Homework 10 (June 30, 2004)

Problem 1: Anomalous dimension of c_F

Up to order $1/m_Q$ the HQET Lagrangian has the form

$$\mathcal{L} = \overline{Q}_v(iv.D)Q_v + \frac{1}{2m_Q}\overline{Q}_v(iv.D_\perp)^2 Q_v - c_F(\mu)\frac{g}{4m_Q}\overline{Q}_v\sigma^{\mu\nu}G_{\mu\nu}Q_v\,.$$

Derive the HQET Feynman rules at order $1/m_Q$ and draw the diagrams one needs to compute to determine the anomalous dimension of the coefficient c_F. Discuss at the level of the diagrams whether the $1/m_Q$ kinetic energy operator can mix with the magnetic moment operator. Write down the color structure of each of the diagrams contributing to the anomalous dimension of the c_F and argue that the anomalous dimension is proportional to $C_A = 3$. (At this point it is helpful carefully look at the diagrams and to remember current conservation in QED.) You don't have to actually compute any of the loop integrations.

Problem 2: Renormalization of the HQET at order $1/m_Q^2$

In the paper hep-ph/9708306 by Bauer and Manohar the LL renormalization of the HQET Lagrangian up to order $1/m_Q^2$ is carried out. This is actually a reading assignment which should encourage you to reproduce some of the results shown in the paper. The only tricky part of the paper is that it treats the time-ordered products of the $1/m_Q$ operators as individual operators. This is just a notational trick to formulate insertions of two $1/m_Q$ operators in the framework of a linear (!) renormalization group equation involving operators at order $1/m_Q^2$. The Wilson coefficient of such a time-ordered product is just the product of the Wilson coefficients of the operators that appear in it. You might actually appreciate how nicely this simple notational trick works in practice.

a) Find the renormalization group equation for c_F and solve it.

b) Derive the reparametrization constraint that relates the Wilson coefficients of the operators O_S with the order $1/m_Q$ magnetic moment operator.

c) Reproduce Eq.(9) which relates the Darwin operator O_D to the operator O_1^{hl} using the gluon equation of motion. This eliminates the operator O_1^{hl} from the anomalous dimension matrix shown in Eq.(14). Derive the anomalous dimension matrix for the operator basis where O_1^{hl} is eliminated. Note that in this paper the anomalous dimension matrix for the operators and not for the Wilson coefficients is shown.

d) Find the solution for c_D and c_F using the matching conditions $c_D(\mu = m_Q) = c_S(\mu = m_Q) = 1$.