Homework 8 (June 16, 2004)

Problem 1: Reparametrization invariance in HQET

Consider the following order $1/m_Q$ heavy-to-light vector currents

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\begin{aligned}
O_1 &= \frac{1}{2m_Q}\bar{q}\gamma^\mu i\not{D} Q_v\,, \\
O_2 &= \bar{q} v^\mu i\not{D} Q_v\,, \\
O_3 &= \bar{q} iD^\mu Q_v\,, \\
O_4 &= \bar{q}(-iv.\overleftarrow{D})\gamma^\mu Q_v\,, \\
O_5 &= \bar{q}(-iv.\overleftarrow{D})v^\mu Q_v\,, \\
O_6 &= \bar{q}(-i\overleftarrow{D}^\mu) Q_v
\end{aligned}
$$

having the coefficients B_1 to B_6 respectively. Use reparametrization invariance to determine which of the coefficients are in fixed by the coefficients C_1 and C_2 of the two leading order heavy-to-light vector currents $\bar{q}\gamma^\mu Q_v$ and $\bar{q}v^\mu Q_v$. Note that the light quark and the gluon fields do not transform under reparametrization transformations because they are equivalent in each v-sector. The easiest way to proceed is to determine linear combinations of currents at leading and subleading order in $1/m_Q$ that are, up to a phase, reparametrization invariant at order $1/m_Q$. You can ignore overall phases.

Derive generic rules how the heavy quark Lagrangian $\mathcal{L}_{\rm HQET} = \sum_v \mathcal{L}_v(Q_v(x), v^\mu, iD^\mu)$ transforms under reparametrization up to order $1/m_Q$. Construct a new heavy quark field, a new velocity label and a new covariant derivative which are reparametrization invariant (up to a possible overall phase for the heavy quark field).

In HQET, write down the reparametrization invariant versions (up to order $1/m_Q$) of the five heavy quark bilinear operators (heavy-heavy currents) $\bar{Q}\Gamma Q$ ($\Gamma = \{1, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, \sigma^{\mu\nu}\}$) known from full QCD. You just have to match at leading order in $1/m_Q$ and then extend the operators by order $1/m_Q$ terms to obtain invariant combinations using the rules derived above.

Problem 2: Heavy quark limit and the decays $(D_1, D_2^*) \to (D, D^*) + \pi$

The members of the $s_l = 3/2$ doublet (D_1, D_2^*) can decay by means of a single pion emission into the two members of the $s_l = 1/2$ ground state doublet (D, D^*). The decay it governed by the strong interaction. So in addition to parity and angular momentum conservation, also heavy quark spin symmetry can be applied in the limit $m_c \to \infty$. Parity restricts the orbital angular momentum L of the emitted pion ($P(\pi) = -1$). Determine the relative amplitudes and the relative decay partial widths for the four possible decays using a Clebsch-Gordan analysis in the heavy quark limit. You need to use the Wigner-Eckard theorem and heavy quark spin symmetry to obtain the expressions for each of the possible partial waves (L).

Determine the allowed partial waves using parity and angular momentum conservation. Remember that that the various partial waves contribute incoherently to the decay rates.

The largest source of heavy quark symmetry breaking arises from the fact that the charm mass is actually not really infinite. This leads to sizeable differences in the phase spaces available for the pion for the various decay channels. The phase space of the pion for partial wave L is to a good approximation proportional to $|\mathbf{p}_\pi|^{2L+1}$ in the (D_1, D_2^*) rest frame, $\mathbf{p}_\pi$ being the pion momentum. Include the corrections arising from this effect into the relative size of the four decay rates and compare to the experimental numbers.

Compare the previous experimental numbers to those for the analogous decays of the $s_l = 1/2$ doublet (D_0^*, D_1*) into the ground state doublet plus a pion which have been seen. Explain the difference.