Homework 9 (June 23, 2004)

Problem 1: Heavy-to-light current in HQET

Consider the following heavy-to-light matrix elements of the vector and axial vector currents (in the full theory)

$$\begin{aligned}
\langle V(p',\epsilon)|\bar{q}\gamma_\mu\gamma_5 Q|P^{(Q)}(p)\rangle &= -if^{(Q)}\epsilon^*_\mu - i(\epsilon^*.p)\,[a_+^{(Q)}(p+p_+)_\mu + a_-^{(Q)}(p-p')_\mu]\,,\\
\langle V(p',\epsilon)|\bar{q}\gamma_\mu Q|P^{(Q)}(p)\rangle &= g^{(Q)}\epsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu}(p+p')^\lambda(p-p')^\sigma\,,
\end{aligned}$$

where $p = m_{P(Q)}v$ and $|P^{(Q)}\rangle$ is the pseudo-scale heavy meson ground state. V is a light vector meson with polarization ϵ that does not contain a heavy quark (ρ, K^*, etc.) The form factors $f^{(Q)}$, $a_\pm^{(Q)}$ and $g^{(Q)}$ are functions of $y = v.p'$. Use the normalization of the heavy meson states in HQET to show that in the limit $m_{b,c}\to\infty$

$$\begin{aligned}
f^{(b)}(y) &= (m_b/m_c)^{1/2}\,f^{(c)}(y)\,,\\
g^{(b)}(y) &= (m_c/m_b)^{1/2}\,g^{(c)}(y)\,,\\
a_+^{(b)}(y)+a_-^{(b)} &= (m_c/m_b)^{3/2}\,[a_+^{(c)}(y)+a_-^{(c)}]\,,\\
a_+^{(b)}(y)-a_-^{(b)} &= (m_c/m_b)^{1/2}\,[a_+^{(c)}(y)-a_-^{(c)}]\,.
\end{aligned}$$

Make the ansatz

$$\langle V(p',\epsilon)|\bar{q}\Gamma Q|P^{(Q)}(p)\rangle = \mathrm{Tr}\Big[M_V(\epsilon^*,p',v)\Gamma\,\frac{1+\not{v}}{2}\,i\,\gamma_5\Big]$$

and use Parity invariance of the matrix element to derive the most general form of the (Dirac) matrix M_V. You need to know the parity of the meson states and how the quark fields and ϵ^*, v and p' transform under P. You can then derive a relation between the Parity-transformed M_V and the original one.

Show that heavy quark symmetry (for $m_Q\to\infty$) does not reduce the number of independent form factors present in the full theory matrix elements, but that the form factors can be related to the form factors of the tensor current

$$\begin{aligned}
\langle V(p',\epsilon)|\bar{q}\sigma_{\mu\nu}Q|P^{(Q)}(p)\rangle &= -ig_+^{(Q)}\epsilon_{\mu\nu\lambda\sigma}\,\epsilon^{*\lambda}(p+p')^\sigma - ig_-^{(Q)}\epsilon_{\mu\nu\lambda\sigma}\,\epsilon^{*\lambda}(p-p')^\sigma\\
&\quad -ih^{(Q)}\epsilon_{\mu\nu\lambda\sigma}(p+p')^\lambda(p-p')^\sigma(p+p')^\lambda(\epsilon^*.p) \qquad (1)
\end{aligned}$$