Homework 1 (April 19, 2004)

Problem 1

a) Determine the muon decay rate $\mu \to e^- \bar{\nu}_e \nu_\mu$ in the limit $M_w >> m_\mu >> m_e = 0$ in the Born approximation. Express the result in terms of the Fermi constant G_F. The result is proportional to a particular power of m_μ. Can one guess this scaling behavior without explicit computation?

b) Apply the results to the inclusive semileptonic decay rate of a B meson in the Standard Model ($B \to e^- \bar{\nu}_e + \text{anything}$). The decay of the B meson is very well approximated by the decay of a free bottom quark into another quark and $e^- \bar{\nu}_e$. Note that there is more than one possible quark flavor in the final state.

Problem 2: Leptonic B decays in a top-less model

This is ancient history, but still interesting as an exercise. Assume that there is no top quark in the Standard Model and that the left-handed bottom quark is a SU(2) gauge singlet. Leave the particle content unchanged otherwise.

a) Assign the hypercharge of the left-handed bottom quark and write down the most general mass terms the quarks can have for the gauge eigenstates. Determine the charged- and neutral-current couplings of the quarks to the gauge Bosons $W^\pm$, Z^0 and A.

b) Change basis to the mass eigenstates and think about how the charged- and neutral-current interactions get modified. What happens to the interactions with the photon? Use the result to show that there are FCNC's mediated by the Z^0 boson involving left-handed fields. Assume that in the basis of mass eigenstates the left-handed quark doublets have the form

$$U(u,L) \begin{pmatrix} u' \\ d'' \end{pmatrix}, \qquad U(c,L) \begin{pmatrix} c' \\ s'' \end{pmatrix},$$

with the ansatz $d'' = c_1 c_2 d' + c_1 s_2 s' + s_1 b'$, $s'' = c_3 c_{2+4} d' + c_3 s_{2+4} s' + s_3 b'$, and where $s_1 = \sin(\theta_1), \ldots$, $s_{2+4} = \sin(\theta_2 + \theta_4)$ and $U(u,L), \ldots$ are unitary matrices. Determine the couplings to $W^\pm$ and Z^0.

c) Compute the ratio of partial widths

$$\frac{\Gamma(B \to l^+ l^- + \text{anything})}{\Gamma(B \to l^+ \nu + \text{anything})}$$

at the Born level just in terms of the coupling constants ignoring possible differences in the phase space for the decays, but including the overall dependence on the W and Z boson masses. (For the Z-boson exchange there are also couplings involving right-handed lepton fields. For deriving the ratio you can treat them as being left-handed, keeping your fingers crossed that this gives the correct result. If you are ambitious, you can check.) Assume that

the rates are given by the decay of a free b quark.

d) Estimate the ratio using the data for $|V_{ud}|$, $|V_{us}|$, $|V_{cd}|$, $|V_{cs}|$ and the number for the weak mixing angle. You may in fact use the orthogonality of the d'' and s'' states first and use then contraints from the CKM elements above. Compare your result with numbers you might find in the PDG (http://pdg.lbl.gov/). Is there a chance?