Homework 7 (June 9, 2004)

Problem 1: $b \to s\gamma$ and QCD equations of motion

After integrating out the top, W, Z and Higgs, a number of local operators are induced which mediate the flavor changing (electric) charge-neutral process $b \to s\gamma$. The operators that are induced include the local 4-quark operators discussed in class and the following local 2-quark operators, which we, however, did not discuss in class:

$$\begin{aligned}
O_1 &= \bar{s}_L \not{D} D_\mu D^\mu b_L \,, \\
O_2 &= \bar{s}_L D_\mu \not{D} D^\mu b_L - \frac{1}{2}\bar{s}_L D_\mu D^\mu \not{D} b_L \,, \\
O_3 &= \bar{s}_L D_\mu \not{D} D^\mu b_L \,, \\
O_4 &= \bar{s}_L \not{D}\not{D}\not{D} b_L \,, \\
O_5 &= g\bar{s}_L T^A \gamma^\mu b_L (D^\nu G_{\mu\nu})^A \,, \\
O_6 &= g G^A_{\mu\nu} \bar{s}_L T^A \gamma^\mu D^\nu b_L \,, \\
O_7 &= g \tilde{G}^A_{\mu\nu} \bar{s}_L T^A \gamma^\mu D^\nu b_L \,, \\
O_8 &= e\, m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu} \,, \\
O_9 &= g\, m_b \bar{s}_L \sigma^{\mu\nu} T^A b_R G^A_{\mu\nu} \,, \\
O_{10} &= m_b \bar{s}_L \not{D}\not{D} b_R \,.
\end{aligned}$$

Here, $D_\mu = \partial_\mu + igT^A A^A_\mu + ieQA_\mu$ is the covariant derivative (in the SU(3) fundamental representation) acting on the quark fields and $(D^\mu G_{\mu\nu})^A = (\delta^{AB}\partial^\mu + g f^{ABC} A^{\mu B}) G^C_{\mu\nu}$.

a) Write down the combined QCD-QED Lagrangian (i.e. only operators up to dimension-4) and derive the equations of motion for the gluon and quark fields. Note that this is a classic problem. So, for the QED equations of motion remember your courses on electrodynamics. The QCD equations of motion are derived in an analogous way.

b) You can take the classic equations of motion for the dominant dimension-4 action to reduce the dimension-6 operators shown above to linear combinations of O_8 and O_9 and the 4-quark operators treated in class. One can in fact prove that this eliminates the 2-quark operators also at the quantum level. Take $m_s = 0$, but keep the bottom quark mass nonzero. You will find the identity $2D^\mu = \{\gamma^\mu, \not{D}\}$ quite useful. For O_7 you need the identity with three gamma matrices

$$\gamma_\alpha \gamma_\beta \gamma_\nu = g_{\alpha\beta}\gamma_\nu + g_{\beta\nu}\gamma_\alpha - g_{\alpha\nu}\gamma_\beta - i\epsilon_{\alpha\beta\nu\eta}\gamma^\eta \gamma_5 \,.$$

Problem 2: Heavy antiquarks in HQET

In the $m_Q \to \infty$ limit, show that the propagator for a heavy antiquark with momentum $p_{\bar{Q}} = m_Q v + k$ is

$$\frac{i}{v.k + i\epsilon}\left(\frac{1 - \not{v}}{2}\right)$$

while the heavy antiquark-gluon vertex is

$$ig(T^A)^T v_\mu \, .$$

Note that one wants to formulate the effective theory for antiparticles having positive energy. So, particle and antiparticle dynamics are decoupled and there is an independent fermion flow for particles and antiparticles.