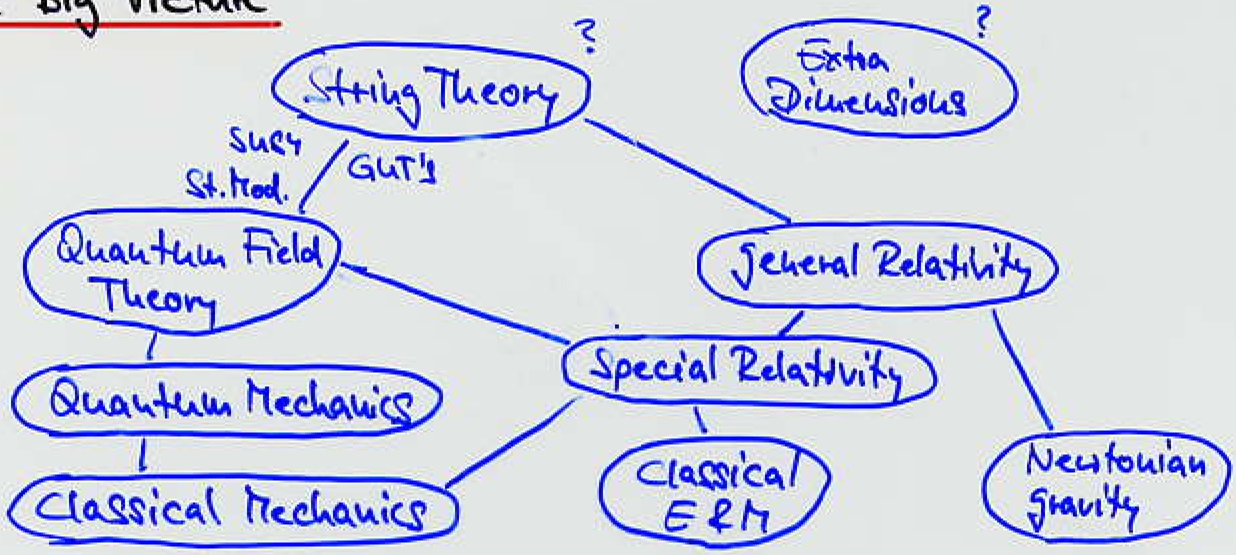
1st lecture → Overview

The Big Picture



- Much of work in particle physics ⟶ more & more general theories
  "unification"
  "Theory of Everything"
  e.g. $\underbrace{\text{E-M + weak + strong}}_{SU(3)\times SU(2)\times U(1)\ \text{SM}}$
- direction: smaller & smaller distances, "NEW Physics"
  → experiments decide, which way to go

In these lectures: I will talk about going in a "different" direction, which, however, does not contradict the search for new physi[cs]

→ effects of new physics often indirect, small
established theory needs to be understood much better

e.g. CKM matrix elements from B decays
→ QCD needs to be understood to higher precision

e.g. search for new particles & resonances (Tevatron/LHC)
⇒ background needs to be controlled

→ QCD IS the established theory of strong interaction[s]
but: it is not fully understood due to non-perturbative effects.

Our Focus: Finding the most economic framework that captures the physical system we want to describe

→ e.g. B mesons, H-atom, jet physics, etc...

Context: Quantum Field Theory → In this context the approach is called **Effective Field Theory (EFT)**

Relevant Questions

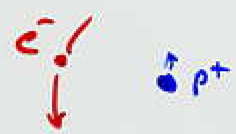
1. What are the relevant degrees of freedom (dof's) ?
2. What are the symmetries that constrain interactions ?
3. What is the appropriate expansion ? (power counting)

Comment: Concept of EFT's is very general !
Valid and relevant for all QFT applications, not just for some specific problems

→ examples for EFT's :

- Chiral pert. th. ; nuclear theory, ...
- QED, QCD
- Standard Model, MSSM, GUT's, ...

$\boxed{\text{A sketchy example: hydrogen atom}}$



→ obvious: all physics contained in the SM

but: in practice using the SM action way too cumbersome !

→ many unclear issues : bound state physics, proton ↔ quarks, top ?, Higgs ?, ...

→ SM contains all physics, but much of it seems entirely irrelevant for the H-atom.

→ How to distinguish what is relevant and what irrelevant? Can there be s.th. in the SM entirely irrelevant ?

→ 2 ways to proceed (in practice always a mixture!)

(A) Top-Down Approach

- Take SM action → identify the "resonating" (= "close to mass-shell" = relevant) dof's in the H-atom.
- Integrate out all other (= "off-shell") dof's.
  ⇒ (a) modifies the SM interactions of the resonating dof's
  (b) leads to new interactions of the resonating dof's

  → **Result: Effective Action**
- Identify the power counting and sort all terms in the effective action in powers of the expansion parameter

$$\mathcal{L}_{SM} \longrightarrow \sum_n \mathcal{L}^{(n)}_{EFT}$$

$\mathcal{L}^{(0)}_{EFT}$: leading order
$\mathcal{L}^{(1)}_{EFT}$: sub-leading order
⋮

(B) Bottom-Up Approach

Situation: underlying theory unknown or relevant dof's cannot be determined from the SM dof's (e.g. non-perturbative)

- Write down the relevant dof's and the symmetries that have to be satisfied by the interactions (Lorentz, gauge, internal, …)
- Identify the powercounting and write down most general set of interactions consistent with symmetries up to desired order → $\sum_n \mathcal{L}^{(n)}_{EFT}$
- Couplings unknown and have to be obtained by comparison with experiment or determined non-perturbatively (e.g. lattice)

→ let's come back to the hydrogen ($\hbar = c = 1$)

relevant dof's:
- non-relativistic electron (Pauli spinor)
- non-relativistic (static) proton
- photon with $E \sim \vec{p} \sim m_e \alpha^2$ ("ultrasoft")

powercounting:
- electron velocity $v \sim \alpha \approx (137)^{-1} \ll 1$
- inverse proton mass m_p^{-1}
- — " — mass of other SM particles

[– photon with $E \sim \vec{p} \sim m_e \alpha$ ("soft")]

Leading order Lagrangian

$D^0 = \partial_t + ig A^0$

$$\mathcal{L}^{(0)} = \sum_{\vec{p}} \left\{ \psi_p^{e^-+} \left(iD^0 - \frac{\vec{p}^2}{2m_e} \right) \psi_p^{e^-} + \chi_p^{p^++} (iD^0) \chi_p^{p^+} \right\} \quad , V_c = -4\pi\alpha$$
$$- \sum_{p,p'} \left\{ \frac{V_c}{(\vec{p}-\vec{p}')^2} \psi_{\vec{p}'}^+ \psi_{\vec{p}} \chi_{-\vec{p}'}^+ \chi_{-\vec{p}} \right\} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

Equation of motion

$\rightarrow (e^-p^+)(e^-p^+)$ – 4-point function

$$\left(-\frac{\vec{\nabla}_r^2}{2m_e} - \frac{\alpha}{|\vec{r}|} - E \right) G(\vec{r}, \vec{r}', E) = \delta^{(3)}(\vec{r}-\vec{r}')$$

"Schrödinger Eq."

$\rightarrow$ poles in G at $E_n^{(0)} = -\frac{m_e \alpha^2}{2n^2}$

$\rightarrow$ α can be determined from H-spectrum analysis

Higher-order effects

e.g. relativistic corrections $\quad \delta\mathcal{L}^{(2)} = \sum_p \psi_p^+ \frac{\vec{p}^4}{8m_e^3} \psi_p \quad$ kinetic E.

$\delta\mathcal{L}^{(2)} = \sum_{p,\vec{p}'} \frac{V_D}{m_e^2} \psi_{p'}^+ \psi_p \chi_{-p'}^+ \chi_{-p} \quad$ Darwin

recoil corrections $\quad \sum_{p,p'} \frac{a}{m_p^2} \psi_{p'}^+ \psi_p \chi_{-p'}^+ \chi_{-p}$

retardation, "Lamb shift" $\quad \rightarrow A^\mu$, ultrasoft photon

$\rightarrow$ described by subleading contributions of the eff. action $\mathcal{L}^{(i>0)}$

→ a few more issues on EFT's ....

Examples for EFT's – Top-Down Approach

Underlying theorie: Standard Model

Fermi theory: weak interactions at low energies

→ W, Z, t, H integrated out

→ describes weak decays of hadrons

Heavy Quark effective theory: dynamics of hadrons with 1 heavy qu

→ small component of heavy quark field integrated out

→ separation of perturbative & non-perturbative QCD effects

Non-relativistic QCD: dynamics of mesons with a heavy $Q\bar{Q}$ pa

→ similar to HQET, but much more complicated

Soft-Collinear EFT: for processes with highly energetic hadrons with $E_H \gg m_H$

Examples for EFT's – Bottom-Up Approach

Standard Model → (renormizable) LO part of the EFT at the weak scale

Einstein's Th. of Relativity → LO part of the low energy EFT for "Quantum gravity"

Chiral perturbation theory → low energy theory of π, K interaction

Renormizable, Nonrenormizable Theories & Powercounting

→ In (local) QFT's there are loop corrections

Traditionally one distinguishs 2 kinds of theories.

- **Renormizable theories**

  → $\mathcal{L}$ contains only field operators with dim. ≤ 4 — ϕ^4, $\bar\psi \not{\partial} \psi$, $\bar\psi \phi \psi$, ϕ^2, $\bar\psi\psi$

  → all UV divergences can be absorbed into fields & couplings already present in the Lagrangian

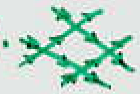
  e.g. Standard Model, MSSM, QED, QCD, ϕ^4-theory

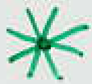
- **Non-renormizable theories**

  → $\mathcal{L}$ contains field operators with dim > 4 — Fermi theory e.g. $(\bar\psi\Gamma\psi)(\bar\psi\Gamma\psi)$ 

  → at higher loop order one can construct interactions with more and more # of external particles that are UV divergent

  ⇒ more and more operators with dim >4 needed for renormalization

  e.g.  $\sim \int \frac{d^4k}{\not{k}\not{k}\not{k}\not{k}} \sim \int \frac{dk}{k} \sim \ln\Lambda$

   dim 12 operator needed (8-fermion operator)

⇒ Sometimes non-renorm. theories are considered less predictive than renormizable ones!

→ All EFT's are non-renormizable in the traditional sense.

→ But: EFT's **are** predictive to any precision you want. because of the powercounting scheme.

e.g. Fermi theory: power-counting is simple dimens. analysis

→ $\int d^4x\, \mathcal{L}$ is dimensionless, $[\psi] = m^{3/2}$

⇒ $\delta\mathcal{L}_F \simeq G_F(\bar\psi\psi)(\bar\psi\psi)$ ⇒ $G_F \sim M_W^{-2}$

 $\sim G_F^4 (\bar\psi\psi)^4$ suppressed by M_W^{-8}

→ at order M_W^{-2} we neglect anything of order M_W^{-n}, $n>2$

→ at any given order in the power counting EFT's have the action $\sum_{n=0}^{m} \mathcal{L}^{(n)}$ with a finite # of operators

$\boxed{\text{Message: EFT's are renormizable in a more modern sense.}}$

Comment 1:

Sometimes $\mathcal{L}^{(0)}_{EFT}$ only contains operators with dim ≤ 4

e.g. SM, QED, QCD, MSSM, ...

→ any of these theories is the LO action of an EFT !

→ Considering only $\mathcal{L}^{(0)}_{EFT}$ might fool you because (traditionally) renormizable theories appear to describe physics up to arbitrarily high energies and appear to be more fundamente

Comment 2:

The high energy scales that have been integrated out appear as inverse powers multiplying the higher dimensional operators in the effective action. → always $[\int d^4x\, \mathcal{L}] = m^0$

Interactions that contain an inverse power of a mass scale always have dimension > 4 !

→ only these operatos can tell you where the expansion of the EFT breaks down

e.g. Fermi theory : expansion in powers of G_F breaks down for energies of $\mathcal{O}(M_W)$

Comment 3:

The 4-fermion operators of the Fermi theory belong to the dim 6 Lagrangian of the EFT that describes the strong interaction for energies $< M_W$. The dim 4 Lagrangian is just $\mathcal{L}_{QCD}$!

Summation of large logarithms → Top-Down only!

→ Sometimes in SM computations there are widely different scales that lead to large logarithms

$\sim \# \alpha_s \ln\left(\frac{M_W^2}{m_q^2}\right)$ → p.th. breaks down

→ By integrating out the large energy scale dof's and switching to an EFT one can sum the large logs using RGE's

$\sim \# \alpha_s \left[\frac{1}{\epsilon} + \ln\frac{\mu^2}{m_q^2} \cdots\right]$

$\Rightarrow \delta\mathcal{L} \sim C(\mu)\, G_F\, (\bar\psi\Gamma\psi)(\bar\psi\Gamma\psi)$

$$\frac{d}{d\ln\mu^2} C(\mu) = \#\, \alpha_s(\mu)\, C(\mu)$$

"anomalous dimension" of the 4-fermion operator

→ <u>Matching</u> fixes $C(\mu = M_W)$ (initial condition)

Solution of RGE for $C(\mu = m_q)$ sums terms $\left(\alpha_s \ln\frac{M_W^2}{m_q^2}\right)^n$ for <u>all</u> n

<u>Comment</u>

"<u>Leading Log approximation</u>" (LL) : $C(m_q) \sim \sum \left(\alpha_s \ln\left(\frac{M_W}{m_q}\right)\right)^n$

→ <u>need</u>: Born matching & one-loop anomalous dimension

"<u>Next-to-leading log approximation</u>" : $C(m_q) \sim \sum [1, \alpha_s] (\alpha_s \ln)^n$

→ <u>need</u>: 1-loop matching & Two-loop anomalous dimension

etc.

Summary Box 1st lecture

- EFT : provides description of a system using <u>only</u> the relevant (= resonating) dof's.
  → make the physics that governs the system most transparent

- For construction of an EFT you need to
  - identify the <u>relevant dof's</u>
  - identify the <u>symmetries</u> that constrain interactions
  - identify the <u>powercounting</u>
  - do everything correctly !

- EFT's are non-renormizable in the traditional sense but "power counting - renormizable"

- EFT's can be derived <u>top-down</u> or <u>bottom up</u>

- The (traditional) non-renormizable interactions (dim > 4) of an EFT tell when the EFT expansion breaks down

- All known renormizable QFT's that are phenomenologically relevant are leading order actions of EFT's.

- EFT's can be used to sum logarithms.

- EFT's are the only systematic way to separate perturbative and non-perturbative effects in QCD.

$\Downarrow$ Coefficients of <u>operators</u>

$\Downarrow$ matrix elements <u>of operators</u>

<u>Fermi theory</u> :

$$C(\mu) \times \langle D e \bar{\nu}_e | \times | \bar{B} \rangle$$