Problems

Sheet 6 and 7, 19.01.2006

Problem 1: Quantum numbers of the light ($J^P = 0^-$) pseudo-scalar meson octet

a) Give the relation between the physical Goldstone boson fields ($\pi^\pm, \pi^0, \ldots$)and the Cartesian Goldstone components ($\pi^a, a = 1, \ldots, 8$) using the relation

$$\phi(x) = \sum_{a=1}^{8} \lambda^a \pi^a(x) = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix} .$$

Determine the quark content of the physical Goldstone bosons. Remember how the quantum numbers of the Goldstone states are fixed.

b) Determine the isospin (I) and strangeness (S) and electric charge (Q) operators and determine the (I, S, Q) quantum numbers of the physical Goldstone boson states. Remember that these operators are just linear combinations of quark number operators and related to the unbroken symmetries of low energy QCD. Draw the Goldstone bosons into the (I, S)-plane and indicate the lines of constant electric charge.

Problem 2: Goldstone boson and quark masses

(a) Accounting for the finite light quark masses induces masses for the Goldstone bosons. Compute the physical Goldstone boson masses accounting for the difference between m_u and m_d as a perturbation; compute the masses to $\mathcal{O}(\epsilon)$ precision ($\epsilon \equiv \frac{B_0}{4}\frac{(m_u - m_d)^2}{(m_s - \bar{m})}$, $\bar{m} \equiv \frac{1}{2}(m_u + m_d)$). You can use the results obtained in class, but need to apply another diagonalization due to $\pi^0 - \eta$ mixing.

(b) Use the formulae to extract the quark mass ratio $m_u : m_d : m_s$ from the physical meson masses. Note that the masses of the Goldstone bosons with electric charge differ from the neutral ones due to electromagnetic corrections. The mass shifts should be proportional to the square of the electric charges and approximately the same for pions and kaons. So, only use combinations of mesons masses where the leading order electromagnetic effects cancel. Try to also use combinations where this is not the case to see how large electromagnetic effects are. Can one consistently explain the electromagnetic mass shifts of the charge mesons by a crude model $\Delta m_\pm = \pm\Lambda$?

Problem 3: Leptonic and Semileptonic decays

(a) Compute the charged pion leptonic partial width $\Gamma(\pi^+ \to \mu^+\nu_\mu)$ from the matrix element derived in class. Compare the result to the data given in the PDG and determine the pion decay constant F_0 using V_{ud} as an input. Predict the leptonic partial width $\Gamma(\pi^+ \to e^+\nu_\mu)$ and compare to the data. What is the physical reason why the electron partial width so much smaller?

(b) Make predictions for the kaon (semi)leptonic partial rates $\Gamma(K^+ \to \mu^+\nu_\mu)$, $\Gamma(K^+ \to e^+\nu_\mu)$ and $\Gamma(K^+ \to \pi^0 e^+\nu_e)$ and compare to the experimental data. For the last rate you can neglect the electron mass. Also have a look on the data for $\Gamma(K^+ \to \pi^0\mu^+\nu_e)$. Why is the situation different to the purely leptonic decay?

Problem 4: Compton Scattering

(a) Implement electromagnetic effects into the chiral effective Lagrangian using the technology we have applied for the weak interactions in class. Compute the two additional terms to the leading order chiral Lagrangian proportional to e and e^2 that result from the procedure. Analyse which elementary interactions of photons with two Goldstone bosons you can get.

(b) Determine the Feynman rules you need to consider for the Compton scattering process $\gamma\pi^+ \to \gamma\pi^+$. Draw the Feynman diagrams for the process and derive the amplitude. Argue that the result is in fact $U(1)_{\rm em}$ gauge invariant. (Answering the questions in part a) might be easier.)