

Problems

Sheet 2, 10.11.2005

Problem 1: Feynman Rules

Determine the Feynman Rules of the following theories:

a) QCD (in R_ξ -gauge):

$$\mathcal{L}_{QCD} = -\frac{1}{4}(F_{\mu\nu}^a)^2 - \frac{1}{2\xi}(\partial^\mu A_\mu^a)^2 + \bar{\psi}(i\not{D} - m)\psi + \bar{c}^a(-\partial^\mu D_\mu^{ac})c^c ,$$

where $D_\mu = \partial_\mu + igT^a A_\mu^a$ and $D_\mu^{ac} = \delta^{ac}\partial_\mu - gf^{abc}A_\mu^b$.

b) Pion Chiral perturbation theory at leading order with photon coupling:

$$\mathcal{L}_2 = \frac{F_\pi^2}{4} \text{Tr}[(D_\mu U)(D^\mu U^\dagger)] + \frac{F_\pi^2}{4} \text{Tr}[U^\dagger \chi + \chi^\dagger U] ,$$

where

$$D_\mu U = \partial_\mu U - (ieA_\mu Q)U + U(ieA_\mu Q) ,$$

$$U = \exp \left[\frac{i\sqrt{2}}{F_\pi} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 & \pi^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 \end{pmatrix} \right] ,$$

$$\chi = 2B \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} , \quad Q = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} .$$

Determine the pion propagators, the pion-photon coupling and the $\pi^+\pi^0 \rightarrow \pi^+\pi^0$ vertex.

What is the dimension of F_π ?

What is the power counting of this theory?

Problem 2: Dimensional Regularization

a) Determine the UV-divergent $\frac{1}{\epsilon_{UV}}$ contribution of the integral in $d = 4 - 2\epsilon$ dimensions

$$\int \frac{d^d q}{(2\pi)^d} \frac{1}{(q^2 + i\epsilon)^2}$$

for $\epsilon \rightarrow 0$.

b) Calculate

$$B_0(\alpha, \beta, p^2) = \int \frac{d^d q}{(2\pi)^d} \frac{1}{[q^2 + i\epsilon]^\alpha [(q+p)^2 + i\epsilon]^\beta}$$

for arbitrary d .