Problems

Sheet 2, 10.11.2005

Problem 1: Feynman Rules

Determine the Feynman Rules of the following theories:

a) QCD (in R_ξ-gauge):

$$\mathcal{L}_{QCD} = -\frac{1}{4}(F^a_{\mu\nu})^2 - \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2 + \bar{\psi}(i\not{D} - m)\psi + \bar{c}^a(-\partial^\mu D^{ac}_\mu)c^c \ ,$$

where $D_\mu = \partial_\mu + igT^a A^a_\mu$ and $D^{ac}_\mu = \delta^{ac}\partial_\mu - gf^{abc}A^b_\mu$.

b) Pion Chiral perturbation theory at leading order with photon coupling:

$$\mathcal{L}_2 = \frac{F_\pi^2}{4}\,\mathrm{Tr}[(D_\mu U)(D^\mu U^\dagger)] + \frac{F_\pi^2}{4}\,\mathrm{Tr}[U^\dagger\chi + \chi^\dagger U] \ ,$$

where

$$\begin{aligned}
D_\mu U &= \partial_\mu U - (ieA_\mu Q)U + U(ieA_\mu Q) \ , \\
U &= \exp\left[\frac{i\sqrt{2}}{F_\pi}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 & \pi^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 \end{pmatrix}\right] , \\
\chi &= 2B\begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \ , \quad Q = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} \ .
\end{aligned}$$

Determine the pion propagators, the pion-photon coupling and the $\pi^+\pi^0 \to \pi^+\pi^0$ vertex.
What is the dimension of F_π?
What is the power counting of this theory?

Problem 2: Dimensional Regularization

a) Determine the UV-divergent $\frac{1}{\epsilon_{UV}}$ contribution of the integral in $d = 4 - 2\epsilon$ dimensions

$$\int \frac{d^d q}{(2\pi)^d} \frac{1}{(q^2 + i\epsilon)^2}$$

for $\epsilon \to 0$.

b) Calculate

$$B_0(\alpha, \beta, p^2) = \int \frac{d^d q}{(2\pi)^d} \frac{1}{[q^2 + i\epsilon]^\alpha [(q+p)^2 + i\epsilon]^\beta}$$

for arbitrary d.