Quantum Energy Inequalities in Quantum Field Theory

Rainer Verch

Institut für Theoretische Physik, Universität Leipzig

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On the Occassion of the 80th Birthday of W. Zimmermann

Outline

Darker than Vacuum: Local Violations of Energy Positivity

Gravitation: Energy Conditions in Classical and Semiclassical Gravity
– Singularities vs Causal Pathologies

QEIs : Limitations on Local Energy Positivity Violations,
Quantum Energy Inequalities,
Dynamical Stability of Quantum Systems

Gravitation: QEIs versus Causal Pathologies,
Time Machines, Wormholes, Warp-Drive ??

Current (Open) Issues and some speculations on the relevance for cosmology

Local Violations of Energy Positivy

Classical Physics (Classical field Theory):

Energy of a (closed) system in a state $\boldsymbol{\xi}$ ($\leftrightarrow$ field configuration, solution of eqns of motion) is given as

$$\boldsymbol{E[\xi] = \int_{\text{space}} \varrho_\xi(t,x)\, d\mu(x) \geq 0}$$

- ♢ $\boldsymbol{\varrho_\xi(t,x) \geq 0}$ **energy density**
- ♢ $\boldsymbol{d\mu(x)}$ some positive measure on config. space

Quantum Physics (QFT):

For a state $\langle\, .\, \rangle_\psi = \langle \psi,\, .\, \psi\rangle$ of the system, the expected total energy is

$$\boldsymbol{\langle H\rangle_\psi = \int \langle \varrho(t,x)\rangle_\psi\, d\mu(x) \geq 0}$$

- ♠ $\boldsymbol{\langle \varrho(t,x)\rangle_\psi}$ **expected energy density can take both signs!**

This occurs also for other density-like quantities in quantum physics!

Energy density in QFT, 1

Epstein, Glaser, Jaffe (1965):

In a Wightman-QFT where the **E**nergy-**M**omentum-**T**ensor $T_{\mu\nu}(t,x)$ is a local quantum field with

$$\langle H \rangle_\psi = \int_{\text{space}} \langle T_{00}(t,x) \rangle_\psi \, dx \,,$$

it is necessary that $\langle T_{00}(t,x) \rangle_\psi$ takes negative values for some spacetime-points (t,x) and states ψ.

Fewster (2005):

If the QFT possesses scaling limits with positive canonical dimension at each (t,x), then $\langle T_{00}(t,x) \rangle_\psi$ is unbounded below, i.e. there is a sequence of vectors ψ_n, $\langle \psi_n , \psi_n \rangle = 1$, so that

$$\langle \psi_n , T_{00}(t,x)\psi_n \rangle \to -\infty \quad (n \to \infty) \,.$$

Energy density in QFT, 2

Simple argument for energy positivity violation:

Assume:

- $T_{00}(f)\Omega \neq 0$ for test-function $f \neq 0$
- $\langle \Omega, T_{00}(f)\Omega \rangle = 0$

for $\Omega = vacuum - vector$; then $\psi_\alpha = \cos\alpha\Omega + \frac{\sin\alpha}{||T_{00}(f)\Omega||} T_{00}(f)\Omega$

$$\Longrightarrow \quad \langle \psi_\alpha, T_{00}(f)\psi_\alpha \rangle = \zeta \sin 2\alpha + \eta(1 - \cos 2\alpha)$$

with $\zeta = ||T_{00}(f)\Omega||$

$$\Longrightarrow \quad \inf_\alpha \langle \psi_\alpha, T_{00}(f)\psi_\alpha \rangle = \eta - \sqrt{\eta^2 - \zeta^2} < 0$$

Casimir-Effect

Two parallel (conducting) plates are subject to a distance-dependent attractive force

similar: plate-sphere system

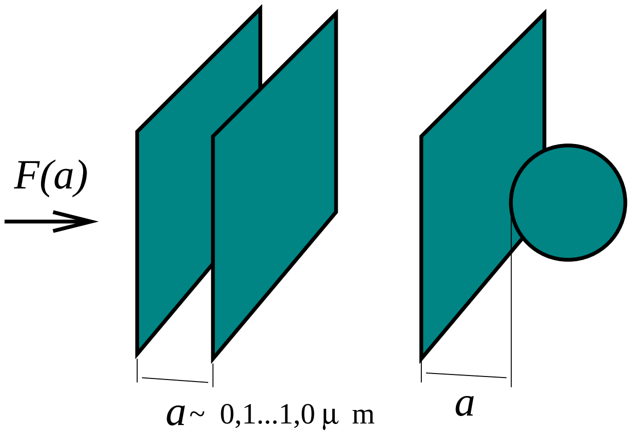
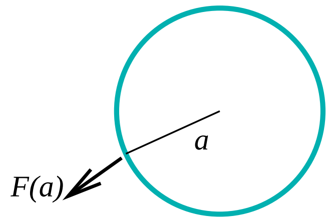


plate-plate system:
$F(a)/A = \frac{\pi^2 \hbar c}{240 \cdot a^4}$
measurements:
Sparnaay 1958 ($\pm 100\%$),
Bressi et al. 2002 ($\pm 15\%$)

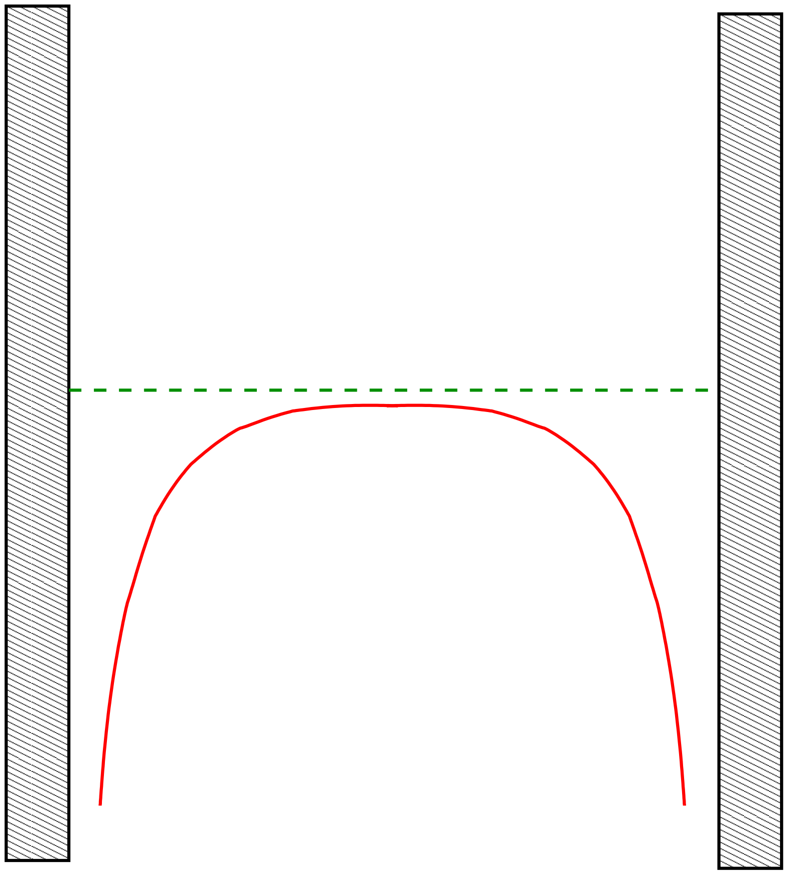
plate-sphere system:
$F(a)/A = \frac{\pi^3 \hbar c R}{360 \cdot a^3}$
measurements:
Lamoreaux, PRL **78**:1997 ($\pm 1\%$), Mohideen et al., PRL **81**:1998

Bei einer Sphäre sagt die Theorie eine expandierende Kraft voraus!



Casimir-Effect

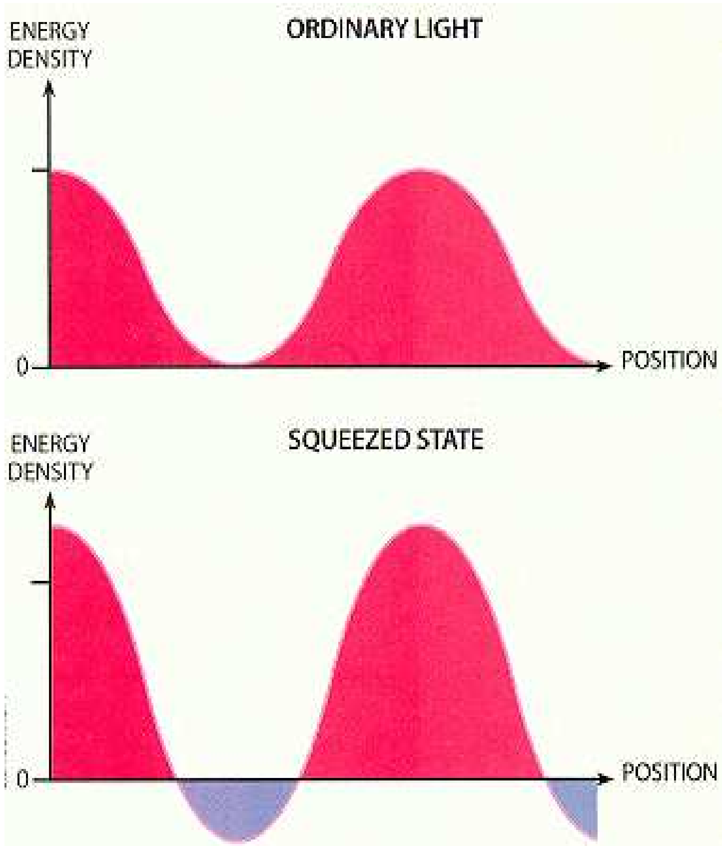
The "Casimir-vacuum" has a negative energy density compared to the global Minkowski-vacuum state



"Darker than Vacuum" states in Quantum Optics

In quantum optics:

Superpositions of vacuum + 2 photon states (2-photon coherent states) have been produced where the energy density $\sim \langle : \boldsymbol{E^2} : (\boldsymbol{t}, \boldsymbol{x}) \rangle$ is lower than the vacuum value.



Gravity

Space-Time structure: a Lorentzian manifold $\boldsymbol{M}\ ,\ \boldsymbol{g}$

- $\boldsymbol{M}$: 4-dim manifold, “catalogue of events”
- $\boldsymbol{g}$: Lorentzian metric, describes motion of light and matter

Einstein’s field eqns of gravity:

curvature terms of metric
describe gravity

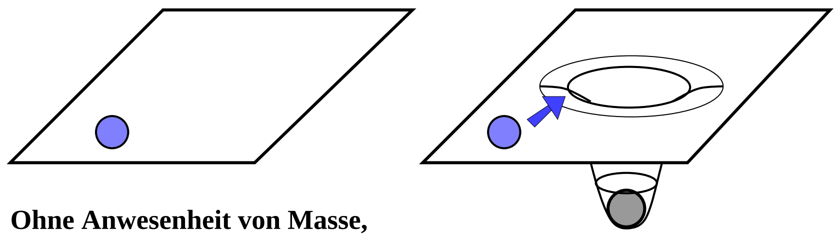
$$R_{\mu\nu}(\mathbf{x}) - \frac{1}{2} g_{\mu\nu}(\mathbf{x}) R(\mathbf{x}) = \frac{8\pi G}{c^2} T_{\mu\nu}(\mathbf{x})$$

energy-momentum tensor
of matter/energy distribution

Fundamental proposition of Einsteinian gravity:

“Presence of energy/matter curves spacetime geometry, spacetime curvature determines motion of energy/matter”

Analogie: Massenkugel auf Gummimembran



Ohne Anwesenheit von Masse,
Raumzeit ohne Krümmung
Testteilchen kräftefrei

Anwesenheit einer Masse
bewirkt Krümmung
Testteilchen "folgt der Krümmung"

Classical Gravity

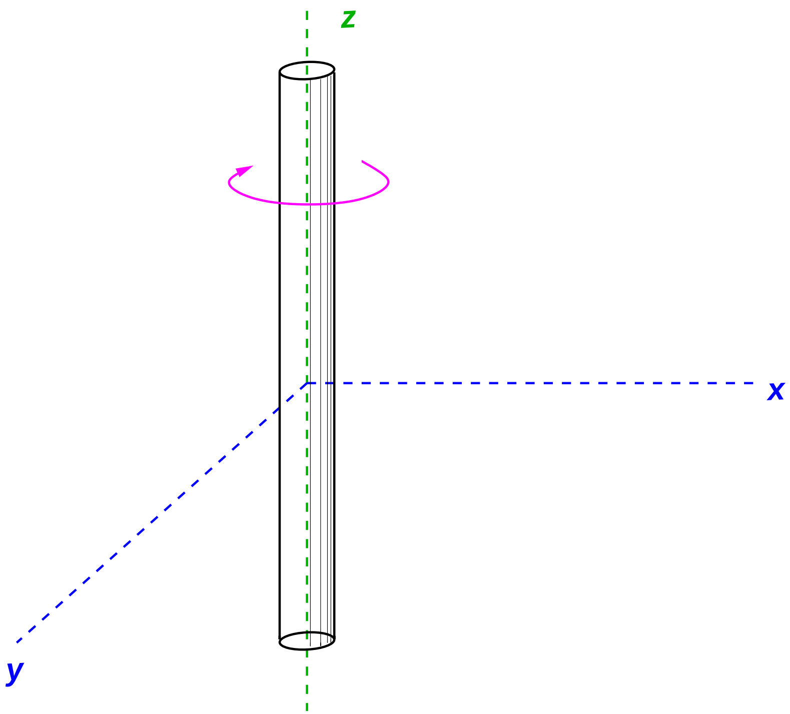
Qualitative behaviour of solutions to Einsteins eqns of gravity:

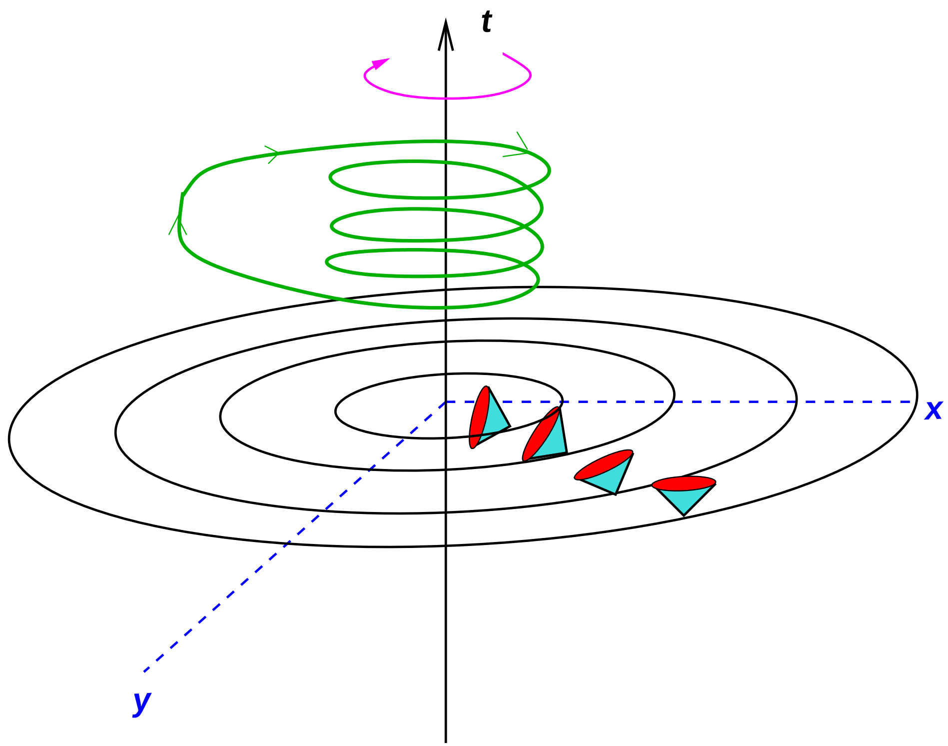
$\rho_{\text{class}}(t, x) = T_{00}(t, x) \geq 0$ positivity of energy density of <u>classical</u> matter/radiation

$\Longrightarrow$

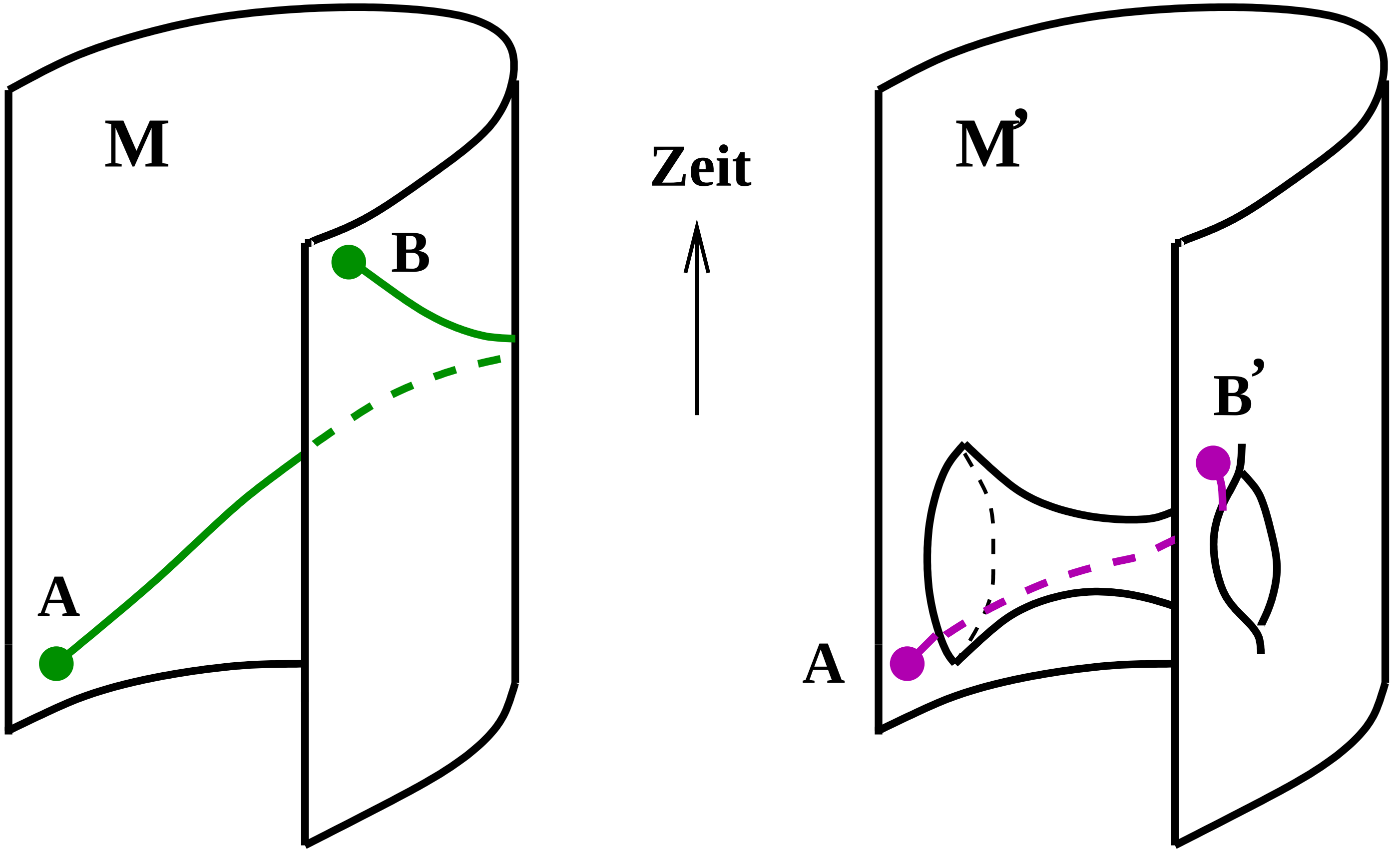
- Gravity is always attractive
- Black holes appear generically as final states of large matter aggregates
- More generally: Spacetime singularities occur generically
- Absence of causal pathologies, e.g.
  - "Time machines" $\leftrightarrow$ spacetimes with closed timelike curves
  - "Superluminal travel" $\leftrightarrow$ "Designer-spacetimes"
    with wormholes or warpdrive-metrics

“Time machine” — Tipler’s rotating cylinder



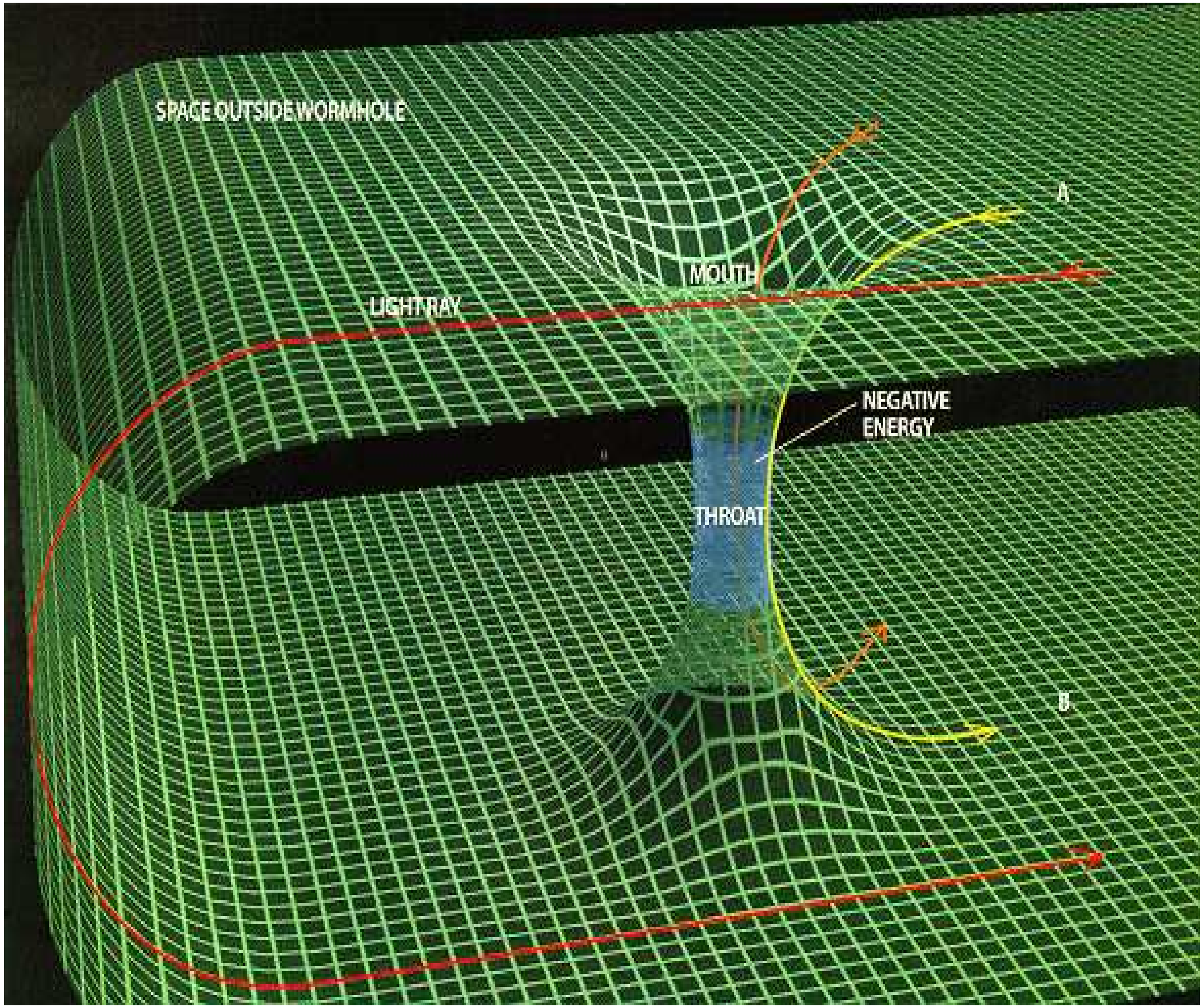


Schneller Reisen mit Wurmloch

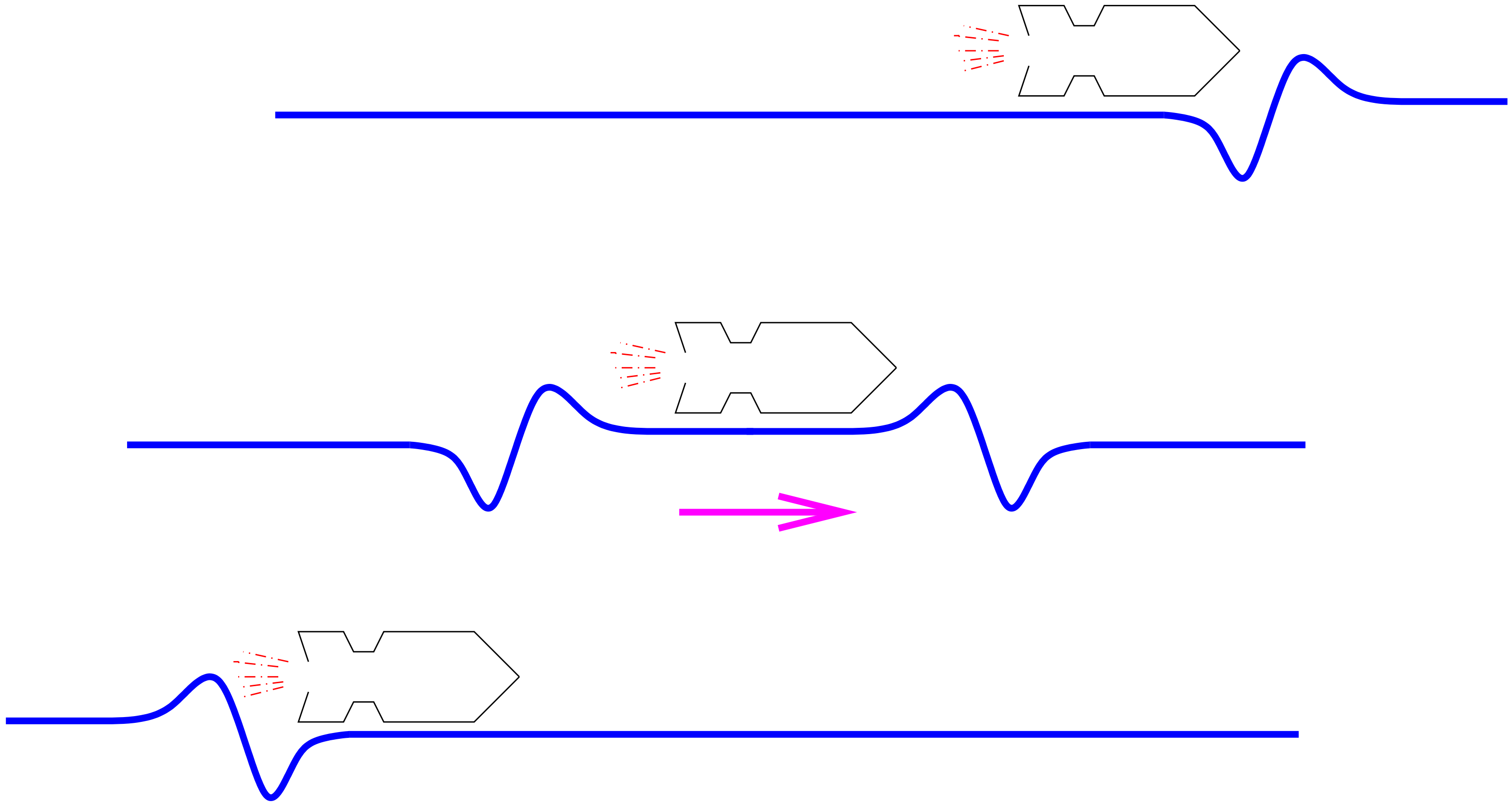


Wurmloch = "Tunnel" in
zusätzlicher Raumdimension

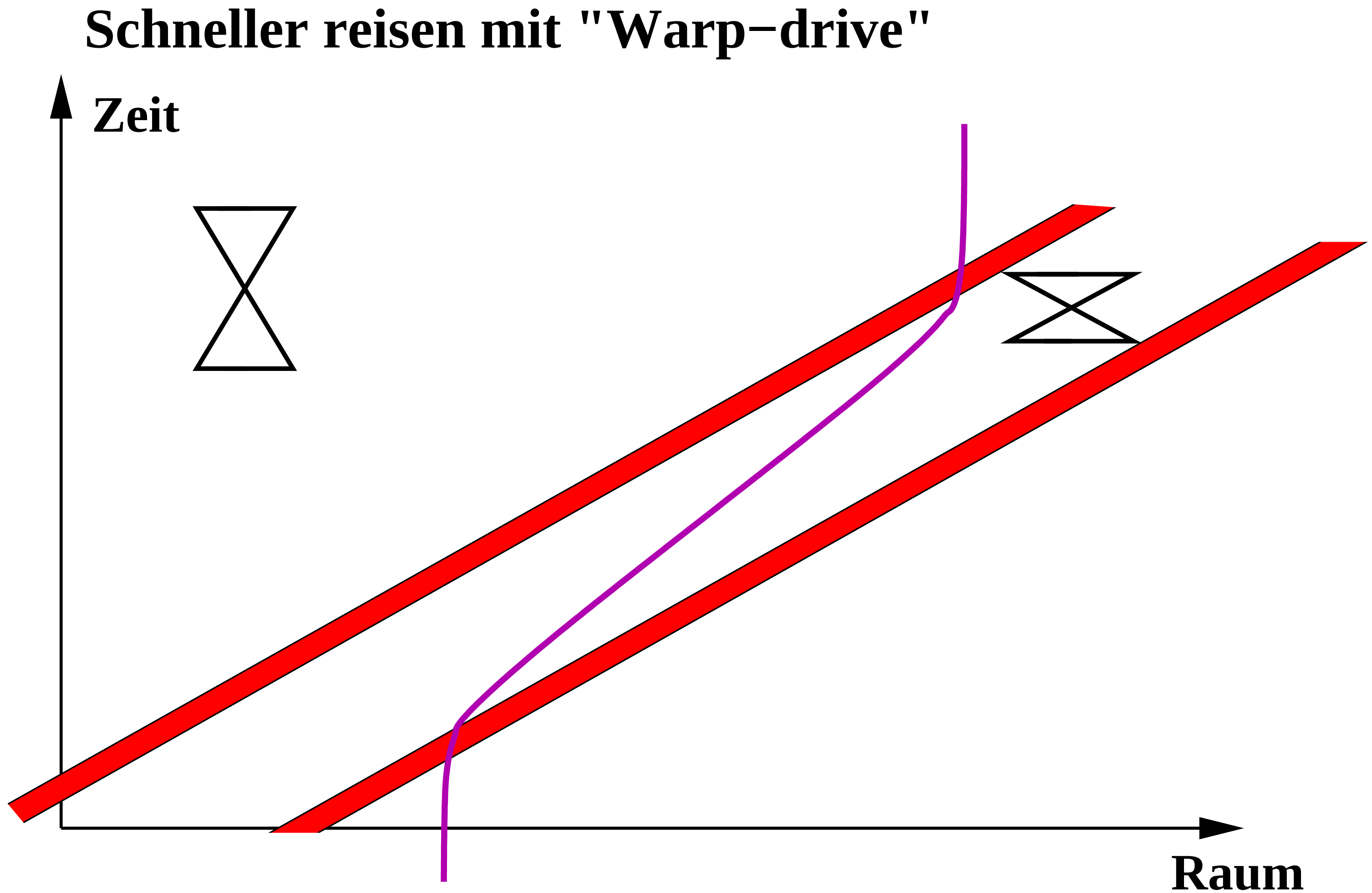
faster travel by warpdrive



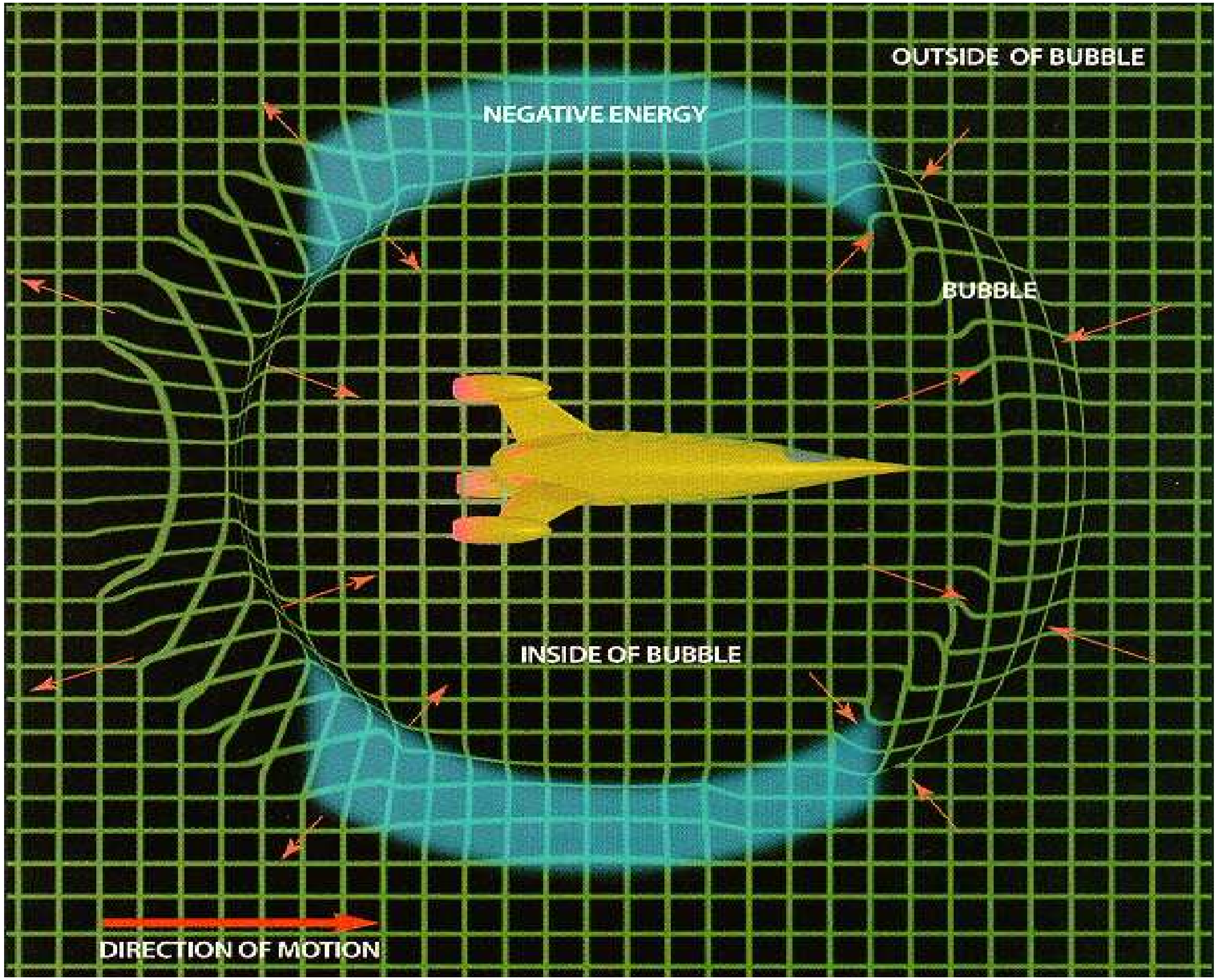
faster travel by warpdrive



faster travel by warpdrive



faster travel by warpdrive



QFT – global stability

- pointwise quantities like expectation values of energy densities

$$\langle \varrho(t, \boldsymbol{x}) \rangle_\psi$$

no longer positive (typically even unbounded from below as function of the state)

- The dynamical evolution of a quantum field system is determined by a Hamilton operator $\boldsymbol{H}$ with respect to an inertial frame,
and one demands that the system fulfills the

principle of global dynamical stability:

$$\boldsymbol{H} \geq \text{a lowest eigenvalue } E_0$$

and

existence of a ground state $\boldsymbol{\Omega_0}$ (“vacuum”)

with respect to every global inertial frame.

Remark: This is typical for elementary systems on Minkowski spacetime

QFT – global stability?

- in curved spacetime (presence of classically described gravitational fields):

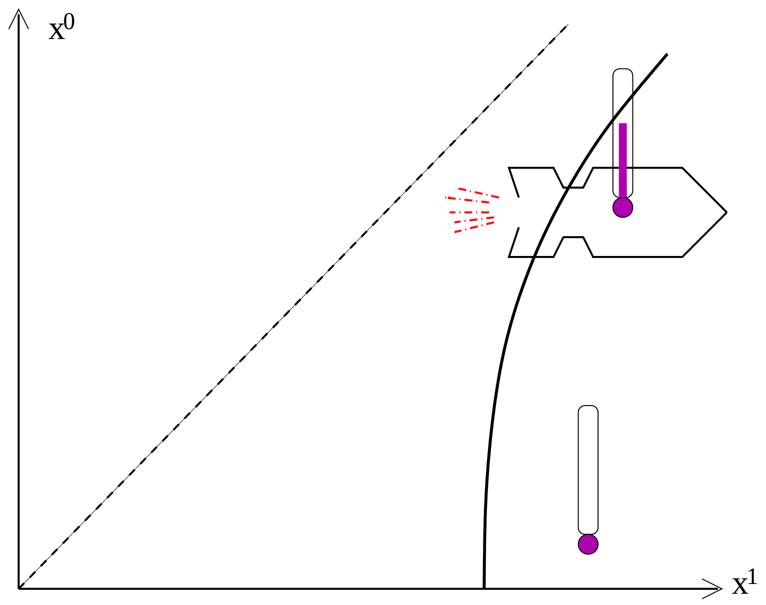
  for quantum fields there is in general no unambiguous global Hamiltonian describing the dynamics (non-existence of global inertial frames)

$\longrightarrow$ how to formulate dynamical stability?

$\longrightarrow$ the concepts of “particle” oder “vacuum” are observer-dependent!

Unruh-Effekt: an observer with constant acceleration registers the inertial vacuum state as thermal ensemble at temperature

$$\boldsymbol{T_a = \frac{\hbar a}{2\pi k_B c}}\,, \quad \boldsymbol{a} = \text{acceleration}$$



QFT – microlocal stability

For QFT on general curved spacetimes, there are local replacements of the principle of local stability,

⋆ microlocal spectrum condition ($\boldsymbol{\mu SC}$)

⋆ quantum energy inequalities (QEIs)

- microlocal spectrum condition:
  The spectral high-energy bevavour of the correlation functions of a quantum field $\boldsymbol{\Phi_M}$ on an arbitrary spacetime M,

$$\langle \boldsymbol{\Phi_M(x_1)\cdots\Phi_M(x_n)}\rangle_{\boldsymbol{\psi}} \ ,$$

  shall — infinitesimally close to the spacetime points $\mathbf{x}_1, \ldots, \mathbf{x}_n$ — coincide with the spectral high-energy behaviour of a quantum field $\boldsymbol{\Phi_0}$ on Minkowski spacetime.

  (This condition is formulated mathematically precisely with the methods of microlocal analysis.)

Local Thermal Equilibrium States

A new condition proposed by Buchholz-Ojima-Roos (2001), Buchholz (2003), Buchholz-Schlemmer (2006):

Local Thermal Equilibrium States (here simplified):

Let Φ be the quantized linear scalar field on a spacetime (M, g).

A state ω is a **2nd order LTE-state at $x \in M$ if**:

$$\omega(:\Phi^2:(x)) \quad \text{and} \quad \lim_{y \to x} (\nabla_x \nabla_x - \nabla_x \nabla_y - \nabla_y \nabla_x + \nabla_y \nabla_y)\,[\omega(\Phi(x)\Phi(y)) - \eta_M(x,y)]$$

coincides (at x) with the corresponding quantities of a thermal equilibrium (KMS) state at inverse temperature $\beta = \beta(x)$ on Minkowski spacetime.

$$\omega(:\Phi^2:(x)) = \lim_{y \to x} (\omega(\Phi(x)\Phi(y)) - \eta_M(x,y))$$

with

$$\eta_M = \text{symmetric Hadamard parametrix}$$

QFT – local stability

- quantum energy inequalities:
  For a quantum field $\Phi_M(\mathbf{x})$ on an arbitrary spacetime M with energy density $\rho_\Phi(\mathbf{x})$ it is required:

  $$\int_\gamma \langle \rho_\Phi(s) \rangle_\psi f^2(s) ds \geq c_\gamma(f) > -\infty$$

  for all states ψ, timelike curves γ, and C_0^∞-weighting functions $f \geq 0$.

  I.e.: the averaged energy density of the quantum field along arbitrary timelike curves should possess state-independent lower bounds.

- limiting case: ANEC = Averaged Null Energy Condition

  $$\liminf_{\lambda \to 0} \int_\gamma \langle \rho_\Phi(s) \rangle_\psi f^2(\lambda s) ds \geq 0$$

Question:

How do μSC and QEIs relate to each other? Do they correspond to the principle of global stability on spacetimes with global time symmetry?

QFT – dynamical stability at three scales

microlocal spectrum condition

general spacetimes
linear quantum fields

stationary
spacetimes,
linear quantum
fields

QEIs

dynamical systems on static
spatially closed spacetimes

existence of
ground states
or passive states

[C.J. Fewster, H. Sahlmann, R. Verch]

QEI variants

The EMT of a quantum field $\Phi_M(\mathbf{x})$ on a spacetime M is defined in the form

$$\langle : T_{\mu\nu} : (\mathbf{x})\rangle_{\psi|\varphi} = \lim_{\mathbf{y}\to\mathbf{x}} \sum_P \langle (P\Phi_M)^*(\mathbf{x})(P\Phi_M)(\mathbf{y})\rangle_\psi - \langle (P\Phi_M)^*(\mathbf{x})(P\Phi_M)(\mathbf{y})\rangle_\varphi$$

QEIs for weighted averages of

$$\langle : T_{\mu\nu} : (\mathbf{x})\rangle_{\psi|\varphi}$$

have ψ independent lower bounds depending on the reference state φ.
Such QEIs are called difference QEIs.

One can define $\langle : T_{\mu\nu} : (\mathbf{x})\rangle_{\psi|\varphi}$ choosing instead of a refence state φ a functional η_M constructed locally from the geometry of the spacetime M (the choice of η_M is canonical).
Then the weighted averages of

$$\langle : T_{\mu\nu} : (\mathbf{x})\rangle_{\psi|\eta_M}$$

have lower bounds depending only on the local geometry of M, i.e. on the metric and curvature quantities.
Such QEIs are called absolute covariant QEIs.

QEI results

- Difference QEIs have been derived in various forms of generality for the quantized Klein-Gordon field, Dirac field, and free E.M. field on Minkowski spacetime and general globally hyperbolic spacetimes (as consequence of the μSC).
  [**L. Ford, T. Roman, M. Pfenning, C.J. Fewster, S. Eveson, C. Smith, E. Flanagan, D. Vollick, R. Verch**]
- Absolute covariant QEIs have recently been developed for the quantized Klein-Gordon field on general globally hyperbolic spacetimes
  [**C.J. Fewster, C. Smith, M. Pfenning**]
- Absolute, sharp QEIs have been obtained for chiral conformal quantum field theories in 1 + 1 dim. Minkowski spacetime
  [**C.J. Fewster and S. Hollands**]
- Difference QEIs have been applied to the squeezed "darker than vacuum states" considered in quantum optics, with the conclusion that there are bounds on the (magnitude $\times$ duration) of energy negativity.
  [**P. Marecki**]
  (Note: Detection of "darker than vacuum" states rests on homodyne detection – there is much room for further investigation...)

more QEI results

- Absolute QEIs hold for LTE (local thermal equilibrium) states of the linear scalar field

$$(\nabla^a \nabla_a + \xi R + m^2)\Phi(x) = 0$$

with arbitrary conformal coupling ξ.

The lower bound $c_\gamma(f)$ depends on the maximal local temperature

$$\frac{1}{\beta_0} = \max_x \frac{1}{\beta(x)}$$

attained by the states along the curve, so the QEIs hold for sets of states having bounded (local) temperature.

- Remark: Fewster and Osterbrink (2007) have shown that QEIs do not hold in general if $\xi \neq 0$.
- ANEC for LTE states: ANEC holds for LTE states for $\xi = 0$ (always) and for $\xi = 1/4$ and $1/8$ if temperature increase along (complete) lightlike geodesics sufficiently bounded.

Semiclassical Gravity

$$R_{\mu\nu}(\mathrm{x}) - \frac{1}{2} g_{\mu\nu}(\mathrm{x}) R(\mathrm{x}) = \frac{8\pi G}{c^2} \left(T_{\text{klass},\mu\nu}(\mathrm{x}) + \langle : T_{\mu\nu} : (\mathrm{x}) \rangle_\psi \right)$$

is the semiclassical generalization of Einstein's equations.

- dynamical stability admits that

$$\inf_{\text{states } \psi} \langle : \rho_\Phi : (\mathrm{x}) \rangle_\psi = -\infty$$

  at each point x of spacetime M.

- Hence, spacetime geometries with causal pathologies like

  • time-machine scenarios • wormholes • warp-drive metrics

  could occur as solutions to the semiclassical Einstein–equations.

- Question:
  Can they really occur for a suitable choice of state ψ?
  Are they realistic? At which scale?

Dynamical stability inhibits causal pathologies

ad “time machines”:

In QFTs obeying the microlocal spectrum condition, solutions to the semiclassical Einstein-equations cannot possess closed timelike curves.

[**B. Kay, M. Radzikowski, R. Wald**]

ad wormholes:

Difference QEIs imply considerable constraints for the possibility of macroscopic traversable wormholes as solutions of the semiclassical Einstein equations, since extreme amounts of negative energy would have to be concentrated in microscopic space domains. The argument involves approximations which are justified in the case of macroscopic wormholes.

[**L. Ford and T. Roman, C.J. Fewster and T. Roman**]

ad warpdrive:

Again, difference QEIs imply severe constraints on the possiblity to have warpdrive geometries as solutions to the semiclassical Einstein equations. Macroscopic warpdrive scenarios appear impossible by a similar reasoning as in the case of wormholes.

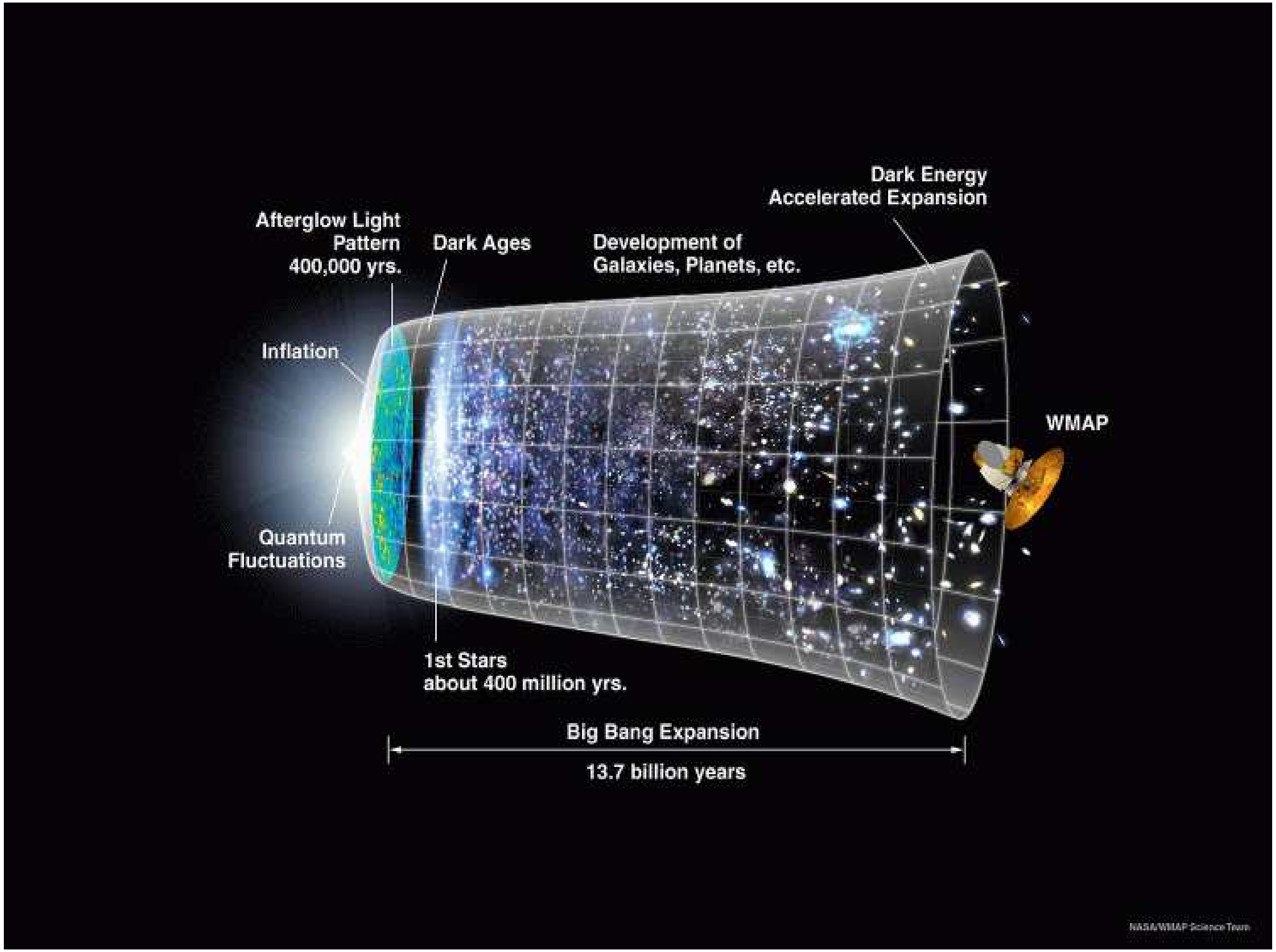
[**L. Ford and M. Pfenning**]

Current (open) issues

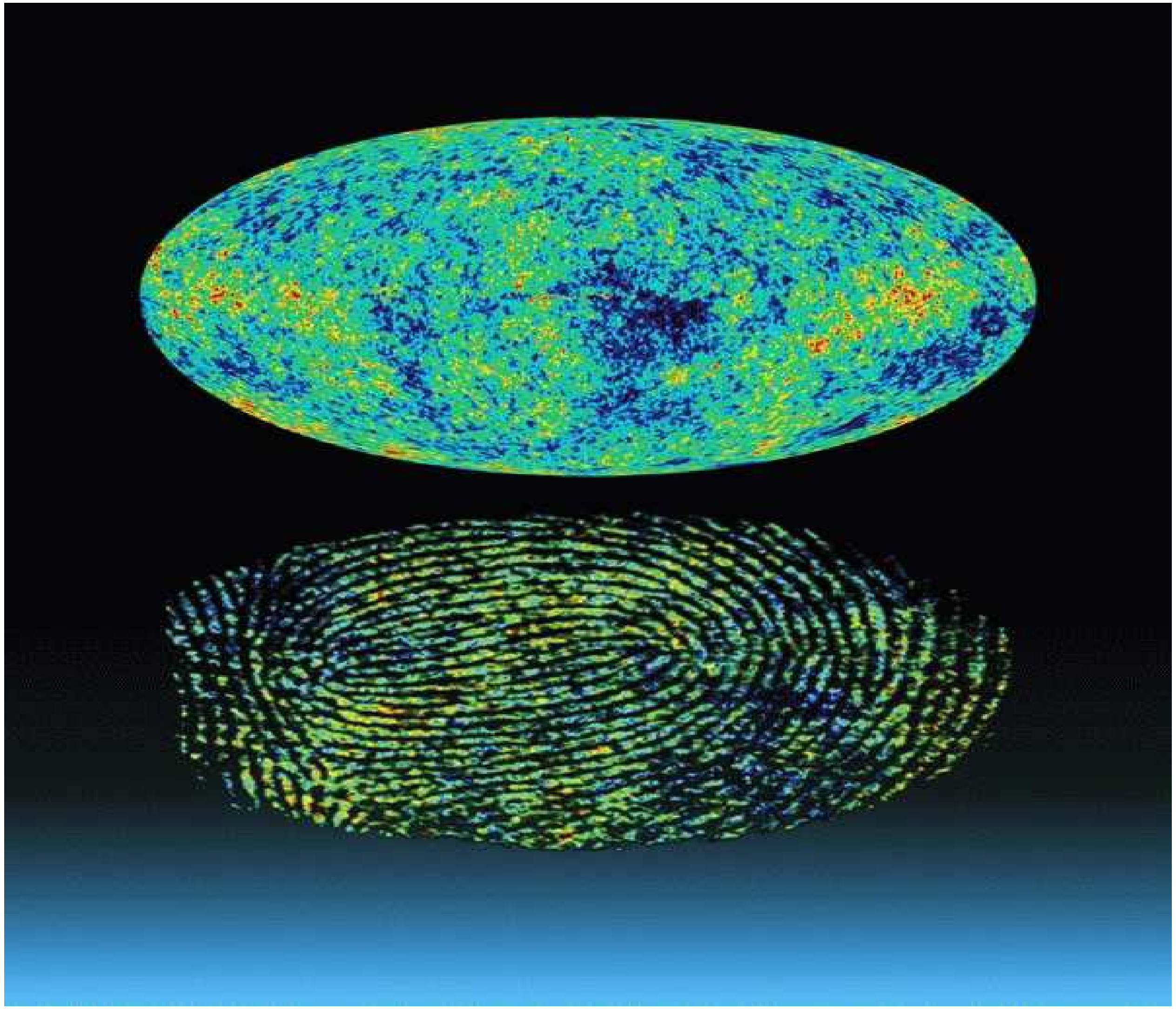
The given arguments against solutions to the semiclassical Einstein-equations do so far not apply to all scales and only to linear quantum fields.

- The issue of validity of QEIs for interacting quantum fields is completely open.
- To restrict wormholes etc at submicroscopic scales, one needs sharp absolute covariant QEIs. This would (partially) clarify the role played by "vacuum energy" contributions in the semiclassical Einstein–equations.
- Existence/Generacy of LTE states?
- Resolution of these matters is of relevance for certain questions in cosmology and quantum gravity.

Universe at all scales



CMB as “fingerprint” of the early universe



CMB variance and curvature