On Spacetime Entanglement

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Black Hole Entropy

- Bekenstein and Hawking: event horizons have entropy!

thermodynamics   relativity   geometry

$$S_{BH} = \frac{k_B\, c^3}{\hbar}\, \frac{\mathcal{A}}{4G}$$

quantum gravity

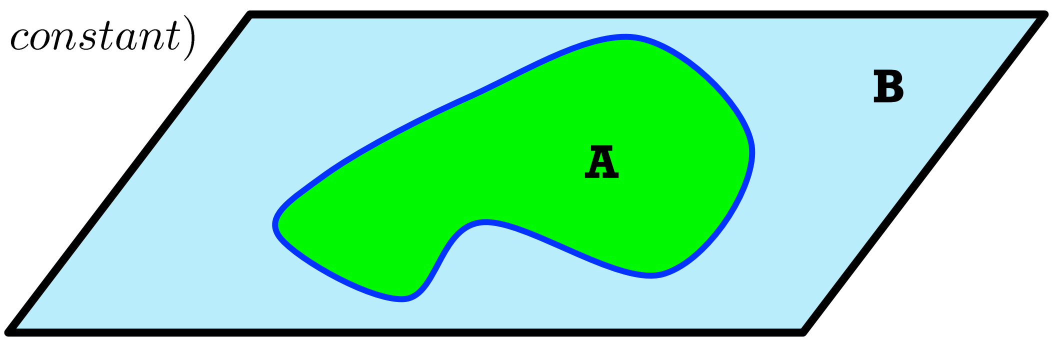
- extends to de Sitter horizons and Rindler horizons
- window into quantum gravity?!?
- different from thermal entropy:
  scales as Area rather than volume!

Entanglement Entropy

- general tool; divide a quantum system into two parts and use entropy as measure of correlations between subsystems
- in QFT, typically introduce a (smooth) boundary or entangling surface Σ which divides the space into two separate regions
- trace over degrees of freedom in "outside" region
- remaining dof are described by a density matrix ρ_A

⟶ calculate von Neumann entropy: $S_{EE} = -Tr\left[\rho_A \log \rho_A\right]$

$(t = constant)$



- Sorkin et. al showed that for the event horizon the leading term leads the area law.

Conjecture :

In a theory of quantum gravity, for **any** sufficiently large region with a smooth boundary in a smooth background there is an entanglement entropy with leading term as area law:

- $$S_{\mathrm{EE}} = \frac{\mathcal{A}_{\Sigma}}{4G_N} + \cdots$$

Bianchi & Myers, arXiv:1212.5183

Our goal is to study this conjecture using RS2 model. We are also interested to find the corrections to above.

How to construct RS2?

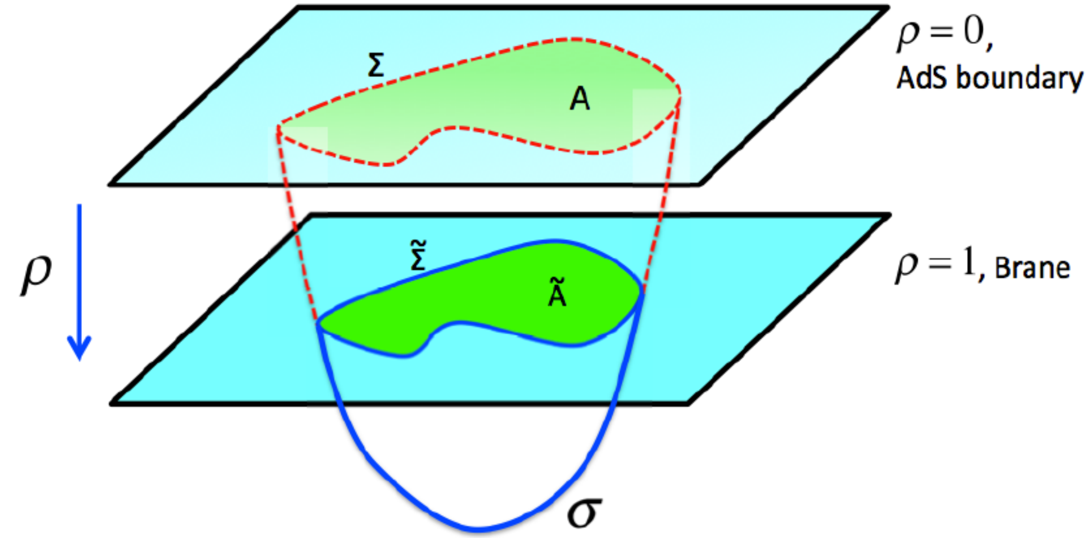
- Take two copies of 5D AdS spacetime and glue them together along a cut-off surface at some large radius and insert a 3-brane at this junction.

- The standard 4D gravity will arise at long distances on the 3-brane as induced gravity.

- Due to AdS/CFT duality, we have two copies of strongly coupled CFT with a UV cut-off $\delta = \tilde{L}$ coupled to induced gravity and any other matter on the 3-dimensional brane.

- This easily extends to an arbitrary dimensions to produce gravity on d-brane located at $\rho = \rho_c$.

AdS/CFT & RS2



The key difference between the standard AdS/CFT and RS2 model is that the bulk geometry is cut off at some fin: $\rho = \rho_c = 1$.

Therefore, in order the metric expansions converge effectively we require both the background geometry on the brane and entangling surface are smooth (weakly curved) on the scale of AdS curvature:

$$\delta^2 \, R^{ij}{}_{kl}[\overset{(0)}{g}] \ll 1$$

$$\delta \, K^i_{ab} \ll 1$$

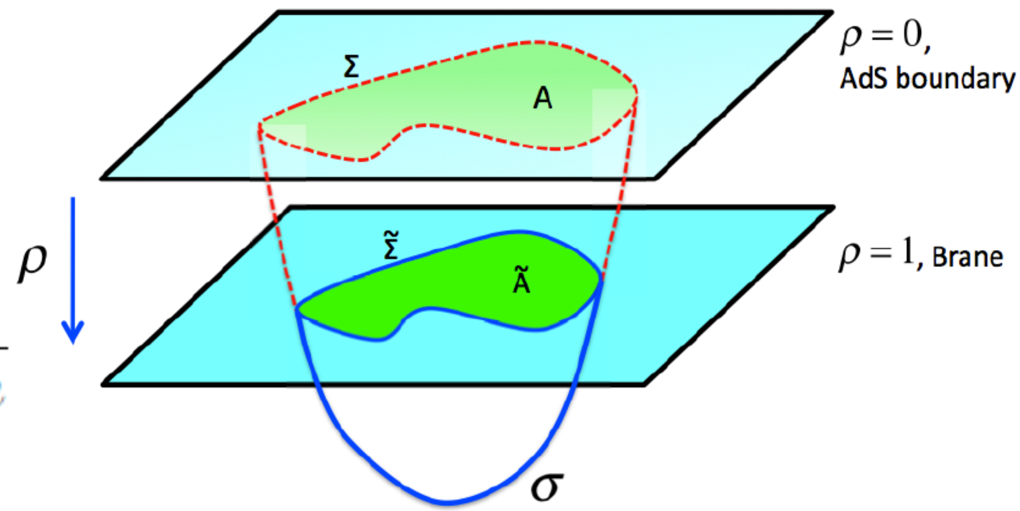
Holographic EE

To evaluate EE associated with general entangling surfaces on the d-brane we use holographic prescription by RT however in the context of RS2 model

Ryu-Takayanagi prescription:

$$S(V) = \underset{\sigma\sim A}{\text{ext}} \frac{\mathcal{A}(\sigma)}{4G_{d+1}}$$

$$\mathcal{A}(\sigma) = 2\int d^{d-2}y\, d\rho\, \sqrt{h}$$



$$S_{\rm JM} = \frac{1}{2G_{d+1}} \int_\sigma d^{d-2}y\, d\rho \sqrt{h} \left[1 + \frac{2\,L^2\,\lambda}{(d-2)(d-3)}\,\mathcal{R}\right] + \frac{2\,L^2\,\lambda}{(d-2)(d-3)G_{d+1}} \int_{\tilde{\Sigma}} \mathcal{K}$$

Bulk and Boundary action

$$I^{\rm GB}_{bulk} = \frac{1}{16\pi G_{d+1}} \int d^{d+1}x \sqrt{-G} \left[\frac{d(d-1)}{L^2} + \mathcal{R} + \frac{L^2 \lambda}{(d-2)(d-3)} \chi_4 \right] + I^{\rm GB}_{surf}$$

$$\chi_4 = \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma} - 4\,\mathcal{R}_{\mu\nu}\mathcal{R}^{\rho\sigma} + \mathcal{R}^2$$

Integrating out the extra dimension we get the induced action on the brane

$$I^{\rm GB}_{ind} = \int d^dx \sqrt{-\tilde{g}} \left[\frac{R}{16\pi G_d} + \frac{\kappa_1}{2\pi} \left(R_{ij}R^{ij} - \frac{d}{4(d-1)} R^2 \right) + \frac{\kappa_2}{2\pi} C_{ijkl}C^{ijkl} + \mathcal{O}(\partial^6) \right]$$

where effective Newton constant on the brane is

$$\frac{1}{G_d} = \frac{2\,\delta}{d-2} \frac{1 + 2\lambda f_\infty}{G_{d+1}}$$

$$\kappa_1 = \frac{\delta^3}{4(d-2)^2(d-4)} \frac{1 - 6\lambda f_\infty}{G_{d+1}} \qquad \kappa_2 = \frac{\delta^3}{4(d-2)(d-3)(d-4)} \frac{\lambda f_\infty}{G_{d+1}}$$

EE of a general region

Applying derivative expansion to the intrinsic and extrinsic curvature in JM entropy while assuming smooth geometry for brane and entangling surface, and integrating out extra direction

$$S_{\rm EE} = \frac{\mathcal{A}(\tilde{\Sigma})}{4G_d} + \kappa_1 \int_{\tilde{\Sigma}} d^{d-2}y\sqrt{\tilde{h}}\left[2R^{ij}\,\tilde{g}^{\perp}_{ij} - \frac{d}{d-1}\,R - K^i K_i\right] + 4\kappa_2 \int_{\tilde{\Sigma}} d^{d-2}y\sqrt{\tilde{h}}\left[\tilde{h}^{ac}\tilde{h}^{bd}C_{abcd} - K^i_{ab}K_i{}^{ab} + \frac{1}{d-2}K^iK_i\right] + \mathcal{O}(\partial^4)$$

Compare to:

$$S_{\rm Wald} = \frac{\mathcal{A}(\tilde{\Sigma})}{4G_d} + \kappa_1 \int_{\tilde{\Sigma}} d^{d-2}y\sqrt{\tilde{h}}\left[2R^{ij}\,\tilde{g}^{\perp}_{ij} - \frac{d}{d-1}\,R\right] + 4\kappa_2 \int_{\tilde{\Sigma}} d^{d-2}y\sqrt{\tilde{h}}\tilde{h}^{ac}\tilde{h}^{bd}C_{abcd} + \mathcal{O}(\partial^4)\,.$$

Conclusion/Further work

- ✓ We showed that the area law conjecture by Bianchi & Myers is realized in RS2 model assuming the smooth geometry for the brane and entangling surface.

- ✓ We also calculated the first leading corrections to the area law for the entanglement entropy of gravity theories including the quadratic curvatures with two independent coefficients.

- ✓ The entanglement entropy of a general surface coincide with Wald entropy if this surface is a Killing horizon.

- ✓ We are trying to generalize the results to include the most general quadratic term, i.e. 3 independent coefficients and ambitiously to higher orders in curvature. Also we would like to check the conjecture in more general cases.