Homework 2 (April 28, 2004)

Problem 1

The QED Lagrangian in R_ξ-gauge has the form

$$\mathcal{L}_{\rm QED} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)(\partial_\nu A^\nu) + \bar{\Psi}(i\not{D} - m)\Psi\,,$$

where the field strength and the covariant derivative have the form $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ and $D^\mu = \partial^\mu + ieQ_f A^\mu$, respectively. Derive the Feynman rules.

Problem 2: Right-handed neutrios

Add three right-handed neutrinos N_R^i to the Standard Model. Then a Majorana-type mass term is allowed of the form

$$\mathcal{L}_M = \bar{N}_R\, i\not{\partial} N_R - \frac{1}{2}\bar{N}_R^c M N_R - \frac{1}{2}\bar{N}_R M^* N_R^c\,,$$

where M is a complex symmetric matrix.

a) Show that possible antisymmetric contributions in M do not contribute.

b) Write down the most general dimension-4 operators that couple N_R to Standard Model fields. Use the gauge symmetries.

c) Transform all operators with N_R to Majorana mass eigenstates with $N_i = N_i^c$. Use the theorem that for any complex, symmetric matrix M there is a unitary matrix X ($X^\dagger X = 1$) such that $M = XDX^T$ where D is a diagonal matrix with real non-negative entries. How many free parameters exist now in the lepton sector? Is there CP-violation in the lepton sector?

d) Take the masses M_i large with respect to the electroweak scale. Integrate out the right-handed neutrinos at tree level using the equations of motion for the Majorana fields. Note that N and $\bar{N}$ for Majorana fermions are not independent! Show that in the limit of large M_i the leading term of the result has the form of the dimension-5 operator built from Standard Model fields discussed in the lectures.