Renormalisation of unstable particles in quantum field theories

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Introduction: the Quantum Universe

Particle Physics
Experiments
Accelerators
Underground

Quantum Field Theory
(Standard Model)

Astronomy Experiments
Telescopes
Satellites

Standard Cosmology Model

$10^{-18}$ m

$10^{26}$ m
What is the quantum structure of the vacuum?

• The recent discovery of a Higgs boson hints at a non-trivial structure of the vacuum, i.e. of the lowest-energy state in our universe.

• The discovered particle provides access to studying the quantum structure of the vacuum!

• How can a Higgs boson be as light as 125 GeV?
  • A new symmetry of nature → Supersymmetry?
  • A new fundamental interaction of nature → composite Higgs?
  • Extra dimensions of space → impact on gravity on small scales?
  • Multiverses → anthropic principle?
Description of fundamental interactions via quantum field theories

External particles: “on-shell”, fulfill relativistic relation between energy, 3-momentum and mass of a free particle

\[ p^2 \equiv p^\mu p_\mu = E^2 - |\vec{p}|^2 = m^2 \]  

[units: \( \hbar = c = 1 \)]

Internal particles: off-shell, \( p^2 \neq m^2 \), described by propagator

\[ \Delta(p) \sim \frac{i}{p^2 - m^2} \quad \text{(in lowest order)} \]
Perturbative evaluation of quantum field theories (gauge theories)

Expansion in coupling constant: \( \alpha \approx \frac{1}{137} \ll 1 \)

\( \Leftrightarrow \) expansion about theory without interaction

lowest order, classical limit

quantum corrections: loop diagrams

\( \mathcal{O}(\alpha) \) relative to lowest order
What can one learn from quantum corrections?

- Inclusion of quantum effects $\Leftrightarrow$ more accurate theoretical predictions

Large loop corrections:
- QCD corrections are often of $\mathcal{O}(100\%)$
- EW enhancement factors: $m_t^2$, $m_t^4$, $\ldots$, large logarithms (involving two very different scales)
- Per mille level corrections needed to match EW precision measurements

- Quantum effects provide sensitivity to the underlying structure of the theory
Electroweak precision physics: high-precision data vs. theory predictions

**EW precision data:**

\[ M_Z, M_W, \sin^2 \theta_{\text{eff}}^{\text{lept}}, \ldots \]

**Theory:**

SM, MSSM, \ldots

↓

Test of theory at quantum level: sensitivity to loop corrections

↓

Indirect constraints on unknown parameters: \( M_H, \ldots \)

Effects of “new physics”?
High-precision physics

1978 Precise measurement of $\sin \theta_W$ @ SLAC via polarized electrons $e^- D \rightarrow e^- X$
→ Prediction of $W$ and $Z$ mass

1983 Discovery of $W$ and $Z$ bosons at SppS

1989- Precise measurement of $W$ and $Z$ @ SLC/LEP → Prediction of top mass

1995 Discovery of top quark at Tevatron

Precise measurement of $W$, $Z$, top @ SLC/LEP/Tevatron → Prediction of Higgs mass

2012 Discovery of Higgs boson

Precise measurement of $W$, $Z$, top, Higgs @ LHC/ILC → Prediction of ???

Planck
Theoretical foundation: renormalisability of gauge theories with spontaneous symmetry breaking

Standard Model Lagrangian as an example:

$$\mathcal{L}_{\text{EW}} (g_2, g_1, v, \lambda, g_f) + \mathcal{L}_{\text{QCD}} (\alpha_s)$$

$$M_W, M_Z, \alpha, M_H, m_f$$

Gauge invariance $\Rightarrow$ theory is renormalisable

[G. ’t Hooft ’71] [G. ’t Hooft, M. Veltman ’72] Nobel prize ’99

$\Rightarrow$ theory can consistently been treated as a quantised field theory:

$\Rightarrow$ quantum effects can be evaluated

For non-renormalisable theory: need additional parameters in each loop order to compensate divergencies
Quantum corrections: regularisation and renormalisation (with a broad brush)

\[ p \rightarrow \infty : \quad \sim \int_0^\infty \frac{q^3 dq}{q^4} = \int_0^\infty \frac{dq}{q} \rightarrow \infty \]

⇒ integral diverges for large \( q \)!

⇒ theory in this form not physically meaningful

⇒ further concept needed: renormalisation

Renormalisable theories: infinities can consistently be absorbed into parameters of theory
Two step procedure: regularisation

Regularisation:

theory modified such that expressions become mathematically meaningful

⇒ “regulator” introduced, removed at the end

e.g. cut-off in loop integral

\[
\int_0^\infty d^4q \rightarrow \int_0^\Lambda d^4q; \quad \Lambda \rightarrow \infty \text{ at the end}
\]

technically more convenient: dimensional regularisation

\[
\int d^4q \rightarrow \int d^D q, \quad D = 4 - \varepsilon; \quad D \rightarrow 4 \text{ at the end}
\]
Two step procedure: renormalisation

Renormalisation:
original “bare” parameters replaced by renormalised parameters + counterterms

reparametrization:
\[ g_0 = g + \delta g \]

bare parameter \quad renormalised parameter \quad counterterm

Renormalisable theory:
divergences compensated by counterterms
Two aspects of renormalisation

- Absorption of divergences

- Determination of physical meaning of parameters order by order in perturbation theory
Higher-order contributions to the propagator

\[
\frac{i}{p^2 - m^2} + \frac{i}{p^2 - m^2} + i \Sigma(p^2) + \frac{i}{p^2 - m^2} + i \Sigma(p^2) + \frac{i}{p^2 - m^2} + \frac{i}{p^2 - m^2} + \cdots
\]

\[
= \frac{i}{p^2 - m^2 + \Sigma(p^2)}
\]

⇒ Pole of the propagator: \( M^2 \)

Renormalised self-energy (denoted below by \( \hat{\Sigma} \))

\[
M^2 - m^2 + \Sigma(M^2) = 0
\]

For a stable particle: \( \Sigma(M^2) \) is real

If \( \Sigma(M^2) \neq 0 \) ⇒ Pole shifted by higher-order contributions
Example: mass renormalisation (stable particle)

Renormalisation of the mass parameter: $m_0^2 = m^2 + \delta m^2$

Physical mass: pole of propagator

inverse propagator up to 1-loop order:

\[
p^2 - m^2 \quad + \quad \Sigma(p^2) \quad + \quad -\delta m^2 + (p^2 - m^2)\delta Z_\Phi \quad + \quad \cdots
\]

Pole of the propagator: $p^2 - m^2 + \Sigma(p^2) -\delta m^2 + (p^2 - m^2)\delta Z_\Phi = 0$
On-shell and MS renormalisation

“On-shell renormalisation”: \[ \delta m^2 = \Sigma(p^2) \bigg|_{p^2=m^2} \]

⇒ pole of propagator for \( p^2 = m^2 \) ⇒ \( m \): “pole mass”

Counterterm contains divergence: expansion in \( \varepsilon \equiv 4 - D \), 1-loop:

\[
\delta m^2 = a \frac{1}{\varepsilon} + b \varepsilon^0 + c \varepsilon + \ldots
\]

divergent \hspace{1cm} \text{finite} \hspace{1cm} \rightarrow 0

for \( D \rightarrow 4 \) \hspace{1cm} \text{for} \ D \rightarrow 4

Other renormalisation prescription:

\[
\delta m^2 = a \frac{1}{\varepsilon} \quad \text{“minimal subtraction” (MS)}
\]
On-shell and MS renormalisation

Slight variant of MS renormalisation: “modified minimal subtraction”, $\overline{\text{MS}}$

$\overline{\text{MS}}$ and MS quantities depend on renormalisation scale $\mu$ (needs to be introduced for dimensional reasons)

$\overline{\text{MS}}$ top quark mass: $m_t(\mu)$, “running mass”

The difference between the pole mass and $m_t(m_t)$ from QCD corrections amounts to about 10 GeV!

The strong coupling is usually given as $\overline{\text{MS}}$ quantity: $\alpha_s(\mu)$, “running coupling”
Zur Formulierung quantisierter Feldtheorien.

H. Lehmann, K. Symanzik und W. Zimmermann

Max-Planck-Institut für Physik - Göttingen (Deutschland)

(ricevuto il 22 Novembre 1954)

Summary. --- A new formulation of quantized field theories is proposed. Starting from some general requirements we derive a set of equations which determine the matrix-elements of field operators and the S-Matrix. These equations contain no renormalization constants, but only experimental masses and coupling parameters. The main advantage over the conventional formulation is thus the elimination of all divergent terms in the basic equations. This means that no renormalization problem arises. The formulation is here restricted to theories which do not involve stable bound states. For simplicity we derive the equations for spin 0 particles, however the extension to other cases (e.g. quantum electrodynamics) is obvious. The solutions of the equations are discussed in a power-series expansion. They are then identical with the renormalized expressions of the conventional formulation. However, the equations set up here are not restricted to the application of perturbation theory.
On the Formulation of Quantized Field Theories - II.

H. Lehmann and K. Symanzik
Institut für Theoretische Physik der Universität - Hamburg

W. Zimmermann
Max-Planck-Institut für Physik - Göttingen

(ricevuto il 9 Maggio 1957)

Summary. — We discuss the concept of causal scattering matrices using retarded multiple commutators of field operators.
BPHZ renormalisation and LSZ formalism

Bogoliubov-Parasiuk-Hepp-Zimmermann renormalization scheme


Dr. Klaus Sibold, Institut für Theoretische Physik Fakultät für Physik und Geowissenschaften Universität Leipzig

The Bogoliubov, Parasiuk, Hepp, Zimmermann (abbreviated BPHZ) renormalization scheme is a mathematically consistent method of rendering Feynman amplitudes finite while maintaining the fundamental postulates of relativistic quantum field theory (Lorentz invariance, unitarity, causality). Technically it is based on the systematic subtraction of momentum space integrals. This distinguishes it from other methods of renormalization. For massless particles the scheme has been enlarged by Lowenstein and is then called BPHZL.

In quantum field theory, the LSZ reduction formula is a method to calculate S-matrix elements (the scattering amplitudes) from the time-ordered correlation functions of a quantum field theory. It is a step of the path that starts from the Lagrangian of some quantum field theory and leads to prediction of measurable quantities. It is named after the three German physicists Harry Lehmann, Kurt Symanzik and Wolfhart Zimmermann.
Higgs and Top mass from Reduction of couplings

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We reduce the couplings in the standard model with one Higgs doublet and \( n \) generations and obtain for three generations 61 GeV and 81 GeV for the mass of the Higgs particle and the top quark respectively. The error is estimated to be about 10–15%.

Did not quite work for the absolute mass values, but the ratio is remarkably close!
Asymptotic in- and out-states

QFT is formulated in terms of asymptotic in- and -out states for \( t \to \pm \infty \): stable particles

LSZ formalism: correct normalisation of S-matrix elements; external particles on-shell + wave-function normalisation factors

Unstable particles:

• Cannot be expressed in terms of asymptotic states

• Decay of an unstable particle: non-zero total width
The problem of unstable particles in QFTs

LHC: production of Higgs, top, W, Z, … ; precise predictions needed for production and decay of unstable particles

One could in principle treat the full process of production and decay and have only stable particles (e, γ, g, u, d, …) as external states: technically inconvenient, difficult to obtain higher-order corrections for $2 \rightarrow n$ process if $n$ is large

But: how to treat the resonance region?
Lowest-order propagator would diverge on-shell, $p^2 \rightarrow m^2$

$$\frac{i}{p^2 - m^2} \rightarrow \infty \text{ for } p^2 \rightarrow m^2$$

Need to introduce width of the unstable particle into the propagator, mixes orders of perturbation theory
Treatment of unstable particles

Very important for practical applications

Long-standing problem (gauge invariance, unitarity, …), still not rigorously solved in QFTs

Some examples:

• Mass and field renormalisation of unstable particles

• Wave function normalisation factors for unstable particles

• Resonance and interference effects
What is the mass of an unstable particle?

Particle masses are not directly physical observables
Can only measure cross sections, branching ratios, kinematical distributions, . . .
⇒ masses are “pseudo-observables”

Need to define what is meant by $M_Z$, $M_W$, $m_t$, . . .:
$\overline{\text{MS}}$ mass, pole mass (real pole, real part of complex pole, Breit–Wigner shape with running or constant width), . . .

⇒ Determination of $M_Z$, $M_W$, $m_t$, . . . involves deconvolution procedure (unfolding)
Mass obtained from comparison data – Monte Carlo
⇒ $M_Z$, $M_W$, $m_t$, . . . are not strictly model-independent
Mass of an unstable (elementary) particle

For an unstable particle:

\[ \Sigma(M^2) \text{ is complex } \Rightarrow \text{ Pole in the complex plane} \]

\[ M^2 - m^2 + \Sigma(M^2) = 0, \quad M^2 = M^2 - iM \Gamma \]

\[ M: \text{ physical mass, } \Gamma: \text{ decay width of the unstable particle} \]

\[ \Rightarrow \text{ The mass of an unstable (elementary) particle is defined according to the real part of the complex pole} \]

Example:

resonant production of the Z boson and its decay (point-like particle!)

\[ e^+e^- \rightarrow \text{hadrons} \]
Expansion around the complex pole for a single resonance

\[ p^2 - m^2 + \hat{\Sigma}(p^2) = (p^2 - M^2) \left\{ 1 + \frac{d \hat{\Sigma}}{dp^2} \right\} \bigg|_{p^2 = M^2} + \ldots \]

\[ p^2 = M^2 \]

→ Breit-Wigner factor with fixed width

→ Field renormalisation and wave function normalisation factor of unstable particle

Note:

Wave-function normalisation factor needs to be evaluated at the complex pole

One-loop field renormalisation:

\[ \delta Z^{(1)} = - \frac{\partial \Sigma(p^2)}{\partial p^2} \bigg|_{p^2 = m^2} \]

Complex quantity, no restriction to Re(…)

Expansion around the complex pole (example: $M_Z$)

Expansion of amplitude around complex pole:

$$A(e^+e^- \to f\bar{f}) = \frac{R}{s - M_Z^2} + S + (s - M_Z^2)S' + \cdots$$

$$M_Z^2 = \overline{M}_Z^2 - i\overline{M}_Z\Gamma_Z$$

Expanding up to $O(\alpha^2)$ using $O(\Gamma_Z/M_Z) = O(\alpha)$

From 2-loop order on:

real part of complex pole, $\overline{M}_Z \neq$ pole of real part, $\tilde{M}_Z^2$

$$\delta \overline{M}_Z^2 = \delta \tilde{M}_Z^2 + \text{Im} \left\{ \Sigma_{T,(1)}(M^2) \right\} \text{Im} \left\{ \Sigma_{T,(1)}(M^2) \right\}$$

gauge-parameter dependent!
Physical mass of unstable particles: real part of complex pole

⇒ Only the complex pole is gauge-invariant

Expansion around the complex pole leads to a Breit–Wigner shape with constant width

For historical reasons, the experimental values of $M_Z$, $M_W$ are defined according to a Breit–Wigner shape with running width

⇒ Need to correct for the difference in definition when comparing theory with experiment
Masses and wave function normalisation factors for mixed states

Extended Higgs sectors: several Higgs states which can mix with each other

Higgs-mass prediction from propagator matrix

Example:

MSSM with complex parameters; two Higgs doublets, three neutral states

Lowest order: $h, H$ (CP-even) and $A$ (CP-odd)
CP violation induced by loop corrections:
Mixing of $h, H, A \rightarrow$ mass eigenstates $h_1, h_2, h_3$
Higgs physics in the MSSM with complex parameters

Five physical states; tree level: \( h^0, H^0, A^0, H^\pm \)

Complex parameters enter via (often large) loop corrections:

- \( \mu \): Higgsino mass parameter
- \( A_{t,b,\tau} \): trilinear couplings
- \( M_{1,2} \): gaugino mass parameter (one phase can be eliminated)
- \( M_3 \): gluino mass \( m_{\tilde{g}} \) + complex phase

\( \Rightarrow \) \( CP \)-violating mixing between neutral Higgs bosons \( h_1, h_2, h_3 \)

Lowest-order Higgs sector has two free parameters

\( \Rightarrow \) choose \( \tan \beta \equiv \frac{v_2}{v_1} \), \( M_{H^\pm} \) as input parameters
**CP-violating Higgs phenomenology: mixing between all three neutral Higgs bosons**

Mixing between $h, H, A$  

⇒ loop-corrected masses obtained from propagator matrix  

$$ \Delta_{hHA}(p^2) = - \left( \hat{\Gamma}_{hHA}(p^2) \right)^{-1}, \quad \hat{\Gamma}_{hHA}(p^2) = i \left[ p^2 \mathbb{1} - M_n(p^2) \right] $$

where (up to sub-leading two-loop corrections)

$$ M_n(p^2) = \begin{pmatrix} m_h^2 - \hat{\Sigma}_{hh}(p^2) & -\hat{\Sigma}_{hH}(p^2) & -\hat{\Sigma}_{hA}(p^2) \\ -\hat{\Sigma}_{hH}(p^2) & m_H^2 - \hat{\Sigma}_{HH}(p^2) & -\hat{\Sigma}_{HA}(p^2) \\ -\hat{\Sigma}_{hA}(p^2) & -\hat{\Sigma}_{HA}(p^2) & m_A^2 - \hat{\Sigma}_{AA}(p^2) \end{pmatrix} $$

⇒ Higgs propagators:

$$ \Delta_{ii}(p^2) = i \frac{1}{p^2 - m_i^2 + \hat{\Sigma}_{\text{eff}}^{ii}(p^2)} $$
Determination of the Higgs masses from the complex poles

\[ \hat{\Sigma}_{ii}^{\text{eff}}(p^2) = \hat{\Sigma}_{ii}(p^2) - i \frac{2\hat{\Gamma}_{ij}(p^2)\hat{\Gamma}_{jk}(p^2)\hat{\Gamma}_{ki}(p^2) - \hat{\Gamma}_{kj}^2(p^2)\hat{\Gamma}_{jj}(p^2) - \hat{\Gamma}_{ij}^2(p^2)\hat{\Gamma}_{kk}(p^2)}{\hat{\Gamma}_{jj}(p^2)\hat{\Gamma}_{kk}(p^2) - \hat{\Gamma}_{jk}^2(p^2)} \]

Complex pole \( \mathcal{M}_i^2 \) of each propagator is determined from

\[ \mathcal{M}_i^2 - m_i^2 + \hat{\Sigma}_{ii}^{\text{eff}}(\mathcal{M}_i^2) = 0, \]

where

\[ \mathcal{M}^2 = M^2 - iM\Gamma, \]

Expansion around the real part of the complex pole:

\[ \hat{\Sigma}_{jk}(\mathcal{M}_{ha}^2) \approx \hat{\Sigma}_{jk}(M_{ha}^2) + i \text{Im} \left[ \mathcal{M}_{ha}^2 \right] \hat{\Sigma}'_{jk}(M_{ha}^2) \]

\( j, k = h, H, A, a = 1, 2, 3 \)
Propagator matrix

\[ \Delta_{hHA} = \begin{pmatrix} \Delta_{hh} & \Delta_{hH} & \Delta_{hA} \\ \Delta_{Hh} & \Delta_{HH} & \Delta_{HA} \\ \Delta_{Ah} & \Delta_{AH} & \Delta_{AA} \end{pmatrix} \]

Without mixing: \( \Delta_{hh}, \ldots \) has a single pole

For \((n \times n)\) mixing: each of the entries \( \Delta_{hh}, \ldots \) in general has \(n\) poles
Finite wave function normalisation factors for amplitudes with external Higgs bosons

Finite wave-function normalisation factors ensure the correct on-shell properties of the S matrix

\[
Z_h = \left. \frac{1}{\partial p^2 \left( \frac{i}{\Delta_{hh}(p^2)} \right)} \right|_{p^2 = M^2_{ha}}
\]

\[
Z_A = \left. \frac{1}{\partial p^2 \left( \frac{i}{\Delta_{AA}(p^2)} \right)} \right|_{p^2 = M^2_{hc}}
\]

\[
Z_{hh} = \left. \frac{\Delta_{hh}}{\Delta_{hh}} \right|_{p^2 = M^2_{ha}}
\]

\[
Z_{AH} = \left. \frac{\Delta_{HA}}{\Delta_{HH}} \right|_{p^2 = M^2_{hc}}
\]

\[
Z_{Hh} = \left. \frac{\Delta_{hh}}{\Delta_{HH}} \right|_{p^2 = M^2_{hb}}
\]

\[
Z_{AH} = \left. \frac{\Delta_{HA}}{\Delta_{AA}} \right|_{p^2 = M^2_{hc}}
\]

Complex quantities, evaluated at complex pole
Finite wave function normalisation factors for amplitudes with external Higgs bosons

WF constants can be written as (non-unitary) matrix $\hat{Z}$,

$$\hat{Z} = \begin{pmatrix} \sqrt{Z_h} & \sqrt{Z_h} Z_{hH} & \sqrt{Z_h} Z_{hA} \\ \sqrt{Z_H} Z_{Hh} & \sqrt{Z_H} & \sqrt{Z_H} Z_{HA} \\ \sqrt{Z_A} Z_{Ah} & \sqrt{Z_A} Z_{AH} & \sqrt{Z_A} \end{pmatrix}, \quad \begin{pmatrix} \hat{\Gamma}_{h_a} \\ \hat{\Gamma}_{h_b} \\ \hat{\Gamma}_{h_c} \end{pmatrix} = \hat{Z} \cdot \begin{pmatrix} \hat{\Gamma}_h \\ \hat{\Gamma}_H \\ \hat{\Gamma}_A \end{pmatrix}$$

Fulfills the conditions

Every assignment between $h_a$, $h_b$, $h_c$, and $h$, $H$, $A$ is possible!

Unit residues

$$\lim_{p^2 \to M_{h_a}^2} \frac{i}{p^2 - M_{h_a}^2} \left( \hat{Z} \cdot \hat{\Gamma}_2 \cdot \hat{Z}^T \right)_{hh} = 1$$

Off-diagonal contributions vanish on-shell

$$\lim_{p^2 \to M_{h_b}^2} \frac{i}{p^2 - M_{h_b}^2} \left( \hat{Z} \cdot \hat{\Gamma}_2 \cdot \hat{Z}^T \right)_{HH} = 1$$

$$\lim_{p^2 \to M_{h_c}^2} \frac{i}{p^2 - M_{h_c}^2} \left( \hat{Z} \cdot \hat{\Gamma}_2 \cdot \hat{Z}^T \right)_{AA} = 1$$
Finite wave function normalisation factors for amplitudes with external Higgs bosons

\[
p^2 = M^2_a \quad \left< \tilde{\Gamma}_{h_a} \right> = \sqrt{Z_a} \begin{pmatrix}
  h_a & h \\
  \hat{Z}_{ah} & \hat{Z}_{ah}
\end{pmatrix} \begin{pmatrix}
  h_a & H \\
  \hat{Z}_{ah} & \hat{Z}_{ah}
\end{pmatrix} \begin{pmatrix}
  h_a & A \\
  \hat{Z}_{ah} & \hat{Z}_{ah}
\end{pmatrix}
\]

\[
+ \ldots
\]

Loop-corrected mass eigenstate

Lowest-order states

⇒ Mixing effects taken into account (via non-unitary matrix)

Correct normalisation of the S matrix
Application of wave function normalisation factors for internal particles

Expansion of the full propagator around a complex pole: dominant contribution from Breit-Wigner factor,

$$\Delta_{a}^{BW}(p^2) := \frac{i}{p^2 - M_{a}^2} = \frac{i}{p^2 - M_{h_{a}}^2 + iM_{h_{a}}\Gamma_{h_{a}}}$$

times wave function normalisation factors:

$$\Delta_{ij}(p^2) \approx \sum_{a=1}^{3} \hat{Z}_{ai} \Delta_{a}^{BW}(p^2) \hat{Z}_{aj}$$

⇒ Approximation of the full off-shell propagator in terms of the on-shell contributions of all complex poles
Application of the propagator approximation

\[ \hat{\Gamma}_h^X h_a \Gamma_{h_a} \]

\[ = \hat{\Gamma}_h^X h_a \Gamma_{h_a} + \hat{\Gamma}_h^X h_a \Gamma_{h_a} + \hat{\Gamma}_h^X h_a \Gamma_{h_a} + \hat{\Gamma}_h^X h_a \Gamma_{h_a} \]

\[ + \sum_{i,j=h,H,A} \hat{\Gamma}_i^X h_a \Gamma_{h_a} \]

\[ \Rightarrow \text{Expression of the full process in terms of the on-shell production and decay of the intermediate states} \]

\[ \Rightarrow \text{Convenient incorporation of higher-order contributions, mixing and interference effects} \]
Propagator approximation vs. full result

\[ \hat{\sigma}(b\bar{b} \rightarrow \tau^+ \tau^-) \text{ with propagator mixing} \]

\[ \Delta_{ij} \ 3 \times 3 \]
\[ \Delta_{BW} \cdot Z \]
\[ \Delta_{BW} \cdot Z \text{ no Int} \]
\[ \Delta_{BW} \cdot Z \text{ h} \]
\[ \Delta_{BW} \cdot Z \text{ H} \]
\[ \Delta_{BW} \cdot Z \text{ A} \]

\[ M_h = 126.20 \ \text{GeV} \]
\[ M_H = 127.55 \ \text{GeV} \]
\[ \Gamma_h = 0.94 \ \text{GeV} \]
\[ \Gamma_H = 1.21 \ \text{GeV} \]

\[ \Rightarrow \text{Very good agreement of propagator approximation with full result} \]

Incorporation of interference effects is crucial.
Phenomenology of extended Higgs sectors

Most obvious possibility: SM-like state at 125 GeV, corresponds to the lightest state of an extended Higgs sector, other Higgs states are heavy (``decoupling region’’)

Heavy additional Higgs states ⇒ decoupling behaviour

Interference effects can be important for heavy Higgs phenomenology

Examples:

• Interference effects of the heavy Higgs signal with the background and with the state at 125 GeV

• MSSM with CP-violation: $h_2$, $h_3$ are typically nearly mass-degenerate, have a large mixing; resonance-type behaviour
Sensitivity to the small signal of an additional heavy Higgs boson in a Two-Higgs-Doublet model (2HDM)

\[ g_{VV} = \sin(\beta - \alpha) g_{\text{SM}}^{H\nu \bar{\nu}}, \quad g_{VV} = \cos(\beta - \alpha) g_{\text{SM}}^{H\nu \bar{\nu}}, \quad V = W^\pm, Z \]

\[ e^+ e^- \rightarrow \nu \bar{\nu} u\bar{d}d, \quad \sqrt{s} = 1 \text{ TeV} \]
\[ \text{Pol}(e^+, e^-) = (0.3, -0.8) \]
\[ 2\text{HDM}, \quad s_{\beta-\alpha} = 0.95 \]
\[ m_h = 125 \text{ GeV}, \quad m_H = 400 \text{ GeV} \]
\[ \int \mathcal{L} dt = 500 \text{ fb}^{-1} \]

⇒ ILC: Potential sensitivity beyond the kinematic reach of Higgs pair production
Higgs mixing: possible interference effects

Total cross section:

\[ \sigma_{\text{tot}} = \sigma(b\bar{b}H) + \sigma(b\bar{b}A) \] (incoherent sum)

holds only in the \( CP \)-conserving case

**But:** in reality we don’t know whether \( CP \) in the Higgs sector is conserved or not

In the general case:

Complex parameters \( \Rightarrow \) loop corrections induce \( CP \)-violation

Two Higgs states, nearly mass degenerate, large mixing

\( \Rightarrow \) Large (destructive) interference possible
Higgs production via gluon fusion in the MSSM with CP-violation: extension of the SusHi code

$$gg \rightarrow h_2 h_3$$, dependence on phase $\phi_{At}$:

[S. Liebler, S. Patel, G. W. ’16]

Only production process shown here

$\Rightarrow$ Full result for $\sigma \times \text{BR}$ needs to incorporate interference contribution
Incorporation of interference contributions

\[
\left| \sum_{a=1}^{3} \frac{b}{b} \right|^{2} \cdot \left| \sum_{a=1}^{3} \frac{g}{g} \right|^{2}
\]

+ higher-order contributions

for bb associated production and gluon fusion process with subsequent decay into \( \tau^+ \tau^- \)
Cross sections with and without interference contributions vs. experimental limits

[Figure 5: Comparison of predicted Higgs cross sections times branching ratio into $\tau^+\tau^-$ with and without the interference and with $2\times2$ and $3\times3$ mixing, for a fixed value of $\tan\beta$, to experimental exclusion bounds. Left column (a, c): production via $b\bar{b}$. Right column (b, d): production via $gg$. Upper row (a, b): strongest interference effect at $\tan\beta = 29$. Lower row: (c) $\tan\beta = 25$, (d) $\tan\beta = 19$. Each plot shows the CMS observed (black, solid) and expected (black, dotted) exclusion bounds at 95% CL at 8 TeV with $R_L = 24.6 \text{ fb}^{-1}$.]

$\Rightarrow$ CP-violating mixing induces resonance-type enhancement + large destructive interference contributions

Renormalisation of unstable particles in quantum field theories, Georg Weiglein, W. Zimmermann Memorial Symposium, MPI Munich, 05 / 2017
Search for heavy Higgs bosons at the LHC: impact of interference effects

Exclusion limits from neutral Higgs searches in the MSSM with and without interference effects:

- CP-violating case, $\phi_{A_t} = \pi/4$
- H, A are nearly mass degenerate: large mixing possible in CP-violating case!
- Incoherent sum is not sufficient!

$\Rightarrow$ Large CP-violating interference effects possible
Field renormalisation for unstable particles + complex parameters: MSSM with complex parameters

Occurrence of imaginary parts:

- From complex parameters
- From absorptive parts of loop integrals
  $\leftrightarrow$ unstable particles

$\Rightarrow$ MSSM with complex parameters:
absorptive parts of loop integrals can contribute to real part of 1-loop quantities

$\Rightarrow$ Consistent renormalisation procedure needed for complex parameters and unstable particles

[A. Bharucha, A. Fowler, G. Moortgat-Pick, G. W. ’12]
[A. Fowler, G. W. ’09]
Renormalisation of the chargino / neutralino sector with complex parameters

Allow field renormalisations to be different for in- and outgoing fermions:

\[
\begin{align*}
\omega_L \tilde{\chi}_i^- &\to (1 + \frac{1}{2} \delta Z^L_{ij}) \omega_L \tilde{\chi}_j^- , \\
\tilde{\chi}_i^- \omega_R &\to \tilde{\chi}_i^- (1 + \frac{1}{2} \delta \bar{Z}^L_{ij}) \omega_R , \\
\omega_R \tilde{\chi}_i^- &\to (1 + \frac{1}{2} \delta Z^R_{ij}) \omega_R \tilde{\chi}_j^- , \\
\tilde{\chi}_i^- \omega_L &\to \tilde{\chi}_i^- (1 + \frac{1}{2} \delta \bar{Z}^R_{ij}) \omega_L ,
\end{align*}
\]

In \(\mathcal{CP}\)-conserving case: can choose a scheme where hermiticity relation holds (up to purely imaginary terms that do not contribute to squared matrix elements at 1-loop)

\[
\delta \bar{Z}_{ij} = \delta Z^\dagger_{ij}
\]

Decomposition of fermion self-energies:

\[
\Sigma_{ij}(p^2) = \not{p} \omega_L \Sigma^L_{ij}(p^2) + \not{p} \omega_R \Sigma^R_{ij}(p^2) + \omega_L \Sigma^{SL}_{ij}(p^2) + \omega_R \Sigma^{SR}_{ij}(p^2)
\]

Renormalisation of unstable particles in quantum field theories, Georg Weiglein, W. Zimmermann Memorial Symposium, MPI Munich, 05 / 2017
Field renormalisation conditions (1-loop)

Vanishing of off-diagonal contributions and unit residues:

\[
\hat{\Gamma}^{(2)}_{i,j}\tilde{\chi}_i(p)|_{p^2=m^2_{\tilde{\chi}_j}} = 0, \quad \bar{\tilde{\chi}}_i(p)\hat{\Gamma}^{(2)}_{i,j}|_{p^2=m^2_{\tilde{\chi}_i}} = 0
\]

\[
\lim_{p^2 \to m^2_{\tilde{\chi}_i}} \frac{1}{p' - m_{\tilde{\chi}_i}} \bar{\tilde{\chi}}_i(p)\hat{\Gamma}^{(2)}_{ii} = i\tilde{\chi}_i, \quad \lim_{p^2 \to m^2_{\tilde{\chi}_i}} \bar{\tilde{\chi}}_i(p)\hat{\Gamma}^{(2)}_{ii} \frac{1}{p' - m_{\tilde{\chi}_i}} = i\tilde{\chi}_i
\]

Additional conditions needed for the general case with \(C\mathcal{P}\) violation:

Loop-corrected propagator should have the same Lorentz structure in the on-shell as at tree level (\(\rightarrow\) vanishing of \(\gamma_5\) contributions)

\[
\hat{\Sigma}^{SL}_{ii}(m^2_{\tilde{\chi}_i}) = \hat{\Sigma}^{SR}_{ii}(m^2_{\tilde{\chi}_i})
\]

Exploit additional freedom:

\[
\delta Z^R_{ii} - \delta \bar{Z}^R_{ii} = \delta \bar{Z}^L_{ii} - \delta Z^L_{ii}
\]
Chargino / neutralino field renormalisation in the MSSM with complex parameters

- In general $\delta \tilde{Z}_{ij} \neq \delta Z_{ij}^\dagger$
  - Hermiticity relation does not hold, but it can be shown that the CPT theorem is fulfilled

- If absorptive parts of the loop integrals are discarded from the field renormalisation constants
  - Hermiticity relation is restored, but correct on-shell properties are spoiled
  - Additional mixing contributions needed to obtain the correct on-shell properties
Conclusions

The ground-breaking work of Wolfhart Zimmermann continues to have a big impact on our present-day applications of quantum field theory for describing the fundamental laws of nature and confronting this description with the latest experimental results.

Window for probing possible effects of new physics

Treatment of unstable particles in spontaneously broken gauge theories: both challenging and phenomenologically very rich.

Many issues related to gauge invariance, unitarity, etc. have to be solved in order to reach the level of precision that will be needed for the comparison with experimental results in the coming years.

Vibrant field with many new results.