Yang - Mills QFT in 4 dim

Foundation of the Standard Model

Only QFT viable in 4 dim

What happen to QCD strings

in absence of quarks ?

Geometric model for Knots

n - field

$\vec{n} = (n^1, n^2, n^3)$ $\quad \sum (n^i)^2 = 1$

static : $\mathbb{R}^3$

$n_\infty = (0,0,1)$ $\quad \mathbb{R}^3 \cup \infty \sim S^3$

$\vec{n} : S^3 \to S^2$

Knot - preimage of a point on S^2.

$$H_{ik} = (\partial_i \vec{n} \times \partial_k \vec{n}, \vec{n}) =$$

$$\varepsilon_{abc}\, \partial_i n^a\, \partial_k n^b\, n^c$$

Preimage of the volume form on S^2

$$H = \sin\theta \, d\theta \wedge d\varphi$$

Magnetic field, lines of force

Hopf number

$$H = dC \qquad Q = \frac{1}{4\pi^2} \int_{R^3} H \wedge C$$

Gauss intersection number

$$\mathcal{H} = \int \left(a^2 (\partial_k n)^2 + b^2 H_{ik}^2 \right) d^3x$$

second order

fourth order

$[a^2] = L^{-2}$, $[b^2]$ = dimensionless

$$\mathcal{H} > c\,|Q|^{3/4}$$

Proposed in mid 70-ties

In the end of 90-ties revived

due to Antti Niemi

Hietarinta - Salo

Battye - Sutcliffe

$Q = 1$ unknot

$Q = 4$ link

$Q = 7$ trefoil

Structure happened to be

rather universal!

LGGP $\qquad D = 3$

ψ_1, ψ_2, A: $\quad \nabla_\mu \psi = \partial_\mu \psi + i A_\mu \psi$

$\delta \psi = i \lambda \psi \quad , \quad \delta A_\mu = - \partial_\mu \lambda$

$$\mathcal{H} = |\nabla\psi_1|^2 + |\nabla\psi_2|^2 + F_{ik}^2 + V(|\psi|^2)$$

$$F_{ik} = \partial_i A_k - \partial_k A_i$$

$$\rho^2 = |\psi_1|^2 + |\psi_2|^2$$

$$\vec{n} = \frac{1}{\rho^2}(\bar\psi_1, \bar\psi_2)\,\vec{\tau}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$C_i = A_i + \frac{1}{\rho^2} J_i \qquad J_i = i\,\vec{\psi}_a \partial_i \psi_a - cc$$

Gauge invariant variables

$$4 + 3 \to 1 + 2 + 3$$

Phase disappears

$$\mathcal{H} = \rho^2 (\partial n)^2 + (\partial_i C_k - \partial_k C_i + H_{ik})^2$$

$$+ (\partial\rho)^2 + \rho^2 C_i^2 + V(\rho^2)$$

$$\langle \rho^2 \rangle = \Lambda^2$$

Meisner effect

Knot - like excitations !

Now to YM , $G = SU(2)$

$$A_\mu^a \quad , \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \varepsilon^{abc} A_\mu^b A_\nu^c$$

$$\mathcal{L}_{YM} = \frac{1}{4} F_{\mu\nu}^2$$

MAG

$$A_\mu^3 = A_\mu \ ; \qquad A_\mu^1 + i A_\mu^2 = B_\mu$$

$$\nabla_\mu = \partial_\mu + i A_\mu$$

$$\nabla_\mu B_\mu = 0$$

$$\mathcal{L}_{YM} \to \mathcal{L}_{YM} + \frac{1}{2} |\nabla_\mu B_\mu|^2 = \mathcal{L}_{MAG}$$

$$\mathcal{L}_{MAG} = \frac{1}{2}|\nabla_\mu B_\nu|^2 +$$

$$+ \frac{1}{4}\Big(\underbrace{\partial_\mu A_\nu - \partial_\nu A_\mu}_{F_{\mu\nu}} + \frac{1}{2i}\left(\bar{B}_\mu B_\nu - \bar{B}_\nu B_\mu\right)\Big)^2$$

$$+ \frac{1}{2i} F_{\mu\nu}\left(\bar{B}_\mu B_\nu - \bar{B}_\nu B_\mu\right)$$

Main Ansatz

$$B_\mu = \psi_1 e_\mu + \psi_2 \bar{e}_\mu$$

$$e_\mu^2 = 0 \qquad \bar{e}_\mu e_\mu = 1$$

$$\delta_E B_\mu = i\lambda B_\mu, \qquad \delta_E A_\mu = -\partial_\mu \lambda$$

$$\delta_M \psi_1 = -i\omega\psi_1, \qquad \delta_M \psi_2 = i\omega\psi_2$$

$$\delta_M e_\mu = i\omega e_\mu$$

$$\gamma = g - 1$$

$$\Gamma_\mu = i(\bar{e}_\nu \partial_\mu e_\nu)$$

$$\delta_M \Gamma_\mu = \partial_\mu \omega$$

$$C_\mu = A_\mu + \frac{1}{\rho^2} J_\mu$$

$$\psi_1, \psi_2 \to \rho^2, \vec{n}$$

$\rho^2 = \bar{B}_\mu B_\mu$

$$\bar{B}_\mu B_\nu - \bar{B}_\nu B_\mu = \rho^2 n_3 (\bar{e}_\mu e_\nu - \bar{e}_\nu e_\mu)$$

Grassmann $(4,2)$

$$\rho, \vec{n}, e_\mu, C_\mu$$

$$1 + 2 + 4 = 5 - 1 + 4 = 11 = 12 - 1$$

$$G \sim S^2 \times S^2$$

$$\mathcal{L}_{MAG} = (\partial\rho)^2 +$$

$$(\partial_\mu C_\nu - \partial_\nu C_\mu + H_{\mu\nu} + K_{\mu\nu})^2$$

$$+ \rho^2 C_\mu^2 + \rho^2\left((\partial n)^2 + (D_\mu e_\nu)^2\right)$$

$$+ \rho^2 (n_+ Q_- + n_- Q_+)$$

$$+ (1 - n_3^2)\rho^4 - \rho^4$$

$$\langle \rho^2 \rangle = \Lambda^2$$

Condensate of dim 2.

Connection with dimensional transmutation?

Conformal interpretation