Taming the Landau Ghost in NCQFT

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to **Prof. Wolfhart Zimmermann**

Congratulations to your 80th birthday

Introduction

- Introduction: RG flows, Axioms, problems,...
- Euclidean scalar fields, modified action + Wulkenhaar
- Taming the Landau ghost + Wulkenhaar
- Induced Gauge Model, BRST approach + Wohlgenannt; Blaschke, Schweda
- Fermions: spectral triple + Wulkenhaar
- Minkowski wedge local fields + Lechner
- Conclusions

History, Axioms, problems

- 50.... success of ren. pert. th.
- 56.... ghost and triviality Landau....
- 69 Bogoliubov, Parasiuk, Hepp, Zimmermann
- 72 t´Hooft, Veltman ren. of nonabelian gauge th.
- 73 Gross, Wilczek, Politzer asymptotic freedom, QCD
- 74 RG flow Wilson,... summability, safety
- Axioms W., OS.,covariance, spectrum condition, causality

- cure problems of ren. pert. exp. (IR,UV,convergence)
- require Borel summability
- take into account qu. gravity effects

Project

merge general relativity with quantum physics through noncommutative geometry

History

- Limited localisation of events in space-time

$$D \geq R_{ss} = G/c^4 hc/\lambda \geq G/c^4 hc/D \tag{1}$$

  gives Planck lenght as a lower limit to localization
  Heisenberg, Snyder, ...

- 1986 Connes: Noncommutative Geometry
  1992 H. G. and Madore: Regularization using nc manifolds
  1994 Filk: Feynman r.; Doplicher, Fredenhagen, Roberts
  1994.. H.G., Klimcik, Presnajder
  1999 Schomerus: obtained nc models from strings
  2000.. H. G. and Schweda, Wulkenhaar, nc Gauge models
  2004 Renormalization Proof H.G. + Wulkenhaar ...

manifold, deform algebra, keep differential calculus, projective modules, traces, cyclic cohomology, spectral triples
extend ren. pert. theory to nc geometry

nc Higgs model

Formulation

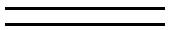
ϕ^4 on nc $\mathbb{R}^4$, $[x^\mu, x^\nu] = i\theta^{\mu\nu}$ antisymmetric,
or equivalently star product

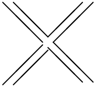
$$(a * b)(x) = \int dy \int dk a(x + \frac{\theta k}{2}) b(x + y) e^{iky}$$

ϕ^4 action

$$S = \int dp (p^2 + m^2) \phi_p \phi_{-p} + \lambda \int \prod_{j=1}^{4} (dp_j \phi_{p_j}) \, \delta(\sum_{j=1}^{4} p_j) e^{-\frac{i}{2} \sum_{i<j} p_i^\mu p_j^\nu \theta_{\mu\nu}}$$

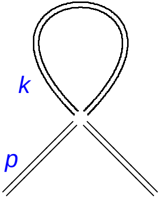
Feynman rules

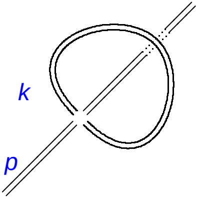
 $\frac{1}{p^2+m^2}$

 $\frac{\lambda}{4!} e^{-\frac{i}{2} \sum_{i<j} p_i^\mu p_j^\nu \theta_{\mu\nu}}$

IR/UV mixing

One-loop two-point function planar and nonplanar contributions:





$$= \frac{\lambda}{6} \int dk \frac{1}{k^2 + m^2} \qquad = \frac{\lambda}{12} \int dk \frac{e^{ip^\mu k^\nu \theta_{\mu\nu}}}{k^2 + m^2} \; p \overset{\sim}{\to} 0 \; \frac{1}{\tilde{p}^2}$$

planar graphs: renormalize BPHZ .., nonplanar graphs "finite" (regularized) insert into bigger graph: mixing
Minwalla, van Raamsdonk & Seiberg, 1999
– detailed treatment: power-counting theorem for ribbon graphs Chepelev & Roiban 1999/2000
– proposals: resummation, supersymmetry (all integrals exist, but unbounded as momenta $\longrightarrow$ 0), . . .
expand use SW map..... QED_Θ is not renormalizable
satisfactory solution: modify action

Theorem

H. G. and R. Wulkenhaar ϕ^4 model modified,
IR/UV mixing: short and long distances related
Theorem: Action

$$S = \int d^4x \Big(\frac{1}{2}\partial_\mu\phi\star\partial^\mu\phi + \frac{\Omega^2}{2}(\tilde{x}_\mu\phi)\star(\tilde{x}^\mu\phi) + \frac{\mu_0^2}{2}\phi\star\phi + \lambda\phi\star\phi\star\phi\star\phi\Big)(x)$$

for $\tilde{x}_\mu := 2(\theta^{-1})_{\mu\nu}x^\nu$
is perturbativly renormalizable to all orders in λ, 3 proofs,
Rivasseau et al: Multiscale analysis in matrix base and in position space
Action has Langmann-Szabo position-momentum duality
$S\big[\phi;\mu_0,\lambda,\Omega\big] \mapsto \Omega^2 S\big[\phi;\frac{\mu_0}{\Omega},\frac{\lambda}{\Omega^2},\frac{1}{\Omega}\big]$

use oscillator base reformulate as dynamical matrix model
interaction becomes matrix product no oscillations

$$S = (2\pi\theta)^2 \sum_{m,n,k,l \in \mathbb{N}} \Big(\frac{1}{2} \phi_{mn} \Delta_{mn;kl} \phi_{kl} + \lambda \phi_{mn} \phi_{nk} \phi_{kl} \phi_{lm} \Big)$$

Propagator complicated! Asymmetric, quasilocal
Wilson-Polchinski approach to nonlocal matrix models
require cutoff independence, add power series of interactions
graphs drawn on Riemann surface of genus g $2 - 2g = L - I + V$
Proof Power counting rule H. G., R. Wulkenhaar
amplitude bounded by $\Lambda^{4-N+4(1-B-2g)}$
use "quasilocality" of propagator to estimate ribbon graphs

all nonplanar graphs irrelevant
planar graphs with more than 4 legs irrelevant
4leg graph log. divergent, 2leg graph quadr. divergent
need 4 rel/marginal parameters!

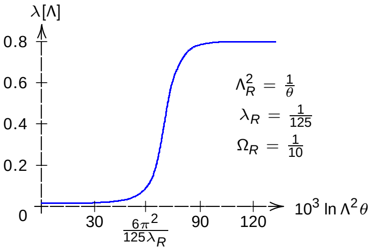
β function

evaluate β function, H. G. and R. Wulkenhaar,

$$\beta_\lambda == \frac{\lambda^2_{\text{phys}}}{48\pi^2} \frac{(1-\Omega^2_{\text{phys}})}{(1+\Omega^2_{\text{phys}})^3} + \mathcal{O}(\lambda^3_{\text{phys}})$$

flow bounded, L. ghost killed!
Due to wave fct. renormalization
$\Omega = 1$ betafunction vanishes
$\Omega^2[\Lambda] \leq 1$

($\lambda[\Lambda]$ diverges in comm. case)



- perturbation theory remains valid at all scales!
- non-perturbative construction of the model seems possible!

How does this work?

- four-point function renormalisation with usual sign
- ∃ one-loop wavefunction renormalisation which compensates four-point function renormalisation for $\Omega \to 1$

Induced Gauge Models

Couple gauge field to scalar field,
H. G. and Michael Wohlgenannt,

$$S = \int d^D x \left(\frac{1}{2} \phi \star [\tilde{X}_\nu, [\tilde{X}^\nu, \phi]_\star]_\star + \frac{\Omega^2}{2} \phi \star \{\tilde{X}^\nu, \{\tilde{X}_\nu, \phi\}_\star\}_\star + \frac{\mu^2}{2} \phi \star \phi + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi \right)(x)$$

use covariant coordinates
One loop calculation in four dimensions
quadratic divergent; use Duhamel expansion...

$$\Gamma^\epsilon_{1l} = \frac{1}{192\pi^2} \int d^4x \left\{ \frac{24}{\tilde{\epsilon}\theta}(1-\rho^2)(\tilde{X}_\nu \star \tilde{X}^\nu - \tilde{x}^2) \right.$$

$$\left. + \ln\epsilon \left(\frac{12}{\theta}(1-\rho^2)(\tilde{\mu}^2 - \rho^2)(\tilde{X}_\nu \star \tilde{X}^\nu - \tilde{x}^2) + 6(1-\rho^2)^2((\tilde{X}_\nu \star \tilde{X}^\nu)^{\star 2} - (\tilde{x}^2)^2) + \rho^4 F_{\mu\nu} F^{\mu\nu} \right) \right\},$$

where $F_{\mu\nu} = [\tilde{x}_\mu, A_\nu]_\star - [\tilde{x}_\nu, A_\mu]_\star + [A_\mu, A_\nu]_\star$

- $\mathcal{O}(1/\epsilon) \; + \; \mathcal{O}(\ln\epsilon)$ gauge invariant renormalizable?
- *quantize* $A = 0$ not stable, nontrivial vacuum solutions

BRST approach

H G, D. Blaschke, M. Schweda, M. Wohlgenannt
Generalize BRST complex to nc gauge models with oscillator

$$S = \int F^2/4 + \Omega^2/2 \int (\{\tilde{x}^\mu, A^\nu\} \star \{\tilde{x}_\mu, A_\nu\} + \{\tilde{x}_\mu \bar{c}, \tilde{x}^\mu c\})$$

$$S_{gf} = \int B \star \partial_\mu A^\mu - B \star B/2 - \bar{c} \star \partial_\mu s A_\mu - \Omega^2 \tilde{c}_\mu \star s C_\mu$$

$$C_\mu = \{\{\tilde{x}_\mu \stackrel{\star}{,} A_\nu\} \stackrel{\star}{,} A_\nu\} + [\{\tilde{x}_\mu \stackrel{\star}{,} \bar{c}\} \stackrel{\star}{,} c] + [\bar{c} \stackrel{\star}{,} \{\tilde{x}_\mu \stackrel{\star}{,} c\}]$$

is BRST invariant

$$sA_\mu = D_\mu c, sc = igc \star c, s\tilde{c}_\mu, = \tilde{x}_\mu, sc = B, sB = 0$$

and s square to zero for all fields.
All propagotors are proportional to $(-\Delta + \Omega^2 \tilde{x}^2)^{-1}$ use Mehler kernel, loop calculations indicate: tadpole finite UV/IR mixing?
renormalization?

A spectral triple

H. G. and Raimar Wulkenhaar,
Take Dirac operator on Hilbert space $L^2(R^4) \otimes C^{16}$

$$D_8 = (i\Gamma^\mu \partial_\mu + \Omega \Gamma^{\mu+4} \tilde{x}_\mu)$$

$\mu = 1, ...4$, Γ_k generate 8-dim Clifford algebra $\{\Gamma_k \Gamma_l\} = 2\delta_{kl}$

$$D_8^2 = (-\Delta + \Omega^2 ||\tilde{x}||^2) 1 - i\Omega \Theta^{-1}_{\mu\nu} [\Gamma^\mu, \Gamma^{\nu+4}]$$

compute action of Dirac operator on sections of spinor bundle

$$[D_8, f] * \psi = i[\Gamma^\mu + \Omega \Gamma^{\mu+4}](\partial \mu f) * \psi$$

only 4 dim. differential appears
leads to spectral triple
configuration space dimension 4
phase space dim. 8 Clifford alg. dim., KO dim.,..R.W.

H G and Gandalf Lechner, extended by Buchholz and Summers
Quantum fields over deformed Minkowski space time
NC coordinates: $[\hat{x}^\mu, \hat{x}^\nu] = iQ^{\mu\nu}$, standard form

$$Q = \begin{pmatrix} 0 & \kappa_e & & \\ -\kappa_e & 0 & & \\ & & 0 & \kappa_m \\ & & -\kappa_m & 0 \end{pmatrix}$$

Field operators are defined as tensor product of Weyl operators times creation and/or annihilation operators:

$$\Phi_Q(x) = \int d\mu_p (e^{ipx} e^{ip\hat{x}} \otimes a_p^\dagger + e^{-ipx} e^{-ip\hat{x}} \otimes a_p)$$

operators $a_{Q,p} = e^{-ip\hat{x}} \otimes a_p$ fulfill a twisted algebra,
which I studied in 1979, related to Zamolodchikov -Faddeev algebra.

$$a_{Q,p} a_{Q,p'} = e^{-ipQp'} a_{Q,p'} a_{Q,p},$$

$$a_{Q,p} a^\dagger_{Q,p'} = e^{-ipQp'} a^\dagger_{Q,p'} a_{Q,p} + \omega_p \delta^{(3)}(\vec{p} - \vec{p}')$$

Gives twisted correlation functions,

$$\phi_Q(x_1)\dots\phi_Q(x_N)|0> = \prod_{l<k} e^{-i\partial^\mu_{x_l} Q_{\mu\nu}\partial^\nu_{x_k}} \phi_0(x_1)\dots\phi_0(x_N)|0>$$

similar to correlation fcts. of Chaichian et al, Fiore and Wess
use repr. of fields on Hilbert space (H G 79) $A_Q(p) = e^{\frac{i}{2}pQ\hat{P}} a_p$
where $\hat{P}$ = momentum operator. Study properties of

$$\Phi_Q(x) = \int d\mu_p \left(e^{ipx} A^\dagger_{Q,p} + e^{-ipx} A_{Q,p} \right)$$

temp. distr., Reeh-Schlieder prop., not local, not covariant
treat all deformations, consider relative properties of fields
determine algebra of $A_{Q,p}$ and $A^\dagger_{Q,p}$ for different Q
Transformation properties: adjoint action on $A_{Q,p}$ gives

$$\begin{aligned} U_{y,\Lambda}\Phi_Q(x)U^\dagger_{y,\Lambda} &= \Phi_{(\gamma_\Lambda(Q))}(\Lambda x + y), \ (y,\Lambda) \in \mathbb{P} \\ \gamma_\Lambda(Q) &= \Lambda Q \Lambda^\dagger, \ \Lambda \in \mathcal{L}^\uparrow \end{aligned}$$

consider relative locality properties of fields $\Phi_Q(x)$

Wedges and Wedge local QF

We relate the antisymmetric matrices to Wedges:
$W_1 = \left\{ x \in \mathbb{R}^D | x_1 > |x_0| \right\}$ act on standard wedge by proper Lorentz transformations $i_\Lambda(W) = \Lambda W$. Stabilizer group is $SO(1,1)xSO(2)$, which corresponds to boosts and rotations.

$$\mathcal{W}_0 = \mathcal{L}_+^\uparrow W_1$$

Reflections: $j_\mu : x_\mu \mapsto -x_\mu$
$\mathcal{W}_0$ with $\mathcal{L}$ -action $i_\Lambda : \mathcal{W}_0 \mapsto \Lambda \mathcal{W}_0$ is $\mathcal{L}_+$ - homogenous space.
Get isomorphism $(\mathcal{W}_0, i_\Lambda) \cong (\mathcal{A}, \gamma_\Lambda)$
$\mathcal{A} = \{\gamma_\Lambda(Q_1) | \Lambda \in \mathcal{L}_+\}$ $Q(\Lambda W_1) := \gamma_\Lambda(Q_1)$
Gives correspondence between the set of wedges and antisymmetric matrices.
We define wedge local fields through: $\phi = \{\phi_W | W \subset \mathcal{W}_0\}$ get family of fields, covariance and localization in wedges.

With this isomorphism define $\Phi_W(x) := \Phi_{Q(W)}(x)$.
Transformation properties

$$U_{y,\Lambda}\Phi_W(x)U^\dagger_{y,\Lambda} = \Phi_{\gamma_\Lambda(Q(W))}(\Lambda x + y)$$

Theorem

Let $\kappa_e \geq 0$ the family $\Phi_W(x)$ is a wedge local quantum field on Fockspace:

$$[\phi_{W_1}(f), \phi_{-W_1}(g)](\psi) = 0,$$

for $supp(f) \subset W_1$, $supp(g) \subset -W_1$.

Show that

$$[a_{Q_1}(f^-), a^\dagger_{-Q_1}(g^+)] + [a^\dagger_{Q_1}(f^+), a_{-Q_1}(g^-)] = 0$$

$\vartheta = \sinh^{-1}\left(p_1/(m^2 + p_2^2 + p_3^2)^{1/2}\right)$
Use analytic continutation from R to $R + i\pi$ in ϑ .

Conclusions

- modified actions for bosonic fields yield renorm. models
- RG flows save Landau ghost tamed
- gauge field actions formulated, BRST invariance maintained renormalizability?
- fermions give spectral triple
  renormalizability?
- Goal: NC ren. Standard model?
- family of fields on def. Mink. ST fulfills WEDGE locality

to **Prof. Wolfhart Zimmermann**

Congratulations to your 80th birthday