Problems

Sheet 5, 7.12.2005

Problem 1: Semileptonic and non-leptonic b decay rate

a) Calculate the semileptonic decay rate of a free b quark $\Gamma(b \to ce\bar{\nu}_e)$. Use the effective Hamiltonian discussed in class. (Recycle the computations you already did earlier.)

b) Now Use the effective Hamiltonian discussed in class to compute the non-leptonic free quark decay rate $\Gamma(b \to cd\bar{u})$. Neglect all masses except for the b quark mass. If you are ambitious you can include a nonzero charm mass.

Problem 2: Non-leptonic b quark decays & penguin transitions

(a) Write down all possible decays of a b quark into u, d, s and c quarks. (This includes decays into two or three of the same quarks.) Order the various decay modes into the following three classes:

- Class I: only current-current operators contribute
- Class II: current-current and penguin operators contribute
- Class III: only penguin operators contribute

(b) Determine the parametric size of the various operators (take into account matching conditions and CKM factors).

Problem 3: $b \to s\gamma$ and QCD equations of motion

After integrating out the top, W, Z and Higgs, a number of local operators are induced which mediate the flavor changing (electric) charge-neutral process $b \to s\gamma$. The operators that are induced include the local 4-quark operators discussed in class and the following local 2-quark operators, which we, however, did not discuss in class:

$$
\begin{aligned}
O_1 &= \bar{s}_L \not{D} D_\mu D^\mu b_L\,, \\
O_2 &= \bar{s}_L D_\mu \not{D} D^\mu b_L - \frac{1}{2}\bar{s}_L D_\mu D^\mu \not{D} b_L\,, \\
O_3 &= \bar{s}_L D_\mu \not{D} D^\mu b_L\,, \\
O_4 &= \bar{s}_L \not{D}\not{D}\not{D} b_L\,, \\
O_5 &= g\bar{s}_L T^A \gamma^\mu b_L (D^\nu G_{\mu\nu})^A\,, \\
O_6 &= g G^A_{\mu\nu} \bar{s}_L T^A \gamma^\mu D^\nu b_L\,, \\
O_7 &= g \tilde{G}^A_{\mu\nu} \bar{s}_L T^A \gamma^\mu D^\nu b_L\,, \\
O_8 &= e\, m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}\,, \\
O_9 &= g\, m_b \bar{s}_L \sigma^{\mu\nu} T^A b_R G^A_{\mu\nu}\,, \\
O_{10} &= m_b \bar{s}_L \not{D}\not{D} b_R\,.
\end{aligned}
$$

Here, $D_\mu = \partial_\mu + igT^A A^A_\mu + ieQA_\mu$ is the covariant derivative (in the SU(3) fundamental representation) acting on the quark fields, $(D^\mu G_{\mu\nu})^A = (\delta^{AB}\partial^\mu + gf^{ABC}A^{\mu B})G^C_{\mu\nu}$ and $\tilde{G}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}G_{\lambda\rho}$

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a) Write down the combined QCD-QED Lagrangian (i.e. only operators up to dimension-4) and derive the equations of motion for the gluon and quark fields. Note that this is a classic problem. So, for the QED equations of motion remember your courses on electrodynamics. The QCD equations of motion are derived in an analogous way.

b) You can take the classic equations of motion for the dominant dimension-4 action to reduce the dimension-6 operators shown above to linear combinations of O_8 and O_9 and the 4-quark operators treated in class. One can in fact prove that this eliminates the 2-quark operators also at the quantum level. Take $m_s = 0$, but keep the bottom quark mass nonzero. You will find the identity $2D^\mu = \{\gamma^\mu, \not{D}\}$ quite useful. For O_7 you need the identity with three gamma matrices

$$\gamma_\alpha\gamma_\beta\gamma_\nu = g_{\alpha\beta}\gamma_\nu + g_{\beta\nu}\gamma_\alpha - g_{\alpha\nu}\gamma_\beta - i\epsilon_{\alpha\beta\nu\eta}\gamma^\eta\gamma_5\,.$$