v}'$, $a_i = a_i(v \cdot v')$

$\not{v} H_v^{(Q)} = H_v^{(Q)}$, $H_v^{(Q)} \not{v} = -H_v^{(Q)}$

$\not{v}' \bar{H}_{v'}^{(Q')} = -\bar{H}_{v'}^{(Q')}$, $\bar{H}_{v'}^{(Q')} \not{v}' = \bar{H}_{v'}^{(Q')}$

$$\Rightarrow \boxed{\bar{Q}'_{v'} \Gamma Q_v = -\xi(w) \, Tr\left[\bar{H}_{v'}^{(Q')} \Gamma H_v^{(Q)} \right]}$$

Isgur-Wise function

non-perturbative, uncalculable!

⇒ independent of Γ-structure! ⇐

⇒ depends on choice of hadron states multiplet!

⇒ independent of choice of Q, Q'

* renormalization scale dependent (≙ defined at the scale μ of the $\bar{Q}'_{v'} \Gamma Q_v$ op.)

→ let's compute a few currents: (I used trace.m)

$\epsilon^{\mu\nu\alpha\beta} = -\epsilon_{\mu\nu\alpha\beta}$

vector current (*)

$$\bar{Q}'_{v'} \gamma^\mu Q_v = P_{v'}^{(Q')+} P_v^{(Q)} \xi(w) (v^\mu + v'^\mu) \qquad ps - ps$$
$$+ P_{v',\nu}^{*(Q')+} P_v^{(Q)} \xi(w) \epsilon^{\mu\nu\alpha\beta} v_\alpha v'_\beta \qquad v - ps$$
$$+ P_{v'}^{(Q')+} P_{v,\nu}^{*(Q)} \xi(w) \epsilon^{\mu\nu\alpha\beta} v'_\alpha v_\beta \qquad ps - v$$
$$+ P_{v',\alpha}^{*(Q')+} P_{v,\beta}^{*(Q)} \xi(w) \left[g^{\mu\alpha}(v'-v)^\beta + g^{\mu\beta}(v-v')^\alpha - g^{\alpha\beta}(v+v')^\mu \right] \qquad v - v$$

axial vector current

$$\bar{Q}_{v'} \gamma^\mu \gamma^5 Q_v = P_{v',\nu}^{*(Q')+} P_v^{(Q)} \xi(w)\, i \left[-g^{\mu\nu}(1+w) + (v^\mu v'^\nu + v^\nu v'^\mu) \right] \qquad v - ps$$
$$+ P_{v'}^{(Q')+} P_{v,\nu}^{*(Q)} \xi(w)\, i \left[g^{\mu\nu}(1+w) - (v^\mu v'^\nu + v^\nu v'^\mu) \right] \qquad ps - v$$
$$+ P_{v',\alpha}^{*(Q')+} P_{v,\beta}^{*(Q)} \xi(w)\, i \left[\epsilon^{\alpha\beta\nu\mu} (v_\nu + v'_\nu) \right] \qquad v - v$$

These results will be very useful later

→ back to the normalization issue ($m_b \to \infty$, rest frame; $v = (1, \vec{0})$, $p = m v$)

full QCD

$$2 = \frac{1}{m_B} \langle \bar{B}^{(*)}(p) | \bar{b} \gamma^0 b | \bar{B}^{(*)}(p) \rangle$$

→ we set $k=0$, and remove an overall factor $(2\pi)^3 \delta^{(3)}(0)$

HQET → now use (*)!

$$= \langle \bar{B}^{(*)}(v) | \bar{b}_v \gamma^0 b_v | \bar{B}^{(*)}(v) \rangle \, \hat{C}(\mu)$$

← Wilson coefficient, compensates μ-dep. of the current

pseudoscalar:

$$= 2\, \xi(1)\, v^0\, \hat{C}(\mu) = 2\, \xi(1)\, \hat{C}(\mu)$$

vector: no pol. sum!

$$= -2 \underbrace{\epsilon^*\cdot\epsilon}_{=-1} v^0 \xi(1) \hat{C}(\mu) = 2\, \xi(1)\, \hat{C}(\mu)$$

One find that $\xi(1)\hat{C}(\mu) = 1$

(We will learn see that $\hat{C}(\mu) = 1$ does in fact not run!)

$$\frac{d\, C^{w=1}}{d \ln\mu} = 0$$

$$\boxed{\Rightarrow \quad \xi(1) = 1 \quad \text{for all orders ?}}$$

- due to heavy quark symmetry also true for $B \to D$ transitions!
  "zero recoil" limit

→ truly remarkable, but makes sense physically because the number of heavy quarks is scale-independent

→ Some more heavy-heavy currents

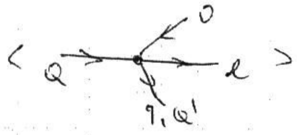
$$\bar{Q}_{v'} Q_v = P_{v'}^{(Q')+} P_v^{(Q)} \xi(\omega)(1+\omega)$$
$$+ P_{v',\alpha}^{*(Q')+} P_{v,\beta}^{*(Q)} \xi(\omega) \left[-g^{\alpha\beta}(1+\omega) + (v^\alpha v'^\beta - v'^\alpha v^\beta) \right]$$

$$\bar{Q}_{v'} \gamma^\nu Q_v = P_{v',\nu}^{*(Q')+} P_v^{(Q)} \xi(\omega)\, i\, (v-v')^\nu$$
$$+ P_{v'}^{(Q')+} P_{v,\nu}^{*(Q)} \xi(\omega)\, i\, (v'-v)^\nu$$
$$+ P_{v',\alpha}^{*(Q')+} P_{v,\beta}^{*(Q)} \xi(\omega)\, i\, \epsilon^{\alpha\beta\mu\nu} v_\mu v'_\nu$$

→ Let's do some more physics now! → Renormalization later!

- B physics is a huge phenomenological field → I cannot do all here!
- Not for all of B physics is HQET the proper EFT!

→ I will mention two applications related to B and D decays mediated by the semi-leptonic 4-quark operators describing electroweak interactions at low energies $\ll m_W$



→ remember: this operator did not pick up large QCD logs $\ln\left(\frac{m_W}{\mu}\right)$ for low scales μ

$$= -i\frac{4G_F}{\sqrt{2}}\underbrace{[\bar{u}_{q,Q'}\gamma_\mu P_L u_Q]}_{\substack{\text{quark current}\\ \text{(non-trivial)}}}\underbrace{[\bar{u}_\ell\gamma^\mu P_L v_\nu]}_{\substack{\text{lepton current}\\ \text{(trivial)}}}\underbrace{C(\mu)}_{=1}\cdot\underbrace{V_{Q,q/Q'}}_{\text{CKM}}$$

$\boxed{\text{Heavy Meson Decay Constants}}$ → again $m_Q\to\infty$ limit! LL approximation

B and D mesons can annihilate into leptons.

e.g. $D^+\to\bar{\ell}\nu$ → observed: $D_s^+\to\bar{\mu}\nu_\mu,\ \bar{\tau}\nu_\tau$ $\quad \mathcal{B}(D_s^+\to\bar{\mu}\nu)=5\cdot10^{-3}$

$B\to\ell\bar{\nu}$ $\qquad D^+\to\bar{\mu}\nu_\mu$ $\quad \mathcal{B}(\quad\bar{\tau}\nu)=6.4\cdot10^{-2}$

$(D^+\to\bar{\mu}\nu)\hat{=}10\cdot10^{-4}$

form factors in QCD: for the $s_\ell^\pi=\frac{1}{2}^-$ ground state $\quad P=D,\bar{B},\ P^*=D^*,\bar{B}^*$

$$\langle 0|\bar{q}\gamma^\mu\gamma^5 Q(\overset{x=0}{0})|P(p)\rangle=-if_P\,p^\mu \qquad \text{pseudoscalar}$$

$$\underbrace{\langle 0|\bar{q}\gamma^\mu Q(0)|P^*(p,\epsilon)\rangle}_{\text{heavy-to-light currents}}=f_{P^*}\epsilon^\mu \qquad \text{vector}$$

heavy-to-light currents in HQET: $\quad$ we need to write down all currents that have the same trafo properties!

$$\bar{q}\gamma^\mu\gamma^5 Q\longrightarrow C_1^{Av}\bar{q}\gamma^\mu\gamma^5 Q_v+C_2^{Av}\bar{q}v^\mu\gamma^5 Q_v \qquad C_i^{Av}=C_i^{Av}(\mu)$$

$$\bar{q}\gamma^\mu Q\longrightarrow C_1^v\bar{q}\gamma^\mu Q_v+C_2^v\bar{q}v^\mu Q_v \qquad C_i^v=C_i^v(\mu)$$

→ obviously we have at tree level $\quad C_1^{v,Av}(m_b)=1,\quad C_2^{v,Av}(m_b)=0$

→ we need to use symmetry arguments again to relate the HQET currents to meson currents

→ we use the standard trick again:

- same Lorentz properties
- heavy quark symmetry properties
- light spin transformation
- parity, ~~time reversal~~, etc

} transformation properties must agree

$$\Rightarrow \bar q \Gamma Q_v \overset{-\mathrm{Tr}[\Gamma Q_v \bar q]}{=} \mathrm{Tr}\left[X \Gamma H_v^{(Q)}\right] \quad , X = a_0 + a_1 \not v \quad , \not v H_v^{(Q)} = H_v^{(Q)}$$

$$\underset{\text{unknown constant}}{=} \frac{a}{2}\mathrm{Tr}\left[\Gamma H_v^{(Q)}\right] \qquad a_i = a_i(v^2) = a_i(1)$$

(analogous to Isgur-Wise fct) can still be μ-dependent!

we get: $\bar q \gamma^\mu Q_v = a P_v^{*(Q)\mu}$; $\bar q v^\mu Q_v = a(v\cdot P_v^{*(Q)}) v^\mu$

$\bar q \gamma^\mu \gamma^5 Q_v = -i a P_v^{(Q)} v^\mu$; $\bar q v^\mu \gamma^5 Q_v = i a P_v^{(Q)} v^\mu$

⇒ now we are ready!

$$\langle 0|\bar q \gamma^\mu\gamma^5 Q|P(p)\rangle = \sqrt{m_P}\,\langle 0|\,C_1^{AV}\bar q\gamma^\mu\gamma^5 Q_v + C_2^{AV}\bar q v^\mu\gamma^5 Q_v|P(v)\rangle$$
$$= \sqrt{m_P}\left[-i a C_1^{AV} v^\mu + i a C_2^{AV} v^\mu\right]$$
$$= \sqrt{m_P}(-i a v^\mu)(C_1^{AV}-C_2^{AV}) \Longrightarrow \boxed{f_P = \frac{a}{\sqrt{m_P}}(C_1^{AV}-C_2^{AV})}$$

$$\langle 0|\bar q\gamma^\mu Q|P^*(p)\rangle = \sqrt{m_{P^*}}\,\langle 0|\,C_1^V\bar q\gamma^\mu Q_v + C_2^V \bar q v^\mu Q_v|P^*(v)\rangle$$
$$= \sqrt{m_{P^*}}\left[C_1^V a\,\epsilon^\mu + C_2^V \cancel{a\, v\cdot\epsilon\, v^\mu}\right] \qquad v\cdot\epsilon = \frac{1}{m_{P^*}} p\cdot\epsilon = 0$$
$$= \sqrt{m_{P^*}}\, a C_1^V \epsilon^\mu \Longrightarrow \boxed{f_{P^*} = a\, C_1^V \sqrt{m_{P^*}}}$$

→ partial width computation

e.g. pseudoscalar
$$\Gamma_P = \frac{G_F^2}{8\pi}|V_{Qq}|^2 f_P^2\, m_\ell^2\, m_P\left(1-\frac{m_\ell^2}{m_P^2}\right)^2$$

helicity suppression

very small rate! difficult to measure
difficult to see for $\ell\nu$ final state

vector mesons → forget it!

will first decay to pseudoscalars (~100%)
by electromagnetic (γ) and strong (π) interaction

→ decay constants for pseudoscalars can be measured

$$\boxed{\frac{f_B}{f_D} = \sqrt{\frac{m_D}{m_B}}\,\frac{C_1^{AV}(\mu,m_b)-C_2^{AV}(\mu,m_b)}{C_1^{AV}(\mu,m_c)-C_2^{AV}(\mu,m_c)}}$$

we choose the same scale for both decays!

matching for D at $\mu=m_c$
for B at $\mu=m_b$

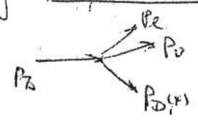
7th 04/12/03 (57)

$\boxed{\text{Semileptonic } B \to D \text{ decays at zero recoil}}$ $\to m_Q \to \infty$ limit!

→ $\bar{B} \to D, D^* + l\nu$ allows measurements of V_{cb}

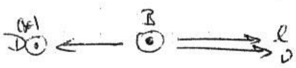
→ a natural variable to analyze the decay: <u>lepton invariant mass</u> q^2

$$q^2 = (p_e + p_\nu)^2 = (p_B - p_{D^{(*)}})^2$$



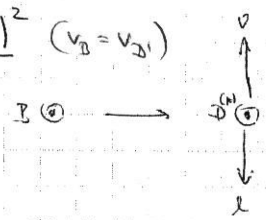
$(m_{e,\nu} = 0)$

$\underline{q^2_{min} = 0}$



"<u>maximal recoil</u>"

$\underline{q^2_{max} = (m_B - m_{D^{(*)}})^2}$ $(v_B = v_{D'})$



"<u>zero recoil</u>" ← $\xi(\omega=1)$ can be used

<u>Can HQET be applied?</u> → HQET describes the dynamics of the "brown muck" around a heavy single quark for momentum transfers of $\mathcal{O}(\Lambda_{QCD}) \ll m_Q$

→ the more appropriate variables to analyze the decay are v: velocity of $\bar{B}$
v': velocity of $D^{(*)}$

<u>momentum of brown muck</u> before decay: $\sim \Lambda_{QCD}\, v$ $\quad v^2 = v'^2 = 1$
" after decay: $\sim \Lambda_{QCD}\, v'$

mom. transfer^2 for brown muck: $(\Delta q_{light})^2 = (\Lambda_{QCD} v - \Lambda_{QCD} v')^2 = 2\Lambda^2_{QCD}(1-\omega)$

HQET is the proper EFT as long as $2\Lambda^2_{QCD}(1-\omega) \ll m^2_{b,c}$

→ <u>we have:</u>
$$\omega = v' v = \frac{p_B\, p_{D^{(*)}}}{m_B\, m_{D^{(*)}}} = \frac{m_B^2 + m^2_{D^{(*)}} - q^2}{2\, m_B\, m_{D^{(*)}}}$$

$\sqrt{2 \cdot 0.6}\, \Lambda_{QCD} \simeq 0.5\,\text{GeV}$ ←

$$1 - \omega = \frac{q^2 - (m_B - m_{D^{(*)}})^2}{2\, m_B\, m_{D^{(*)}}} \quad \Rightarrow \quad -0.6 \le 1 - \omega \le 0$$

$\left.\begin{array}{l} m_B = 5.3\,\text{GeV} \\ m_D = 1.9\,\text{GeV} \end{array}\right\}$ $\dfrac{(m_B - m_D)^2}{2\, m_B\, m_D} \simeq 0.6$

⇒ HQET works for the whole phase space! (in the $m_b, m_c \to \infty$ limit)

<u>heavy-to-heavy currents in HQET</u>

<u>vector:</u> $\bar{c}\gamma^\mu b \longrightarrow C_1^V \bar{c}_{v'}\gamma^\mu b_v + C_2^V \bar{c}_{v'} v^\mu b_v + C_3^V \bar{c}_{v'} v'^\mu b_v$, $\quad C_i^V = C_i^V(\mu)$

<u>axial-v:</u> $\bar{c}\gamma^\mu\gamma^5 b \longrightarrow C_1^{AV} \bar{c}_{v'}\gamma^\mu\gamma^5 b_v + C_2^{AV} \bar{c}_{v'} v^\mu\gamma^5 b_v + C_3^{AV} \bar{c}_{v'} v'^\mu\gamma^5 b_v$, $\quad C_i^{AV} = C_i^{AV}(\mu)$

→ tree level matching conditions (do out!): $C_1^{V,AV}(\mu) = 1$, $\quad C_{2,3}^{V,AV}(\mu) = 0$

→ again use HQ symmetry arguments to relate these quark currents to meson currents

7th 04/19/03 (58)

$B \to D^{(*)}$ form factors ($m_{c,b} \to \infty$)

Vector currents

$$\langle D(p')|\bar c\gamma^\mu b|\bar B(p)\rangle = \sqrt{m_B m_D}\,\langle D(v')|C_1^v \bar c_{v'}\gamma^\mu b_v + C_2^v \bar c_{v'} v^\mu b_v + C_3^v \bar c_{v'} v'^\mu b_v|\bar B(v)\rangle$$

$$= \sqrt{m_B m_D}\,\xi(\omega)\left[C_1^v (v+v')^\mu + C_2^v(1+\omega)v^\mu + C_3^v(1+\omega)v'^\mu\right]$$

$$\langle D^*(p')|\bar c\gamma^\mu b|\bar B(p)\rangle = \sqrt{m_B m_{D^*}}\,\langle D(v')|\ldots|\bar B(v)\rangle$$

$$= \sqrt{m_B m_{D^*}}\,\xi(\omega)\left[C_1^V \epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v_\alpha v'_\beta\right]$$

axial-vector currents

$$\langle D^*(p')|\bar c\gamma^\mu\gamma^5 b|\bar B(p)\rangle = \sqrt{m_B m_{D^*}}\,\langle D(v')|\ \ldots|\bar B(v)\rangle$$

$$= \sqrt{m_B m_{D^*}}\,\xi(\omega) i\left[C_1^{AV}\left(-\epsilon^{*\mu}(1+\omega)+(\epsilon^*\cdot v)v'^\mu\right) + C_2^{AV}(\epsilon^*\cdot v)v^\mu + C_3^{AV}(\epsilon^*\cdot v)v'^\mu\right]$$

$\Rightarrow$ Compare to most general parametrization consistent with parity, time reversal (but without heavy quark symmetry)

$$\langle D(p')|\bar c\gamma^\mu b|\bar B(p)\rangle = \sqrt{m_B m_D}\left[h_+(\omega)(v+v')^\mu + h_-(\omega)(v-v')^\mu\right]$$

$$\langle D^*(p')|\bar c\gamma^\mu b|\bar B(p)\rangle = \sqrt{m_B m_{D^*}}\left[h_V(\omega)\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v_\alpha v'_\beta\right]$$

$$\langle D^*(p')|\bar c\gamma^\mu\gamma^5 b|\bar B(p)\rangle = \sqrt{m_B m_{D^*}}\left[-ih_A(\omega)(1+\omega)\epsilon^{*\mu} + ih_{A_2}(\omega)(\epsilon^*\cdot v)v^\mu + ih_{A_3}(\omega)(\epsilon^*\cdot v)v'^\mu\right]$$

$\Rightarrow$ 6 unknown functions of ω $\to$ can be expressed in terms of $\xi(\omega)$

HQET ($m_Q \to \infty$): only one unknown function $\xi(\omega)$ (due to heavy quark symmetry)

$$\Rightarrow \boxed{\begin{aligned} &h_+(\omega) = h_V(\omega) = h_{A_1}(\omega) = h_{A_3}(\omega) = \xi(\omega)C_i(\mu)\\ &h_-(\omega) = h_{A_2}(\omega) = 0\end{aligned}}$$

Differential decay rates $\qquad r = \frac{m_D}{m_B}\,,\quad r^* = \frac{m_{D^*}}{m_B}$

$$\frac{d\Gamma}{d\omega}(\bar B\to De\bar\nu_e) = \frac{G_F^2|V_{cb}|^2 m_B^5}{48\pi^3}(\omega^2-1)^{3/2}r^3(1+r)^2 F_D(\omega)^2 = \frac{G_F^2|V_{cb}|^2m_B^5}{48\pi^3}A(\omega,r)F_D^2(\omega)$$

$$\frac{d\Gamma}{d\omega}(\bar B\to D^*e\bar\nu_e) = \frac{G_F^2|V_{cb}|^2 m_B^5}{48\pi^3}(\omega^2-1)^{1/2}(\omega+1)^2 r^{*3}(1-r^*)^2\left[1+\frac{4\omega}{1+\omega}\frac{1-2\omega r^*+r^{*2}}{(1-r^*)^2}\right]F_{D^*}(\omega)^2$$

$$= \frac{G_F^2|V_{cb}|^2 m_B^5}{48\pi^3}A^*(\omega,r^*)F_{D^*}^2(\omega)$$

$$F_D(\omega) = \left[h_+ + \frac{1-r}{1+r}h_-\right]^2 = \xi^2(\omega)$$

$$F_{D^*}(\omega) = \left[(1-r^*)^2 + \frac{4\omega}{1+\omega}(1-2\omega r^*+r^{*2})\right]^{-1}\left\{2(1-2\omega r^*+r^{*2})\left[h_{A_1}^2 + \frac{\omega-1}{\omega+1}h_V^2\right] + \left[(1-r^*)h_{A_1} + (\omega-1)(h_{A_1}-h_{A_3}-r^*h_{A_2})\right]^2\right\} = \xi^2(\omega)$$

$\to$ $\boxed{\text{Zero recoil limit: } \omega\to 1\,,\ \xi(1)=1 \Rightarrow \text{no nonperturbative effects!}}$

$\Rightarrow$ clean measurement of V_{cb}

$\boxed{\text{Renormalization in HQET}}$ ($m_Q \to \infty$, LL approximation)

→ we are not going to carry out the full program explicitly as for QED

$$\mathcal{L} = \overbrace{\sum_{Q,v} i \bar{Q}_v^0 v^\mu \left[\partial_\mu + i g^0 A_\mu^0\right] Q_v^0}^{\text{"heavy action"}} + \underbrace{\sum_q \bar{q}^0 i \not{D} q^0 - \frac{1}{4}\left(G_{\mu\nu}^a G_a^{\mu\nu}\right)^0}_{\text{"light action"}} \qquad \underline{\text{unrenorm. Lagrangian}}$$

→ as usual we define

$$A_\mu^0 = Z_A^{1/2} A_\mu \quad ; \quad q^0 = Z_q^{1/2} q$$

$$Q_v^0 = Z_h^{1/2} Q_v \quad ; \quad g^0 = Z_g \, g \, \mu^\epsilon$$

Background field gauge → Abbott, NPB 185 (1981), 189

→ The renormalization of the "light Lagrangian" is $\underline{\text{exactly}}$ as for full QCD. We also adopt $\underline{\text{background field gauge}}$, therefore

$$Z_g = Z_A^{-1/2}$$

⇒ $\alpha_s = \frac{g^2}{4\pi}$ runs just like in QCD!

→ Note: this is not the case for an arbitrary gauge in QCD because of the gauge fixing which breaks gauge invariance!

→ Does the heavy quark field Q_v contribute to the running of the light dof's?

No, because any closed heavy quark loop is identically 0.
Any closed heavy quark loop has the form $\sim \int d^d q \frac{i}{v\cdot(q+p_1)} \cdots \frac{i}{v\cdot(q+p_n)}$.
Take e.g. the rest frame $v = (1, \vec{0})$, then the spatial integration gives zero in dim. reg.

 $= 0$

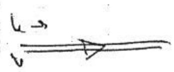
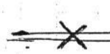
$$Z_A = 1 + \frac{\alpha_s}{4\pi\epsilon}\beta_0 \quad , \quad \beta_0 = \frac{11}{3}C_A - \frac{4}{3}T n_q$$

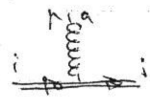
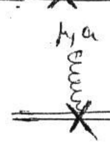
$$Z_g = 1 - \frac{\alpha_s}{8\pi\epsilon}\beta_0 \quad \rightarrow \quad Z_{\alpha_s} = 1 - \frac{\alpha_s}{4\pi\epsilon}\beta_0$$

$$Z_q = 1 - C_F \frac{\alpha_s}{4\pi\epsilon}$$

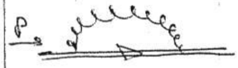
→ heavy quark Lagrangian Feynman rules (from $\mathcal{L}_0$)

$$\mathcal{L}_h = i Z_h \bar{Q}_v v^\mu \left(\partial_\mu + i g \mu^\epsilon A_\mu\right) Q_v$$

propagator:  $\frac{i}{v\cdot k + i\epsilon}$  $i\, v\cdot k\,(\delta Z_h)$

quark-gluon:  $-i g_s v^\mu T_{ji}^a$  $-i g_s v^\mu T_{ji}^a (\delta Z_h)$

Heavy quark 2-point fct



$$= \int d^d q \left[-ig T^A \tilde\mu^\epsilon v_\mu\right] \frac{i}{v\cdot(q+p)+i\epsilon}\left[-ig T^A \tilde\mu^\epsilon\right] v^\mu \frac{-i}{q^2+i\epsilon}$$

$$= -C_F g^2 \tilde\mu^{2\epsilon} \int \frac{d^d q}{[v\cdot(q+p)]_+ [q^2]_+}$$ → use HQET trick

$$= -C_F g^2 \tilde\mu^{2\epsilon} \int_0^\infty \frac{d^d q \, d\lambda}{[v\cdot(q+p)\lambda + q^2]_+^2}$$

$$= -i C_F \frac{\alpha_s}{2\pi\epsilon} v\cdot p \quad \Rightarrow \quad \boxed{Z_h = 1 + C_F \frac{\alpha_s}{2\pi\epsilon}}$$

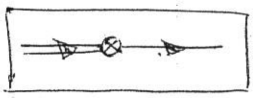
v-independent!
flavor-independent!

$\boxed{\text{Renormalization of the weak effective currents}}$

heavy-to-light currents

e.g. $\bar q \gamma^\mu (\gamma^5) Q_v$, $\bar q v^\mu (\gamma^5) Q_v$ → we do exactly what we have learned!

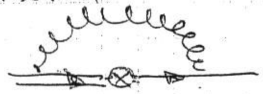
→ $j_\Gamma = C_\Gamma^0 \bar q^0 \Gamma Q_v^0 = Z_{C_\Gamma} Z_q^{1/2} Z_h^{1/2} C_\Gamma \bar q \Gamma Q_v$



any of the relevant Γ structures

$$= \left(1 + \delta Z_{C_\Gamma} + \frac{1}{2}\delta Z_q + \frac{1}{2}\delta Z_h\right) C_\Gamma \bar q \Gamma Q_v$$

we can set external mom. $=0$



$$= \int d^d q \left[-ig T^A \tilde\mu^\epsilon \gamma^\mu\right] \frac{i \not q}{[q^2]_+} C_\Gamma \Gamma \frac{i}{[v\cdot q]_+} \left[-ig T^A \tilde\mu^\epsilon v_\mu\right] \frac{-i}{[q^2]_+}$$

$$= -i C_\Gamma C_F g^2 \tilde\mu^{2\epsilon} \int \frac{d^d q \, \not v \not q \Gamma}{[v\cdot q]_+ [q^2]_+^2}$$ ← Γ structure is not relevant for the result!

Note: integral is scaleless but has IR + UV divergence! ⇒ insert IR regulator by hand or leave external mom. $\neq 0$ to extract UV-divergence

$$= C_\Gamma C_F \frac{\alpha_s}{4\pi\epsilon} \Gamma$$

↪ $\delta Z_{C_\Gamma} - \frac{C_F \alpha_s}{8\pi\epsilon} + \frac{C_F \alpha_s}{4\pi\epsilon} + \frac{C_F \alpha_s}{4\pi\epsilon} = 0$

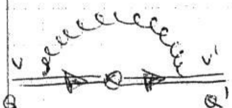
$$\boxed{Z_C^{qQ_v} = 1 - \frac{3}{8} C_F \frac{\alpha_s}{\pi\epsilon}}$$

independent of Γ!
flavor- and v-independent!

heavy-to-heavy currents

$\rightarrow \; j_\Gamma = C^0 \bar{Q}'^0_{v'} \Gamma Q^0_v = Z_{C_\Gamma} Z_h \bar{Q}'_{v'} \Gamma Q_v$ → similar computation

$= (1 + \delta Z_{C_\Gamma} + \delta Z_h)\, C_\Gamma \bar{Q}_{v'} \Gamma Q_v$



$$= \int d^dq \left[-ig T^A \tilde{\mu}^\epsilon v'_\mu\right] \frac{i}{[v' \cdot q]_+} C_\Gamma \Gamma \frac{i}{[v q]_+} \left[-ig T^A \tilde{\mu}^\epsilon v^\mu\right] \frac{-i}{[q^2]_+}$$

$$= -ig^2 \tilde{\mu}^{2\epsilon} C_F \omega \int \frac{d^dq \; C_\Gamma \Gamma}{[v' \cdot q]_+ [v q]_+ [q^2]_+}$$

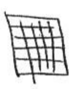
$$= -C_\Gamma C_F \frac{\alpha_s}{2\pi\epsilon} \omega\, r(\omega)\, \Gamma \quad , \quad r(\omega) = \frac{1}{\sqrt{\omega^2-1}} \ln\left(\omega + \sqrt{\omega^2-1}\right)$$

$$\boxed{Z_C^{Q_{v'} Q_v} = 1 + \frac{C_F \alpha_s}{2\pi\epsilon}\left[\omega\, r(\omega) - 1\right]}$$

independent of Γ!
flavor independent!

Comments:

- The anom. dim. of heavy-heavy currents depends on v, v'
  → v, v' are not dynamical variables, but "labels" that distinguish the various boxes of size $\sim 1/\Lambda_{QCD}$ mentioned in the 6th lecture →



  → One of the holy rules of EFT's: "anomalous dimensions never depend on dynamical variables such as momenta." is still o.k. in HQET

- we find: $[\omega r(\omega) - 1] \xrightarrow{\omega \to 1} \frac{2}{3}(\omega-1) - \frac{1}{5}(\omega-1)^2 + \ldots$

→ The anomalous dimension for heavy-heavy currents vanishes in the zero-recoil limit!

→ heavy quark number operator: $\bar{Q}_v \gamma^0 Q_v$ is RG-invariant

⇒ $\boxed{\text{Isgur-Wise function: } \xi(1) = 1}$

physical interpretation:

$\bar{Q}_v \gamma^0 Q_v$: → # of heavy quarks is RG-invariant

$\bar{Q}'_v \Gamma Q_v$: At zero recoil the light dof's do not realize a change in the heavy quark flavor caused by a weak decay. Their dynamics is unaffected by the decay.

interesting: the zero recoil limit $\overset{v=v'}{\omega = 1}$ corresponds to the limit of maximal lepton invariant mass q^2
→ this shows that q^2 is an "inappropriate" variable since it obscures the actual physics

→ let's come back to our two applications:

$$\frac{d}{d\ln\mu^2} = \overbrace{\frac{d\alpha_s}{d\ln\mu^2}}^{-6\alpha_s+\dots}\cdot\frac{d}{d\alpha_s}$$

Meson decay constants

$$0 = \frac{d}{d\ln\mu^2} C^0_{qQ_v} = \frac{d}{d\ln\mu^2}\left(Z_c^{qQ_v} C_{qQ_v}\right) \longrightarrow \frac{d}{d\ln\mu^2} C_{qQ_v} = -\left(Z_c^{qQ_v}\right)^{-1}\left(\frac{d}{d\ln\mu^2} Z_c^{qQ_v}\right) C_{qQ_v}$$

$$= -\frac{3}{8} C_F \frac{\alpha_s}{\pi} C_{qQ_v}$$

$$\Rightarrow \frac{dC}{C} = -\frac{3C_F}{8}\frac{\alpha_s}{\pi} d\ln\mu^2 \rightarrow \ln\left(\frac{C(\mu)}{C(m_Q)}\right) = -\frac{3C_F}{8\pi}\alpha_s\left(-\frac{4\pi}{\beta_0}\right)\frac{d\alpha_s}{\alpha_s^2} = \frac{3C_F}{2\beta_0}\frac{d\alpha_s}{\alpha_s}$$

$$= \frac{3C_F}{2\beta_0}\ln\left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)$$

$C(m_Q)$: matching condition

$$\Rightarrow \boxed{C_{qQ_v}(\mu) = C_{qQ_v}(m_Q)\left(\frac{\alpha_s(m_Q)}{\alpha_s(\mu)}\right)^{-\frac{3C_F}{2\beta_0}} = C_{qQ_v}(m_Q)\left(\frac{\alpha_s(m_Q)}{\alpha_s(\mu)}\right)^{-\frac{6}{33-2n_q}}}$$

$$C_{qQ_v}(\mu) = C_{qQ_v}(\mu, m_Q)$$

matching scale

f_B/f_D at LL order

B: $C_1^{AV}(\mu=m_b, m_b) = 1$ $\quad C_2^{AV} = 0$, same for D meson ($m_b \to m_c$)

$$\frac{f_B}{f_D} = \sqrt{\frac{m_D}{m_B}}\frac{C_1^{AV}(\mu, m_b)}{C_1^{AV}(\mu, m_c)}$$ take e.g. $\mu = m_c$

$$= \sqrt{\frac{m_D}{m_B}} C_1^{AV}(m_c, m_b)$$

$$\boxed{\frac{f_B}{f_D} = \sqrt{\frac{m_D}{m_B}}\left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{-\frac{6}{33-2n_q}\,\}-\frac{3C_F}{2\beta_0}}, \quad n_q = 4}$$

→ "sums up $\left(\alpha_s \ln\left(\frac{m_b^2}{m_c^2}\right)\right)^n$ terms"

$$\frac{\alpha_s(m_b)}{\alpha_s(m_c)} = 1 + \frac{\alpha_s(m_b)}{4\pi}\beta_0 \ln\left(\frac{m_c^2}{m_b^2}\right)$$

→ the logs of $\frac{m_b}{m_c}$ are actually not really large, but this is the correct LL result

↳ the quality of the expansion into LL, NLL, etc is not decided by how large the logs are that are summed up with respect to a different expansion, but by whether the expansion is convergent.

Numbers: $\frac{f_{B^0}}{f_{D^+}} = 0.71$ $\quad \frac{f_{B_s^0}}{f_{D_s^+}} = 0.72$ $\quad$ (take $m_b = 4.7$, $m_c = 1.3$)

recent lattice results (unquenched $n_f = 2$) [Bernard et al, PRD 66, 094501 ( )]

$$\frac{f_B}{f_D} = 0.91 \pm 0.08 \qquad \frac{f_{B_s}}{f_{D_s}} = 0.92 \pm 0.08$$

⇒ there are $\frac{\Lambda_{QCD}}{m_c}$, $\frac{\Lambda_{QCD}}{m_b}$, $\alpha_s(m_c)$, $\alpha_s(m_b)$ corrections which are $O(30\%)$

o.k. ← → but: actual $\frac{1}{m_{b,c}}$ corrections seem to be huge! (Neubert, PRD 46 (1992) 1076)

$\boxed{\text{Semileptonic } \bar{B} \to D, D^* \text{ decay at zero recoil}}$

Experimentally one can measure

$\frac{d\Gamma}{dw}(\bar{B} \to D e \bar{\nu}_e)$ and $\frac{d\Gamma}{dw}(\bar{B} \to D^* e \bar{\nu}_e)$

and determine $|V_{cb}||F_{D^{(*)}}(w)|$

→ at LL approximation one finds: $F_D(w) = F_{D^*}(w) = \xi(w)$

→ in the zero recoil limit: $F_D(1) = F_{D^*}(1) = \xi(1) = 1$ ⟹ No NONPERT. CONTRIBUTIONS!

⟹ the form factor is entirely perturbative at $w=1$

Comment:

$$\frac{1+\slashed{v}}{2}\gamma^\mu\frac{1+\slashed{v}}{2} = \frac{1+\slashed{v}}{2}v^\mu\frac{1+\slashed{v}}{2}$$

- At zero recoil $\bar{c}_v\gamma^\mu b_v = \bar{c}_v v^\mu b_v$ ⟹ only one current mediates the decay

⟹ matching condition: $C^V_{1,w=1}(\mu) = 1 + \frac{\alpha_s(\mu)}{\pi}\left[-2 + \frac{m_b+m_c}{m_b-m_c}\ln\left(\frac{m_b}{m_c}\right)\right]$

$$C^{AV}_{1,w=1}(\mu) = 1 + \frac{\alpha_s(\mu)}{\pi}\left[-\frac{8}{3} + \frac{m_b+m_c}{m_b-m_c}\ln\left(\frac{m_b}{m_c}\right)\right]$$

- Best measured: $\bar{B} \to D^* e\bar{\nu}$:

theory: $F_{D^*}(1) = C^{AV}_{1,w=1}(\sqrt{m_b m_c}) + 0 \times \delta_{\frac{1}{m}} + \delta_{\frac{1}{m^2}}$

all $\frac{1}{m}$ corrections cancel ("Lukes Theorem")

estimates $\simeq -0.05 \pm 0.04$

$$= 0.96 - 0.05 \pm 0.05 = 0.91 \pm 0.04$$

experiment (average from CKM workshop → hep-ph/0304132)

$|V_{cb}||F_{D^*}(1)| = (38.3 \pm 1.0)\cdot 10^{-3}$

⟹ $|V_{cb}| = (42.1 \pm 2.2)\times 10^{-3}$

Only 5% error!